

Contagion from market price impact: a price-at-risk perspective

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Abstract

Overlapping portfolios constitute a well-recognised source of risk, providing a channel for financial contagion induced by the market price impact of asset deleveraging. We introduce a novel method to assess the market price impact on a security-by-security basis from historical daily traded volumes and price returns. Systemic risk within the euro area financial system of banks and investment funds is then assessed by considering contagion between individual institutions' portfolios holdings under a severe stress scenario. As a result, we show how the bias of more homogeneous estimation techniques, commonly employed for market impact, might lead to loss estimates that are more than twice as large as losses estimated with heterogeneous price impact parameters. Another new feature in this work is the application of a price-at-risk measure instead of the average market price impact to evaluate the tail risk of possible market price movements in scenarios of different severity. Our results also show that system-level losses at the tail can be three times higher than average losses using the same scenario.

Keywords: Price impact, fire sales, indirect contagion, overlapping portfolios, quantile regression.

JEL Codes: G01, G12, G17, G23, G32.

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Non-technical summary

Financial institutions investing in common assets are exposed to so-called *indirect contagion risk*, where the marked-to-market loss associated with a decline in value of overlapping portfolios can become a major potential thread to the stability of the financial system. This mechanism exposes banks and other financial intermediaries to market risks even vis-a-vis institutions with whom they have no direct exposure or explicit financial relationship, and constitutes an important channel through which contagion and financial distress can traverse the system. This could start either from hefty deleveraging of one or more distressed financial institutions, seeking to raise liquidity in short windows of time, or outright fire-sales due to sudden asset repricing, and end up hitting even the healthiest and most prudent market participants.

The corresponding systemic risk is therefore a function of two major factors. On the one hand, the amount of overlapping portfolios across the financial system – that is, to which extent financial intermediaries invest in the same assets or asset classes – which increases the interconnectedness of the system across geographical borders, magnifying the complexity of the system and the interdependence of its agents. On the other hand, the price impact associated with the sale of large portfolio fractions, which may either dampen or exacerbate indirect risk sharing depending on the asset classes which constitute the overlaps in investments.

This work exploits historical information on daily traded volumes and prices of thousands of securities held by major participants in the euro area financial markets, in order to estimate price impacts at the level of individual ISINs. This is done by extending recent theoretical findings on *non-linear price impacts* and employing a *non-linear quantile regression* approach. We find bonds are generally associated with lower price impacts, which broadly contributes to a reduction in indirect risk sharing when considered in conjunction with the fact that a large fraction of euro area banks portfolios are made up of government bonds. Nonetheless, price impact weighted overlapping portfolios still show heterogeneous system-wide correlations, and high degrees of indirect risk sharing, albeit lower than what could naively be inferred by looking at nominal portfolios only. Moreover, estimated price impacts are employed in a model of financial contagion with both direct and indirect channels, which has been used before to perform system-wide stress testing. In this context, the full value of a quantile regression approach to estimate price impacts can be exploited by repeating the impact assessment for different quantiles of the price impact distribution. Our results provide numerical evidence of the relevance of tail scenarios in modelling fire sales dynamics where the impact of extreme deleveraging pressures, such as those which the financial system may experience in times of system-wide distress, is assessed across a range of shocks to the system and price impacts.

1 Introduction

In the past years, a growing amount of literature has been documenting the importance of common holdings of assets in spreading market stress via a mechanism called financial contagion: financial institutions may be forced to sell assets to acquire liquidity amidst distressed financial situations. The selling of some securities implies price declines which causes losses to other institutions holding those specific securities mark-to-market in their trading book. This mechanism may lead to a spiral of price declines and further liquidity needs, similarly to Brunnermeier and Pedersen (2009) and Greenwood, Landier, and Thesmar (2015).

The severity of the decline depends on the complexity of portfolio similarities, creating deeply connected overlapping portfolios. In the post-financial crisis era, these overlapping portfolios are recognised as a major potential source of systemic risk. This risk, originating from mechanisms of joint asset deleveraging, depends on a notion of market price impact, which must consequently play a central role in the modelling of the *system-wide* dynamics of the financial system. This provides a compelling motivation for an in-depth understanding of the mechanisms of market price impact, and for deploying analytical models able to capture this source of contagion.

This paper touches upon two strands of the literature. The first is the price impact literature, defined as the change in price as a result of individual trades. One of the earlier research on price impact was performed by Kyle (1985), in which price dynamics are modelled following a linear equilibrium model. In this setup, three market participants influence the price based on asymmetric information. All else being equal, the linear price change is a function of size. The framework introduced by Kyle is a common starting point price impact models developed at a later stage.

Further research regarding the dependence between volume and price changes consistently shows decreasing impact severity per traded increment, resulting in a concave impact (see Almgren et al. (2005), and Tóth et al. (2011)). Other research regarding individual or independent traded volumes (see Lillo, Farmer, and Mantegna (2003), Potters and Bouchaud (2003), Gabaix et al. (2003), Farmer, Patelli, and Zovko (2005)) also concluded concave dependencies between traded volumes and subsequent price changes. Huberman and Stanzl (2004) specify that a linear price change could be described as a linear function of size, provided a constant liquidity and permanent impact, to prevent arbitrage. Moreover, they discover also that in practice market liquidity fluctuates by orders of magnitude, significantly affecting price change dynamics.

Bouchaud, Farmer, and Lillo (2009) study the adaptation of new information into the price of a security and find a slow process causing prices to be affected long after the information became public. Patzelt and Bouchaud (2018) extend this research by analyzing the limit order book and the aggregated price fluctuations observed therein. They find a relatively stable concave impact curve across all intra-day time scales.

The exact shape of the impact curve is further debated in several publications. For example, Bouchaud, Farmer, and Lillo (2009) find strong evidence in favor of a square-

root impact function from segmented meta-orders. While in Cont and Wagalath (2016) the authors develop an econometric framework for the forensic analysis of fire sales episodes, which is characterized by an exponential price impact form. This same exponential function has been implemented in several publications on spillover effects and contagion due to portfolio deleveraging and fire-sales (Cont and Schaanning, 2017; Caccioli, Ferrara, and Ramadiah, 2020; Caccioli, Ferrara, and Ramadiah, 2020)). However, to our knowledge, price impact, by means of capturing quantiles of the entire distribution in combination with an exponential impact function to model contagion from large scale portfolio deleveraging, has not been introduced yet.

The literature regarding contagion through marketable assets is the second subject that this paper investigates. In an important publication by Shleifer and Vishny (2011) the authors show heavily discounted prices of assets that needed to be liquidated in some form of fire-sale. Similar work for the banking sector is done by Khandani and Lo (2008), Cont and Wagalath (2016), and Glasserman and Young (2016), where financial institutions experience forced deleveraging as a results of a financial shock. Also, Cont and Schaanning (2019) analyse the effect of forced portfolio deleveraging on the vulnerability of the European banking system to indirect contagion.

Chaderina, Mürmann, and Scheuch (2018) argue that institutional investors tend to sell liquid assets first causing trades to be focused on a specific selection of assets, which might lead to a more pronounced price drop for these liquid assets than for illiquid ones. Similar to studies by Greenwood, Landier, and Thesmar (2015), the authors assume a linear relation between sold assets and the subsequent price change. However, there is abundant empirical evidence of a sub-linear price impact dependence on traded volumes (see Bouchaud (2010) and Bershova and Rakhlin (2013)).

In contrast to these more traditional methods of point-in-time estimations, Adrian and Brunnermeier (2011), Engle and Manganelli (2004) and the research by Adrian, Boyarchenko, and Giannone (2019) implement quantile regression based value-at-risk analyses. The reason for implementing a quantile regression, rather than a traditional single point forecasting method, is that a quantile regression allows for a distribution of predictions, providing a prediction for the events at the tail of the distribution and not just a median or mean prediction.

The aim of our research is to incorporate the theoretical findings of non-linear price impact relations in combination with a quantile regression approach in order to better explain tail events in the propagation of losses due to contagion from market price impacts. To our knowledge, we are the first in making an attempt to estimate security-level price impact functions from individual historical prices using a non-linear quantile regression. Using these parameters and granular securities holdings statistics, we are able to estimate potential contagion losses in the event of system-wide or individual liquidity needs in the financial system.

Moreover, our work is also related to work on tracing the impact of asset purchase programmes on asset prices. Altavilla, Carboni, and Motto (2015), Eser et al. (2019), Rostagno et al. (2021) have, for instance, investigated the impact of asset purchase

programmes and unconventional monetary policies on financial markets. These studies distinguish between "flow effects" and "stock effects", where the first one is covering the price effect from actual trading, the second one is capturing the impact from investors' beliefs at the announcement of changes in the policy stance. The framework in this paper, using traded volumes, is, therefore closer, to the estimation of flow effects under tail events.

The rest of the paper is structured as follows. In section 2, we review existing price impact estimation methodologies and motivate our own approach using quantile regressions. Section 3 describes the dataset used in this paper. Section 4 shows results of price impact parameter estimations at aggregate level and price impact function behaviour for selected trade volumes. In section 5, we use these parameters in a fire sale contagion model to estimate endogenous market losses in a financial system of banks and investment funds. In this exercise, we assume a constant exogenous redemption shock for the whole investment fund sector leading to fire sales of financial assets in the system, which in turn causes contagion effects due to overlapping portfolios.

2 Methodology

Before delving into the detailed dynamics of the spread of financial contagion via overlapping portfolios, one should have a good understanding of the individual components related to this topic. This begins with the explanation of the general features of a *financial market*, also known as *exchange*. Strictly speaking a financial market is a place where supply of and demand for *financial products* come together. The financial products usually refer to products that provide monetary finance to the supplier of a product. For example a non-financial corporation is looking to expand their business for which it needs some form of financing. Broadly speaking this corporation can turn to a financial market to sell part of the company in exchange for funding (i.e. equity funding) or acquire a loan type product such as a bond. There exist more complex financial products that base their price on the value of a different financial product, these are called derivatives. For the scope of this work, only bond and equity instruments are considered.

Originally, financial markets were physical markets where market participants came together to trade their products. Thanks to technological developments, the modern financial markets are electronic systems, mainly handled by computational algorithms. These markets also developed a new type of trade, referred to as *continuous-time double-auction* mechanism. A double auction market allows both buyers and sellers to submit the number of financial products they desire to trade (i.e. volume) and the preferred price against which they are willing to trade the products. This type of submission of an order is called a *limit order* and is stored in the exchange's *limit order book (LOB)*. A participant placing a limit order is also referred to a liquidity provider (or maker), since they add liquidity to the market. The limit order can either be cancelled from the order book (i.e. removed without it being traded) or accepted by another market

participant, which results in a trade. The market participant, who desires to accept an outstanding limit order, submits a market order. A participant placing a market order is also referred to a liquidity taker (taker for short), since they remove liquidity from the market. At each moment in time, the highest price in the LOB offered for the purchase of a financial product is the best bid price. Conversely, the best ask price refers to the lowest price a product can be bought for. As mentioned above, a limit order contains both a price and a volume (in number of securities), while a market order only specifies a volume to be bought or sold, the price is determined by the best available price at that moment. In case the volume of a market order is larger than the available volume against current best price, the access volume is executed against the next best price.

The above mentioned case shows that market orders might eat into limit order volume, when market orders come in faster than newly submitted limit orders or a substantially large market order has been executed, the best available price in the LOB gets affected, i.e. trades consume liquidity and impact prices. Consequently, buy trades push prices up while sell trades drag prices down (see Bouchaud et al. (2018)). The magnitude of the impact is, therefore, related to the volume size of the traded order. Hence, a market participant has the incentive to carefully select the execution size, in order to minimize execution costs, as a consequence of her trade. A simple solution is to split-up large volumes into smaller sized orders referred to as *metaorders*. Much of the development of price impact functions are build around the execution of several metaorders over a given period of time. The following section explains the methodology behind some common theoretical models that attempt to explain the relation between these metaorders and the subsequent price change.

2.1 Price impact modelling

The foundation of most price impact models is derived from the linear price impact specification described within Kyle’s framework (see Kyle (1985)). In the setup of this model, there are three market participants, the informed traders, uninformed trades, and the market-maker. The informed traders have some knowledge of the fundamental price of a security and try to exploit this information by trading a certain quantity of this security accordingly. The uninformed traders behave like a random variable and do not affect the price. The market-makers adjust their inventory to match the incoming market orders. However, they do not know the fundamental price. Since the informed traders do know the fundamental price, their objective is to maximize profits by taking advantage of mispriced securities. Kyle (1985) assumes that the market-makers fix the price given a linear model of overall trade volumes:

$$p = p_0 + \lambda v. \tag{1}$$

In this context the impact coefficient, denoted by λ (known as Kyle’s lambda) is a parameter inversely proportional to the market depth; the impact on price is then written as:

$$\Psi = \lambda v. \quad (2)$$

The market-maker chooses λ to get $E[p^*|v] = p$, where p^* is the fundamental price. The informed traders are aware of the price fixing by the market-maker. Hence, they choose a trade volume small enough not to disturb the order flow from uninformed traders. This relation is known to provide a reliable description of the impact for small volumes but will tend to overestimate impacts for larger volumes.

The tool presented here draws from the same underlying idea of volume dependent price change; however, it imposes a more general specification that holds for a larger volume domain. There is abundant empirical evidence (see e.g. Bouchaud (2010)) of a sublinear price impact dependence on the volume. Potters and Bouchaud (2003) study the price impact as a response to the execution of a sequence of metaorders. They find a square-root relation between the execution size and the subsequent price change, which is presented as

$$\Psi(Q) = Y \sigma_d \left(\frac{Q}{V_d} \right)^\delta, \quad (3)$$

where Q is the total volume of the *metaorder*, Y is a numerical factor of order one, σ_d describes the daily volatility, V_d denotes the daily traded volume, and δ is the parameter that describes the concave nature of the price impact, which has been found to be $\delta \approx 0.5 < 1$, resulting in a square root impact function. This representation would provide the necessary concave shape but it requires knowledge about the sequence of metaorders Q from individual investors. The square-root impact does not apply to aggregated orders (see Bouchaud et al. (2018)). Since metaorders are unobserved in the analysis of this paper, the square-root impact function is not suitable. The square root nature of the impact function implies the possibility of arbitrarily large shocks that are able to push the price below zero. For that reason an exponential specification is used (see e.g. 4) in line with previous work (Schnabel and Shin (2002), Cifuentes, Ferrucci, and Shin (2005) and Cont and Schaanning (2017)):

$$\Psi_\phi(V) = B_\phi(1 - e^{-V\lambda_\phi/B_\phi}), \quad (4)$$

where λ_ϕ is the price impact coefficient, B_ϕ the corresponding impact boundary for a given security ϕ , and V is the daily traded volume. This allows for a price impact smoothly converging to the boundary B_ϕ . Moreover, notice that (4) is asymptotic to Kyle's model in the limit of small traded volumes¹.

2.2 Model assumptions

To motivate further the impact behaviour of trades on the price change, the following assumptions are made:

¹That is $\Psi_\phi(V) = \lambda V + \mathcal{O}(V^2)$.

- Conditional on one extra sell (buy) trade the price of a security, on average, will move down (up) when everything else is kept constant;
- The price impact function is both sublinear in the traded volume and permanent in time.

See Bouchaud (2010) for a more in-depth discussion. During any given trading day, trades can generally occur in both buy and sell directions. The volumes corresponding to these trades are referred to as the buy or sell initiated volume. Let us now assume that both the daily sell initiated volume v_s and the daily buy initiated volume v_b are identically distributed $v_s, v_b \sim f$. For symmetric price impacts, one can simply write the daily observed price change as

$$R(v_s, v_b) = \Psi(v_b) - \Psi(v_s), \quad (5)$$

where v_i for $i \in (b, s)$ are the signed traded volumes and $R(\cdot, \cdot)$ is the daily price change in percent.

Having access to daily unsigned traded volumes only, one would, however, be able to observe $V = v_s + v_b$. Thus, a lack of knowledge on the individual daily v_s and v_b hinders the possibility to infer the functional form of the price impact Ψ . However, one can attempt to obtain an estimate for it. Trading days, in which the trade imbalance is large, provide the best estimate for the price impact within this framework. In particular, it is straightforward to realise that it is when either $v_s \gg v_b$ or $v_s \ll v_b$ that the observed impact is closest to Ψ . Therefore, an estimate for Ψ can be found in the boundary of the set of data points in the *volumes-impacts* plane.

It is also worth mentioning that relaxing the assumption of symmetry is also possible, conditional on having sufficient data. In other words, there exists a trade-off between the assumption of symmetry and the accuracy on the estimate of Ψ .

Figure 1 provides an example from simulated data points. Assuming ex-ante the functional form of Ψ allows to simulate points in the *volumes-impacts* plane according to (5). The set of points can be observed to be bound by Ψ , and the null impacts can be traced back directly to (i.e. explained by) the realisation of equal buy and sell initiated volumes during the simulated trading day. Notice, however, that the probability of having sharp trade imbalances decreases quickly with increasing volumes. The estimation of Ψ through the bounding curve described above is thus accurate only for (relatively) small volumes.

2.3 Intra-day estimation methods

The price impact models in the report are motivated by theoretical research on financial market behaviour, contagion modeling, and financial stress testing. ...

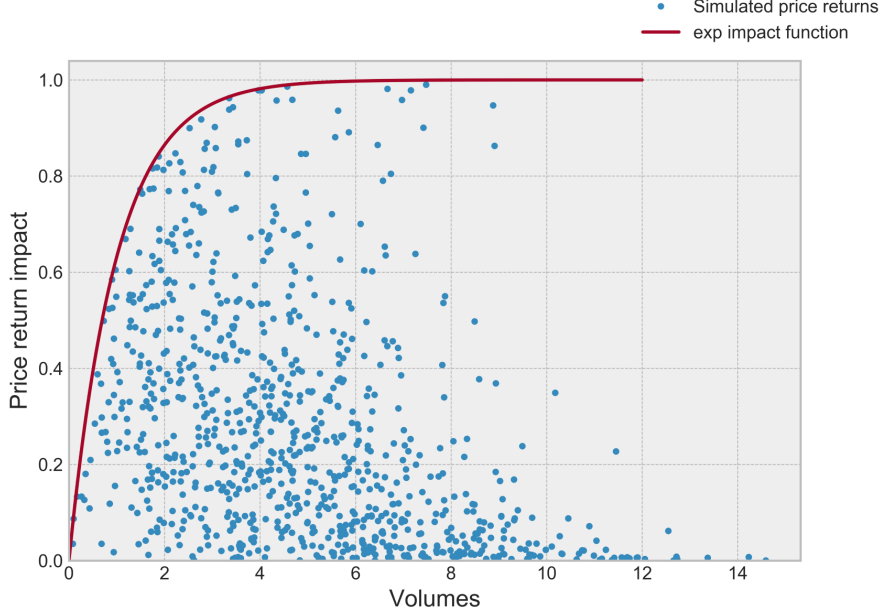


Figure 1: Price impact from simulated volumes: volumes represent sell initiated traded volumes, while returns represent the negative impact on the price return (blue dots), price impact curve (red line).

2.3.1 Linear relation model

The simplest approach to the calibration of a security’s price impact parameter is by estimating a linear OLS over the set of price returns and daily traded volumes. The estimated parameter can be referred to as average impact and is denoted by λ^{Avg} .

$$\Psi_{\phi}(V) = \lambda_{\phi}V, \quad (6)$$

where V are traded volumes in one direction. Closely related to the notion of a price impact, is the market liquidity of a security. Liquidity is not directly observable, however, it can be defined as the ease with which market participants are able to obtain funding from the sell of securities or investment opportunities from the purchase of securities (Brunnermeier and Pedersen, 2009). Conversely, the lack of liquidity, measures the discount (buy trades), or premium (sell trades) associated with the price of a security when a market order is executed (Glosten and Milgrom, 1985). A widespread indicator of illiquidity is the *Amihud illiquidity measure* (Amihud, 2002), capturing the average daily price response to traded amounts:

$$ILL_t = \frac{1}{N} \sum_{t=1}^N \frac{|R_t|}{V_t}, \quad (7)$$

where R_t and V_t are the daily return and volume respectively at day t and N is the number of trading days in a year with non-zero level of trading volume.

2.3.2 Convex hull price impact

For the case of the *Convex Hull*² (CH), given the aforementioned assumptions, the price impact is estimated via the curve bounding the returns in the *volumes-impacts* plane. More specifically, we approach this problem by first computing the CH on the historical set of data points, and then fitting the desired functional form to the first points on the CH (where the estimation is more accurate - see previous section). The calibrated parameter by means of the CH is referred to as the hull lambda or λ^{CH} .

Figure 2 provides an illustration. The CH provides a good estimate for the price impact function (here again chosen ex-ante), especially for small volumes. For comparison, the estimated linear impact is reported as well.

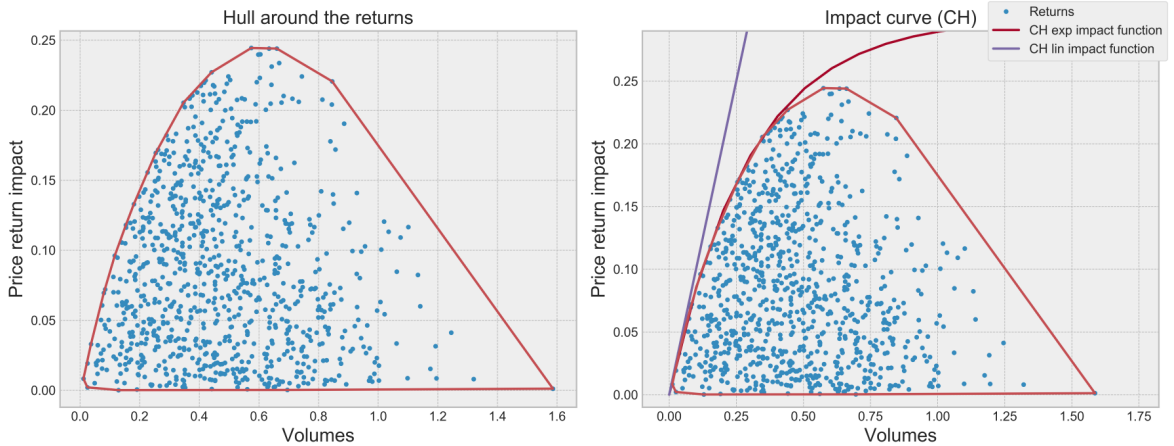


Figure 2: Price impacts from simulated unsigned daily volumes (arbitrary units). The price impact curve is chosen ex-ante and is represented in red. The orange polygon is the Convex Hull.

2.4 Quantile regression with systematic component

A pitfall of the previously described methods is that either they select the most extreme outcomes on the volume-price plane, making them sensitive to outliers, or a linear relationship between prices and volumes is not observed in our database of daily price changes. Furthermore, a key missing component of these methods is the correlation of prices. We solve this problem by including the system-level (market) return as additional factor to explain security-level price changes defined for different security buckets. Our approach is similar to the CoVaR methodology of Adrian and Brunnermeier (2011)³ with the exception that we estimate the q th quantile of the security-level price impact as a function of volumes sold and the system-level return:

$$\hat{R}_{\phi,t}^q = \hat{\beta}_0^q (1 - \exp(-s_\phi V_{\phi,t})) + \hat{\beta}_1^q R_{\text{sys},t}, \quad (8)$$

²The convex hull is the smallest convex polygon bounding a set of points (see Appendix for more details).

³See section 3.1 of the cited paper.

where $s_\phi V_{\phi,t}$ is a scaled traded volume of security ϕ at time t and $R_{\text{sys},t}$ is the system return defined as the weighted average of returns country/sector and country/sector/residual maturity buckets. For background information on quantile regressions, see appendix A.1. Volumes are scaled such that for one single security, all historical volumes range between 0 and 1: $s_\phi \cdot \max_t V_{\phi,t} = 1$. Then, the volume-dependent term is $\hat{\beta}_0^q (1 - \exp(-s_\phi V_{\phi,t}))$ in which s_ϕ is fixed and obtained from the previous equation and $\hat{\beta}_0^q$ is estimated using the quantile regression. Since we know from equation (4) that $\lambda_\phi/B_\phi = s_\phi$, we easily derive $\lambda_\phi = s_\phi \cdot \hat{\beta}_0^q$, which is also reported in our result tables and figures.

This specification makes it possible to isolate the role of individual trade volumes from market sentiment and hence correlation of prices. This setup also encompasses equation (4) and β_0^q can also be interpreted as a bound on the price change from initiated trade volumes. A further useful feature of this method is that we can treat the system return as a variable from a macro-financial scenario, therefore our endogenous price impacts are independent from the scenario and depend only on the quantile chosen ex-ante.

2.5 Aggregation level of estimations

The level of aggregation is particularly important in price impact estimations. Our estimations are based on security-level price changes and, thus, cannot be applied to less granular portfolios. The most obvious reason for this is the different capitalization and outstanding volumes.

Assume we have two securities A and B with market capitalization c_A , c_B and corresponding price impact functions Ψ_A and Ψ_B . The question is how to derive the price impact function Ψ_{A+B} of the aggregate security. If an amount x is sold from the aggregate security, we can assume that $\frac{c_A}{c_A+c_B}x$ and $\frac{c_B}{c_A+c_B}x$ are sold from the individual securities, respectively. This leads to individual price impacts $\Psi_A\left(\frac{c_A}{c_A+c_B}x\right)$ and $\Psi_B\left(\frac{c_B}{c_A+c_B}x\right)$. Thus, the average price impact, weighted by capitalization, becomes

$$\Psi_{A+B}(x) = \frac{c_A \Psi_A\left(\frac{c_A}{c_A+c_B}x\right) + c_B \Psi_B\left(\frac{c_B}{c_B+c_B}x\right)}{c_A + c_B}. \quad (9)$$

This function is a weighted average of exponential functions and does not reduce to an exponential function. Hence, parameters of the aggregate price impact cannot be derived from the individual price impacts.

An alternative solution to the above problem is the construction of artificial securities. This means that given a price history of a number of securities $\{p_{\phi_i,t}\}_{i \in I}$, one can construct an aggregate price history for an index set I as

$$p_{I,t} = \sum_{i \in I} \frac{c_i p_{\phi_i,t}}{\sum_{i \in I} c_i}. \quad (10)$$

Having this aggregate price history, the price impact estimation method is identical.

Country (ISO code)	Number of bonds			Outstanding amount (EUR Bn)		
	FC	GOV	NFC	FC	GOV	NFC
AT	67	17	27	39	173	17
BE	32	34	28	24	247	20
DE	486	433	66	474	1381	48
ES	139	136	19	141	907	12
FR	318	133	200	357	1533	158
IT	166	143	70	106	1616	48
NL	318	24	39	280	276	25
Total	1526	920	449	1421	6132	327

Table 1: Cardinality and outstanding amounts of bonds by country and sector (2020Q4)

3 Data

The empirical analysis in this paper is focused on portfolio holdings of bonds and equities. The security’s characteristics, including the issuer information, are obtained from the ECB’s Centralised Securities Database (CSDB). All securities are identified by their International Securities Identification Number (ISIN). The information collected regarding the issuers are the country of residence, the institutional sector, and the security class. The institutional sector is in line with the 1995 European system of national and regional accounts (ESA95), for which three major groups are selected: *non-financial corporations (S.11)*, *financial corporations (S.12)*, and *general government (S.13)*. The securities are classified based on the ESA95 instrument classes, where the following two major groups are of interest to us: *debt securities AF.3*, and *Equity and investment fund shares/units AF.5*. Furthermore, for bonds the residual maturity, in the form of number of days, is included in the database.

3.1 Bond market data

Market data for bonds are obtained from the ECB internal data source Market Data Provision (MDP). This database includes daily bid and ask quote volumes from over 900 market specialists, as provided by Bloomberg with a recording period between 2018 up to the last quarter of 2020 at a daily frequency. A problem related to the analyses of traded bonds is that the transactions of such securities commonly take place *over-the-counter (OTC)*, which makes it more difficult to obtain prices and volumes. To mitigate this problem, multiple sources, reporting similar securities, are combined to create a more complete data set with a broader coverage. In order to ensure sufficient data quality, these different sources are combined using the highest correlation between the reported prices and volumes. Any misreporting or larger data gaps are reduced by optimizing the reported data from all sources.

Market cap.	Number of equities		Outstanding amount (EUR Bn)	
	FC	NFC	FC	NFC
1b	298	488	123	198
10b	145	330	465	1379
100b	33	110	971	3514
Total	476	928	1560	5090

Table 2: Cardinality and outstanding amounts of equities by capitalization and sector

3.2 Equity market data

Data on equity securities are obtained from the commercial data provider Refinitiv and the ECB Securities Holdings Statistics (SHS). The dataset from Refinitiv consist of bid and ask prices at the end of the day, daily open and close prices, and daily traded volumes. The data spans all trading days for the period 2018 to 2020.

The SHS dataset provides information on security-level holdings of euro area investors. This database includes debt securities (bonds) and equity securities.⁴ In addition, a classification of SHS data by sector (SHS-S), provides information on holdings of securities by euro area countries.

Investors and financial institutions, issuing or holding financial securities, are expected to hold records of securities at the market value. This market value is defined as the mid-price between the bid and ask price where the securities are quoted on markets with a buy-sell spread (also bid-ask spread) (BIS, ECB, and IMF, 2010).

The securities that are held by financial institutions such as banks and investment funds, have a combined observed value of EUR 19 trillion, corresponding to several millions of securities, to which these two sectors are exposed. The distribution of the securities, based on their observed value, follows a power law. This means that a small number of securities cover a large portion of the total observed amount. In order to optimize the trade-off between maximum coverage and best computational performance a sample of the largest 10,000 securities (by observed value) is selected, containing an aggregated market value of EUR 5.62 trillion. This accounts for 30% of the observed valuation covered in the SHS-S dataset.

4 Parameter estimation

In this section, statistical results from the quantile regressions are presented. The results have been grouped by security type (i.e. bonds and equities) and sector (i.e. government, financial, and non-financial). In addition the bonds were separated into

⁴For a more detailed description of the data set, see the Handbook on securities statistics(BIS, ECB, and IMF, 2010)

three maturity buckets short-, medium-, and long-term bonds, and the equities by market capitalization. The importance of grouping these securities together is to identify risk patterns in security portfolios, without the need for estimations for each individual security. Allowing to analyse market risk for larger, more interconnected portfolios.

The rest of this section is structured as follows: firstly, in section 4.1 calibration results are summarised by group and discussed in detail. Estimated parameters are further analysed by exposing the price impact function to different levels of direct shocks. The latter represents a potential direct market shock as a result of theoretical fire-sales. These shocks don't show indirect effect from contagion, which will be included in section 5. Secondly, in section 4.2 the implications of different liquidation horizons are discussed .

4.1 Empirical estimation

Figure 3 provides a visual representation of the estimated quantile regressions on empirical data for a single security ⁵. In the figure, the scatter points represent daily traded volume, denoted in hundred million euro, against daily returns. Each colored line is a price impact function corresponding to an estimated quantile q from the quantile regression, described as $\hat{R}_{\phi,t}^q$ in equation (8).

The unique advantage of estimating several levels of severity by means of the quantile regression approach, in comparison with the convex hull calibration approach in the simulated example can be seen in Figure 1. With the estimated quantile approaching the median, the presence of small impacts relating to larger volumes can be traced back directly to (i.e. explained by) the realisation of equal volumes in both buy and sell directions during a trading day. This also explains the non-symmetrical behaviour around the median, where small imbalance in buy and sell pressure can results in unpredictable returns. In further use of the calibrated parameters this research will focus on the quantile below 0.5. Lastly, as shown in (8), an important factor in the calibration is the inclusion of a market risk component. The correlation between other securities in the sample influence the observed returns up or down.

As each individual security might be exposed to different risks, it is important to analyse results at a higher macro-level. At the same time, this will provide a picture of the robustness of the calibration. In the following subsection the statistical results are discussed at a macro-level grouped by relevant categories. For each category the median and confidence bounds are plotted of the β_i for $i \in [0, 1]$, is the , standard error (SE), and the pseudo R^2 are provided for each calibrated quantile.

⁵Note that positive returns are excluded from the figure, since the focus of this paper is on the trade pressure forcing the price down.

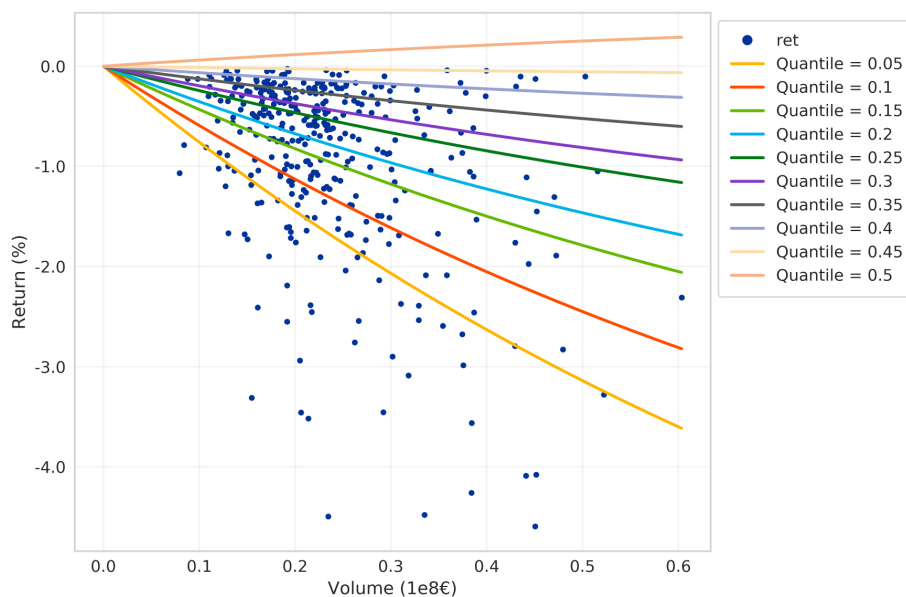


Figure 3: Real data for a specific security and estimated price impact functions for different quantiles.

Note: positive returns have been excluded from figure for visual purposes.

Source: Refinitiv (Eikon).

4.1.1 Parameter calibration security level impact

The instrument-level calibration allows to get a detailed perspective of the market impact at a micro-level. Nonetheless, such a granular perspective also permits to infer the relevance of price impact dynamics at a macro-level. Moreover, the classification of securities, allows to identify common risk factors and potential areas of concentrated risk.

Bond calibration

Regarding the categorisation of bonds, characteristics that are considered are the bonds' maturity, sector, and rating.

Sector: government (GOV), financial corporations (FC), and non-financial corporations (NFC);

Maturity: less than 2 years (2y), between 2 and 5 years (5y), and between 5 and 10 years (10y);

Credit rating: prime, high grade, upper medium grade, lower medium grade, non-investment grade.

Since the price of a bond converges to the face value + coupons over the course of the bonds' maturity, the price volatility tends to decrease over time. Hence, one would expect to observe a decrease in the impact severity, in securities with a shorter residual maturity. Empirically, this effect can be observed when comparing the estimated idiosyncratic risk parameter β_0 for bonds with a residual maturity less than 2 years, to bonds with a residual maturity between 5 and 10 years in Figure 5 where the latter has

an impact parameter, nearly 3 times larger than the former at all quantiles.

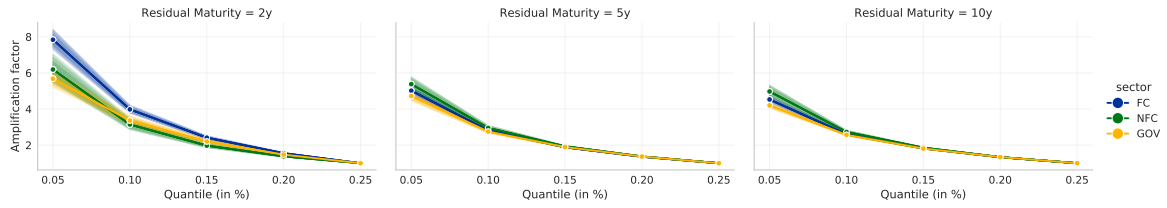


Figure 4: Tail impact multiplier.
Bond impact parameter relative to 25th percentile

While, the difference between sectors seem to be low when looking at the median, the tail (i.e. $q > 0.1$) impacts show significant differences. The sever increase in risk when approaching the tail of the distribution is even more apparent from Figure 4. The multiplication factor of the idiosyncratic impact parameter (i.e. $\frac{\beta_0^{q \leq 25}}{\beta_0^{q = 25}}$) indicates the additional sensitivity in the tail compared to the sensitivity at the median. Bond impacts in the tail can be at least 5 times larger compared to the 25th quantile, see Figure 4. Interestingly to note is that the multiplication factors appear to stabilise with increasing residual maturity. This result is important when analysing the effects of trading strategies or when assessing potential tail events from interconnected market participants.

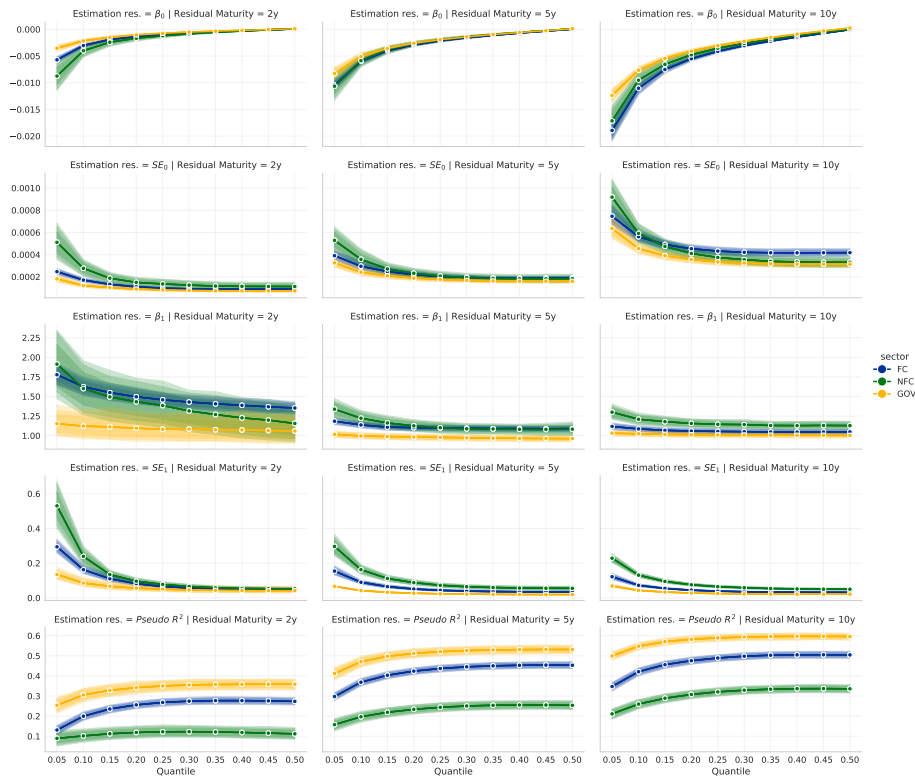


Figure 5: Bond price impact parameter estimations.

Bond security level impacts

Overall, market impacts related to bonds prove to be limited, a result that became clear after exposing several bonds to a direct liquidation shock, see Figure 6 for the results. In this exercises, all bonds are subject to three levels of fire-sale severity governed by the non-linear price impact function as described by (8) with the security level estimated parameters above. There is no contagion mechanism at this stage, hence impacts presented in Figure 6 can be interpreted as direct market impact presented in percentages. Notably, the highest impacts can be observed in the non-financial sector, this sector experiences an average shock 1.4 times higher than shocks to the financial sector and 2.4 higher compared to the government sector. The direct impact for short-term NFC bonds ranges from -0.5% to -0.75% when selling 10 and 100 Mln euro, respectively. An impact that gets roughly 2.5 times larger for long-term bonds.

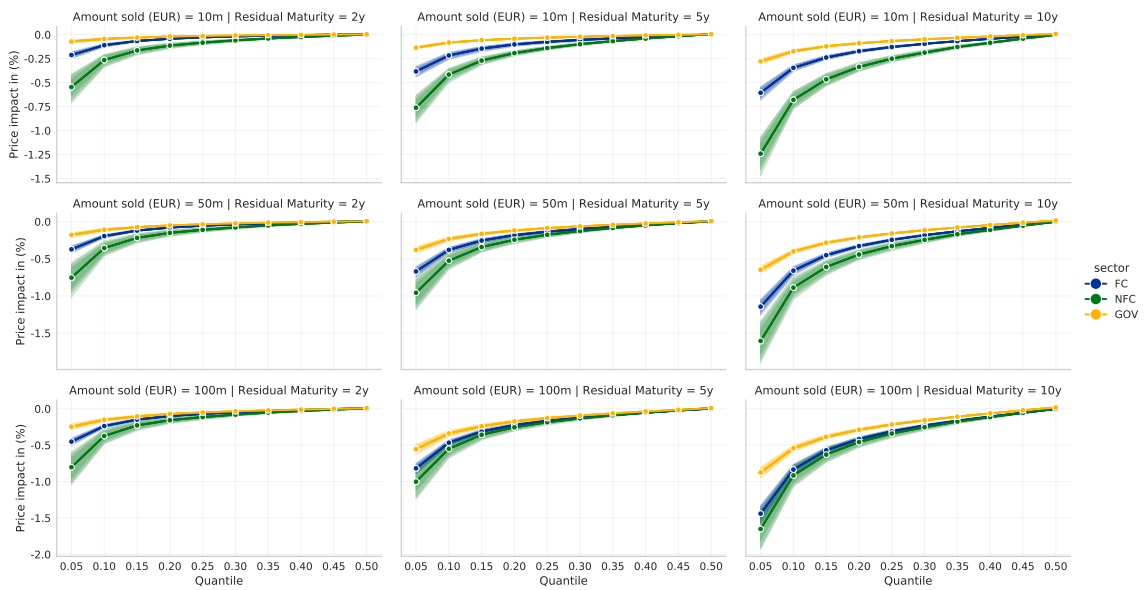


Figure 6: Bond price impacts for selected amounts sold.

The rows represent the amounts sold in Mln EUR, where the amounts are 10M, 50, and 100 Mln EUR.

The columns represent maximum residual maturity in their respective buckets, where 2y: up to 2 years, 5y: between 2 and 5 years, and 10y between 5 and 10 years.

Moreover, the quantile regression shows that impacts estimated at the tail of the distribution can be as much as 3 times larger than shocks around the 25th quantile, commonly estimated by single point estimation models. A quick comparison of government bonds impact parameters (see A.4 for more detailed overview) one can observe that on average impact parameters in the tail are nearly 4 times more severe than impact parameters estimated at the 25th quantile. For bonds with a residual maturity less than 2 years the parameter even exceeds 5 times the size.

One fundamental feature of a bond securities is the credit rating. Rating agencies providing a credit score to individual securities, allow to explore the relevance of price impacts across different rating categories. The credit scores used for the purpose of

this work are obtained from Standard & Poor’s (S&P). The categories considered range from *Prime* (least risky), to *Non-investment grade* (most risky); see table A9 for a more detailed overview of the credit ratings. Due to data coverage issues, the analysis is confined to instruments with credit ratings higher than *Lower medium grade*. Figure A4 shows that there is a significant difference between the levels of credit rating. Based on the results, an average security with a *High grade* credit score, has 17% higher impact, than an average security with a *Prime* rating. while the jump between *High grade* and *Medium grade* rating increases the impact even with 40%, the most significant jump can be observed between *Medium grade* rating and *Non-investment grade* rating of nearly 240%. As usually government bonds are higher rated than non-financial corporation bonds, the lower observed impacts for bonds could be explained by the distribution of credit ratings within the bond category.

These estimation results provide useful insight into the market risk associated with bond securities. While it is clear that there exists much greater tail risk, compared to average observed market impacts, bonds overall market impact potential remains low. This does however strongly depend on the type of bond, as results show a significant dependency on the length of the residual maturity. Furthermore, these findings provide insight in the potential security combination to mitigate market risks, which will be investigated in more detail in section 5.

Equity calibration

Equity securities are grouped by the size of the outstanding market capital of the issuing firm and the firm’s sector, which is either financial corporations (FC) or non-financial corporations (NFC). Similar to results from the bond impact calibration, the non-financial sector has a significantly higher market sensitivity than the financial sector equity. This in part may be explained by the diversification of bank held assets reflected in the bank’s equity. The estimation results for each quantile are presented in Figure 7. Note that as the firm’s market capital increases, so does the resilience against market shocks. It can also be observed that the sectors show increasingly similar impact sizes, among firms with a large market capitalization. As apposed to the estimated parameters for bonds, equity can reach impact coefficients up to -0.5 for NFC equities, compared to the -0.01 for NFC bonds. The equity impact parameters in the tail can become up to 3 times larger, compared to impact parameters around the 25th quantile for medium and large cap. securities and even up to 4.5 time for small cap. securities, see Figure 8. These consistently large multipliers show the potentially underestimated market risks in more common estimation methods. While the equities present smaller multipliers than those observed with bond securities, the much lower impact coefficients nonetheless, could lead to exceedingly larger impacts.

Equity security level impact

Similar to the bond category, the non-financial sector pose higher risks than the financial sector equities, the differences become smaller as the size of the company increases.

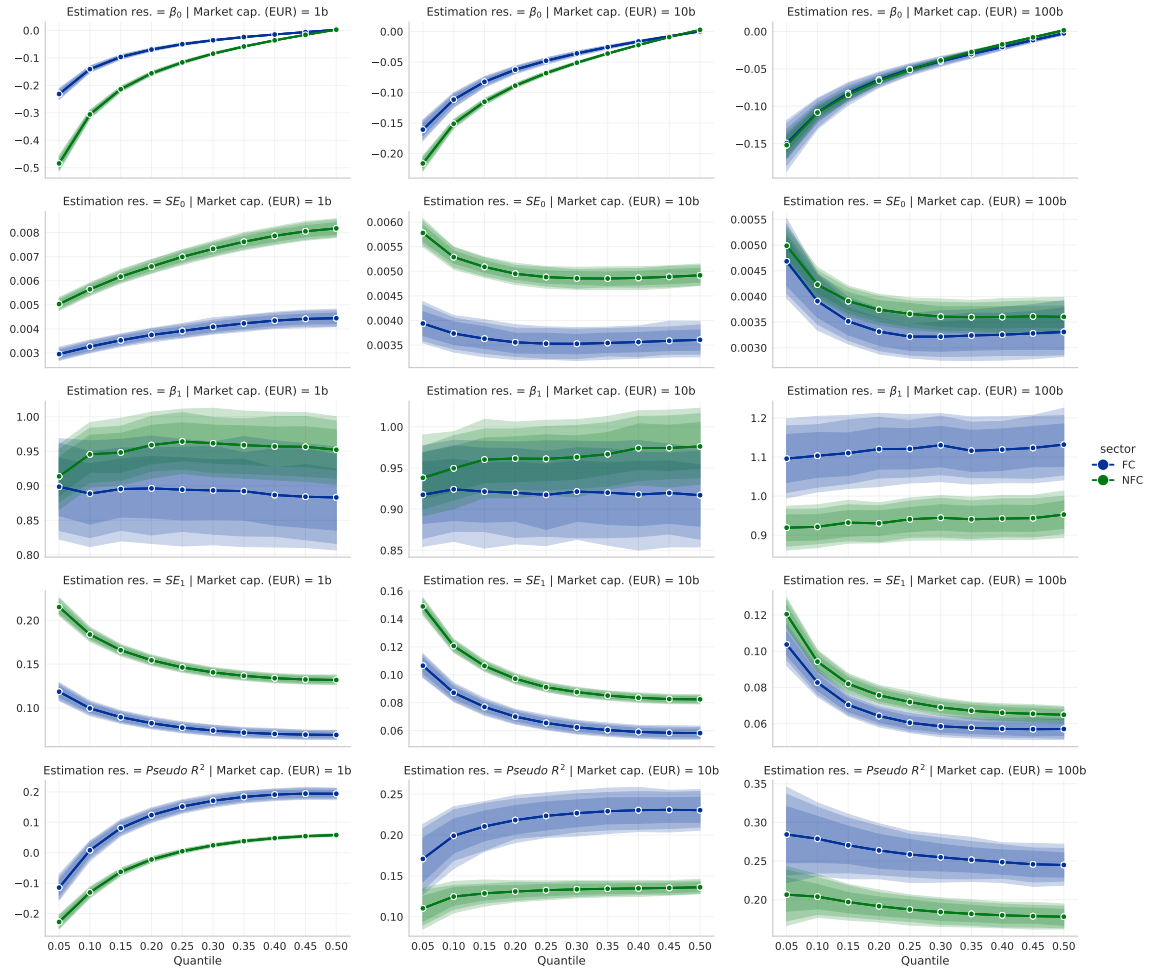


Figure 7: Equity price impact parameter estimations.

The difference between the percentage impact of the two sectors can be as much as twice as large for small cap. equity, Figure 7, while the large cap. equity differences become insignificant. The size of the impact also greatly depends on the size of the company, where small cap. NFCs could experience impacts over 40%, the most severe impact observed in large companies doesn't exceed 12%. This suggests that holding large cap. equity is significantly less sensitive to direct market risk. However, what cannot be concluded from these figures, is the impact due to second round effects. As the large cap. securities can be considered less risky, this also could lead to more concentrated investments, which will be discussed in more depth, in section 5.

4.2 Scaling properties

The discussion in previous sections has focused on daily frequencies. According to different modelling requirements, one might however need to account for longer liquidation horizons. The motivation for trading incrementally over several days rather than a

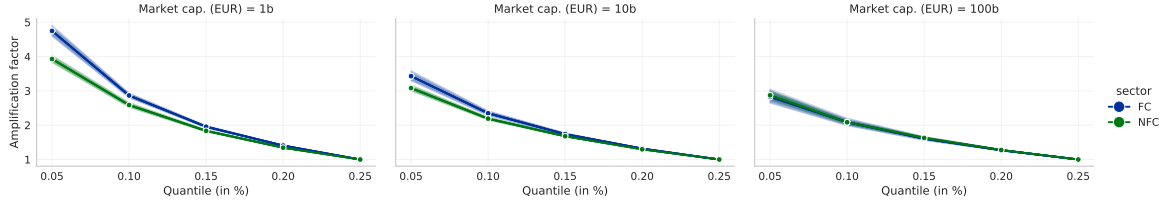


Figure 8: Tail impact multiplier equity

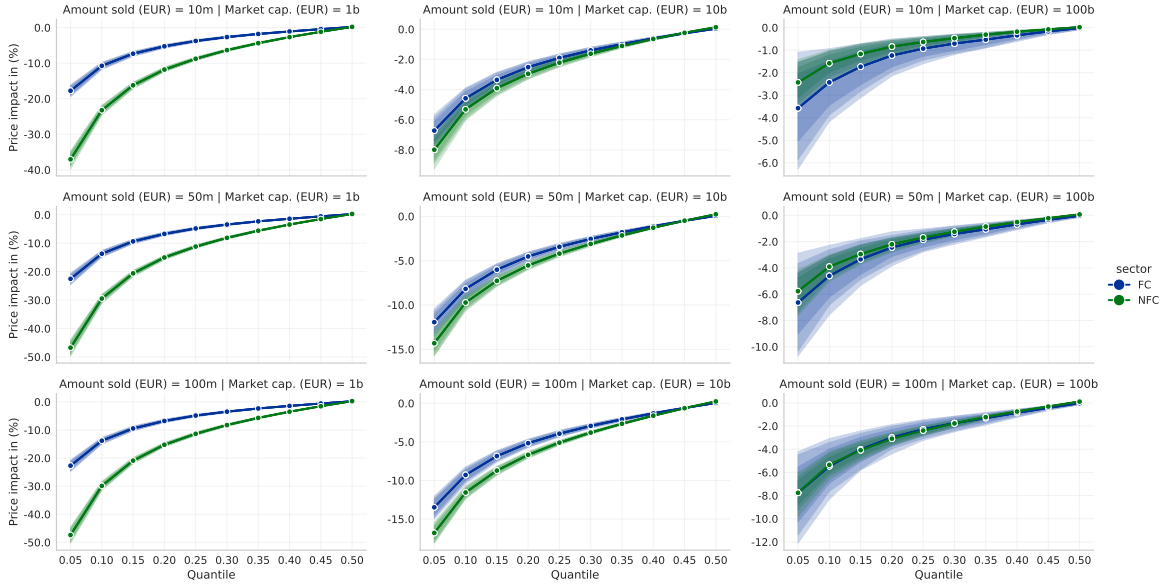


Figure 9: Equity price impacts for selected amounts sold.

large volume at once can be found in market participants' desire to decrease the impact their operations have on the market. Here, liquidation horizon refers to the size of the window over which returns are computed. Window sizes between one and twenty days are considered in the following analysis.

The impact coefficient, λ , can be found to be inversely related to market depths so that the accounting of different liquidation horizons τ can be done by the replacement $\lambda \rightarrow \frac{\lambda}{\sqrt{\tau}}$. For each instrument the price impact parameters are re-computed over the different liquidation horizons. We find the price impact to liquidation horizon relationship is well described by a power law:

$$\lambda_\tau = \lambda_1 \tau^\gamma. \quad (11)$$

Here, γ denotes the scaling factor, and λ_τ is the impact parameter calibrated over a window size of τ days.

Estimating the scaling factor results in a value of $\gamma = -0.6293$ with an $R^2 = 0.9755$. This result justifies the square root scaling presented above, also commonly employed in the literature (Cont and Schaanning, 2017), see Figure 10

The estimation of boundaries over varying liquidation horizons can be modelled through compounding as $B_\tau = 1 - (1 - B_1)^\tau$, where B_τ is the boundary corresponding to a τ days

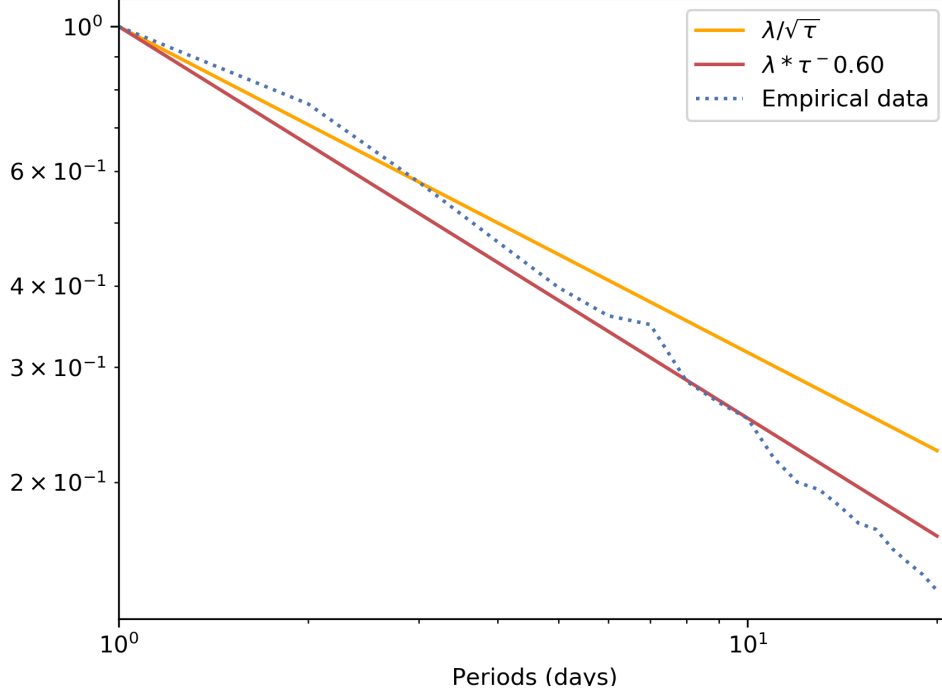


Figure 10: Price impact parameter calibrated over time horizons from 1 to 20 days

window. Alternatively, as done before, one can fit a power law to estimate a general scaling factor from the empirical data as

$$B_\tau = B\tau^\xi \quad (12)$$

where B_τ is the boundary corresponding to the liquidation horizon τ and ξ is the scaling factor. See table 3 for the estimated factor.

The mean of the boundaries over a larger time window (up to 20 days considered here) together with the compounded boundary and the power law fitted boundary can be observed in figure 11. The orange line represents the compounded boundary, the green line corresponds to the boundary based on the power law, and the blue dotted line is the empirical boundary given a liquidation horizon.

Table 3: Calibrated scaling parameter for the boundary

Boundaries: power law	
Scaling factor ξ	0.2397
R^2	0.9552

This parameter can then be used to scale the boundaries for individual securities or at any desired level of aggregation. Modifying equation (4), by including the scaled parameters, results in the following expression for the price impact function:

$$\Psi_\tau = B_\tau(1 - e^{-v\lambda_\tau/B_\tau}) \quad (13)$$

where Ψ_τ is the price impact corresponding to the liquidation horizon of τ days, $\lambda_\tau = \frac{\lambda}{\sqrt{\tau}}$

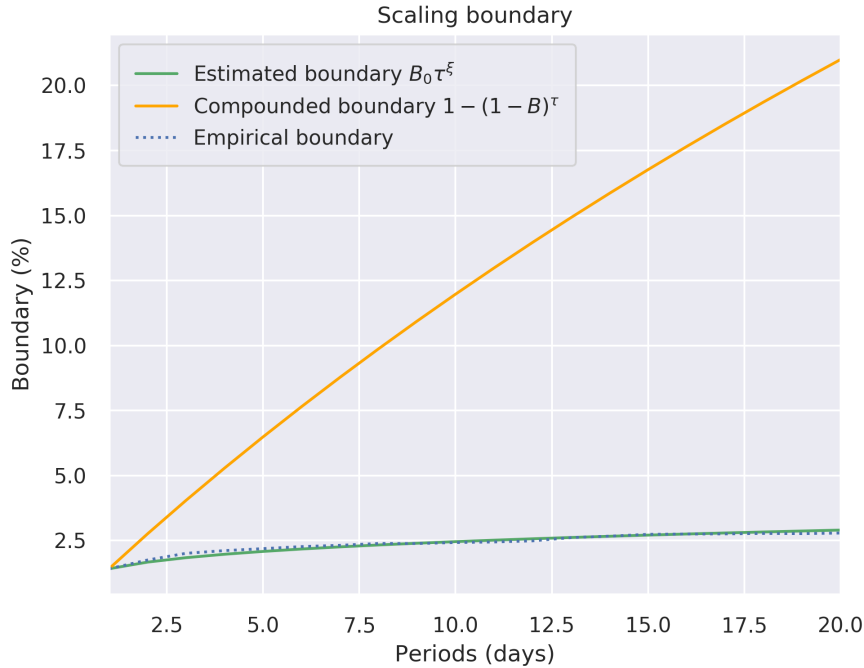


Figure 11: Boundary scaling over a period of 20 days.

and B_τ is the boundary scaled using either the compound method $1 - (1 - B_1)^\tau$ or power law estimation $B\tau^\xi$. Figure 12 provides a visualisation of equation (13) using both boundary scaling methods with a liquidation horizon of 20 days (orange and green lines). The blue line in each figure is generated by equation (4), without the use of any scaling methods, thus can be interpret as a 1 day liquidation horizon.

The 1 day boundary is equal to 1.53%, corresponding to the average 1 day boundary. From this value one can infer a boundary of 26.51% using compounding and 3.13% using the power law approach for the 20 days liquidation horizon. It becomes clear in both figures that the increased liquidation horizon doesn't guarantee a lower impact. The slope of the impact decreases due to the scaled lambda, i.e. the blue line has a much steeper slope than the orange and green lines, resulting in a lower impact. However, the scaled impacts surpass the boundary of the 1 day impact, due to an increased upper bound.

As these results present a theoretical explanation of individual scaling features, this approach doesn't take into account cross-impacts when liquidating multiple securities at the same time, let alone multiple portfolios. Therefore, it is important to be causes with implementation of price impacts over a longer time horizon. In the following section an example of an application is discussed.

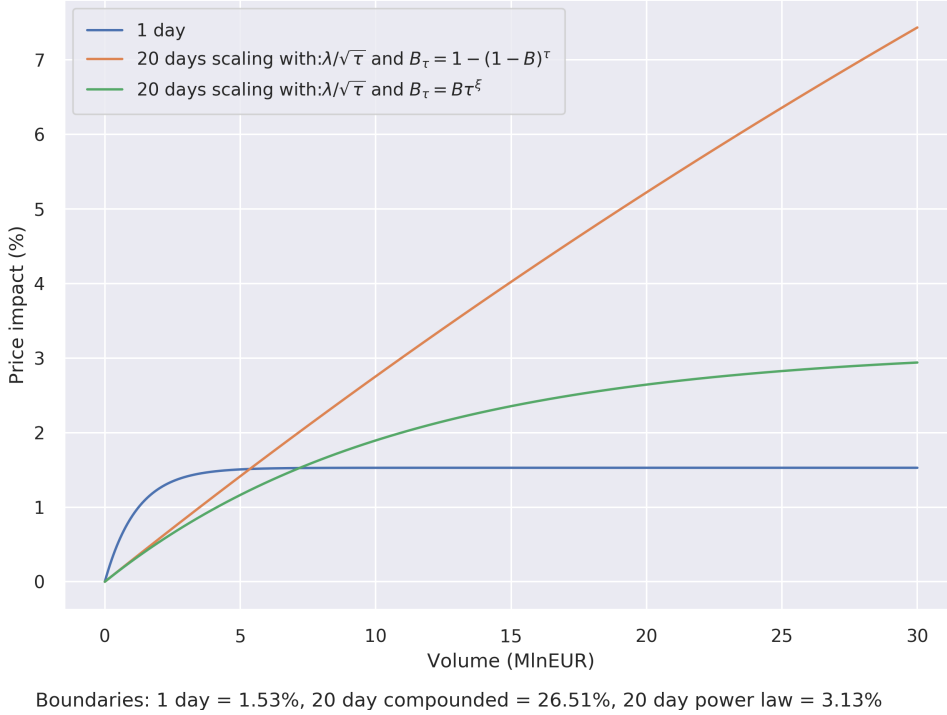


Figure 12: Scaled price impact function

5 Application for fire sales contagion analysis

This section demonstrates the usefulness of granular price impact functions. For this purpose, we use a simplified version of the stress testing model of Sydow et al. (2021), also see appendix A.3 for equations of the fire sale module. The aggregation method of section 2.5 is used at issuer level in the analysis. According to our experience, fire sale losses were driven mostly by investment funds, thus we introduce a constant 5% redemption from all funds, without any other shock to the system. Funds react to this liquidity shock by selling their securities in a pro rata manner. Figure 13 reports system-level losses in percentage of total assets of banks and investment funds. We find that the usage of homogeneous price impact parameters over-estimates fire sale losses up to a factor of more than two. This finding is in line with Cont and Schaanning (2017)⁶.

For sensitivity check, we run the simulations for different redemptions shocks as well, reported in Figure 14.

Overlapping portfolios among different financial institutions are a major source of risk in times of system-wide financial distress, representing an indirect channel for indirect contagion to spread from institutions under pressure to other market participants investing in the same assets. Be it because of asymmetric information on the fundamental value of individual assets or asset category, or because of sudden liquidity pressures, individual institutions might find themselves forced to liquidate large amounts of their portfolios in relatively short windows of time. As we have discussed this process of

⁶See Figures 14-16 in the aforementioned paper.

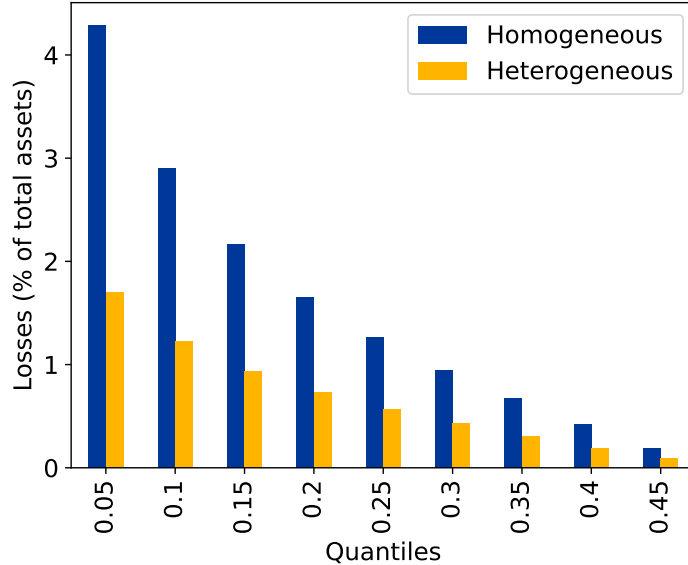


Figure 13: System-level losses for different quantiles of price impact estimations, comparison to homogeneous price impact parameters.

fire sales is associated with an impact on the price of securities subject to liquidation, eventually resulting in marked-to-market losses across the system. Crucially, this mechanism can interest not only market participants directly exposed to the distressed institution, but also seemingly unrelated ones, regardless of geography, or financial health, giving rise to what is often referred to as indirect contagion⁷.

A common analytical tool for assessing the risk of indirect contagion is the matrix of overlapping portfolio. Given a matrix $S \in \mathbb{R}^{N_B \times N_S}$ of N_B institutional ISIN-level portfolios composed of investments in N_S different assets, one can write the overlapping portfolio matrix $\mathcal{O} \in \mathbb{R}^{N_B \times N_B}$ as the matrix with elements

$$\mathcal{O}_{ij} = \sum_k \min \{S_{ik}, S_{jk}\}. \quad (14)$$

Figure 16a presents the overlapping portfolio matrix \mathcal{O} for the 126 largest banking groups in the euro area, showing wide overlapping investments worth multiple tens of billions in the same assets. This suggests a pro-rata liquidation of a bank's portfolio would have far reaching implications for all banks across the system, which would face marked-to-market losses on assets held at market value.

However, what (14) does not consider is the relevance of heterogeneity in price impacts across different securities and asset classes. Much of banks' portfolio of securities are composed of low risk government bonds which can be exchanged for liquidity with the central bank (see Figure 15). As discussed in the preceding section, these assets are generally associated with substantially more contained price impacts which contributes

⁷A further mechanism which contributes to indirect contagion is cross-impact. Cross-impact on price refers to the impact on other securities and assets that might result from the liquidation of one specific asset due to market correlations. For simplicity cross-impact is not considered in this analysis but could be done by in a simplified manner by iterating the system-level return as in equation (8)

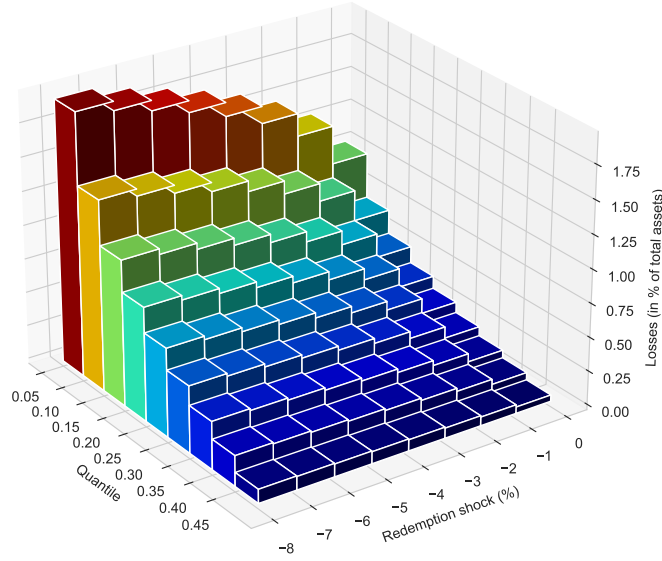


Figure 14: System-level losses for different quantiles and redemption shocks.

to a significant reduction of system-wide risk. A weighted overlapping portfolio matrix can instead be constructed to capture the heterogeneities inherent in the different assets composing the institutions' portfolios. In particular we define the weighted overlapping portfolio \mathcal{O}^W as the matrix with elements:

$$\mathcal{O}_{ij}^W = \sum_k \Psi_k \min \{ S_{ik}^2, S_{jk}^2 \}. \quad (15)$$

Weighted overlapping portfolios \mathcal{O}^W are presented in Figure 16b. The substantial risk reduction played by heterogeneities in price impacts can be observed at a glance, giving more prominence to overlaps in equity holding as opposed to overlaps in safer government bonds which compose the bulk of banks' portfolios in the euro area.

A similar effect can also be observed by looking at the slightly related cosine similarity matrices presented in Figure 17. Specifically, we consider a *nominal* cosine similarity matrix $CS \in \mathbb{R}^{N_B \times N_B}$ defined elementwise as

$$CS_{ij} = \frac{\sum_k S_{ik} S_{jk}}{(\sum_k S_{ik}^2)^{\frac{1}{2}} (\sum_k S_{jk}^2)^{\frac{1}{2}}}, \quad (16)$$

and a *weighted* cosine similarity matrix $CS^W \in \mathbb{R}^{N_B \times N_B}$ defined elementwise as

$$CS_{ij}^W = \frac{\sum_k S_{ik} S_{jk} \Psi_k^2}{(\sum_k S_{ik}^2 \Psi_k^2)^{\frac{1}{2}} (\sum_k S_{jk}^2 \Psi_k^2)^{\frac{1}{2}}}. \quad (17)$$

Notice that both CS and CS^W are bounded in $[0, 1]$ when considering long positions only, but can extend to $[-1, 1]$ when accounting for short positions as well.

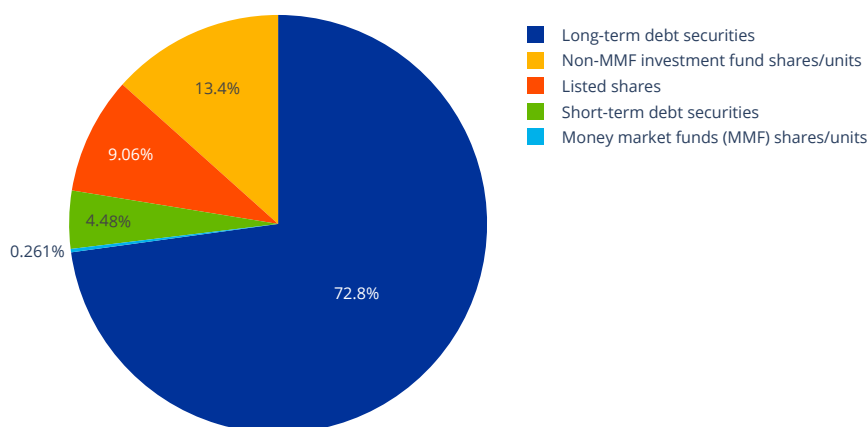


Figure 15: Composition of euro area banks securities portfolios [*Source - Security Holding Statistics*].

Cosine similarity gives a metric of similarity between each pair of portfolios discounting the absolute amount of the overall investment in the portfolio. Therefore, CS and CS^W provide an overview of the degree of diversification in portfolios of euro area banks. The price impact weighted matrix CS^W exhibits a markedly higher degree of diversification across the system, while at the same time revealing stronger similarities for some pairs of banks than what are instead observed in the nominal cosine similarity CS .

6 Conclusion

The aim of this paper was to analyse the effect of large scale portfolio deleveraging on the price of market traded securities and how the corresponding shocks from overlapping portfolios propagate through the financial system. The ability to describe these effects allows banks and other systemically important institutions such as CCPs, to prepare for potential threat to the stability in the financial system. In this paper the systemic risk is described as the combination of overlapping portfolios across the financial system and the price impact associated with the sale of large portfolio fractions, which may either dampen or exacerbate indirect risk sharing depending on the asset classes which constitute the overlaps in investments. Based on security level bank exposures in combination with historical daily prices and trade volumes, held by major participants in the euro area financial markets, we were able to quantify potential cascade effects from large scale deleveraging.

We describe the interconnected behaviour by means of a *non-linear price impact quantile regression* approach. The results presented in the section 4.1 show that in general bonds prove to have much lower price impacts when compared to equity securities allowing investors to mitigate market risk by optimising their portfolios. In contrast to the

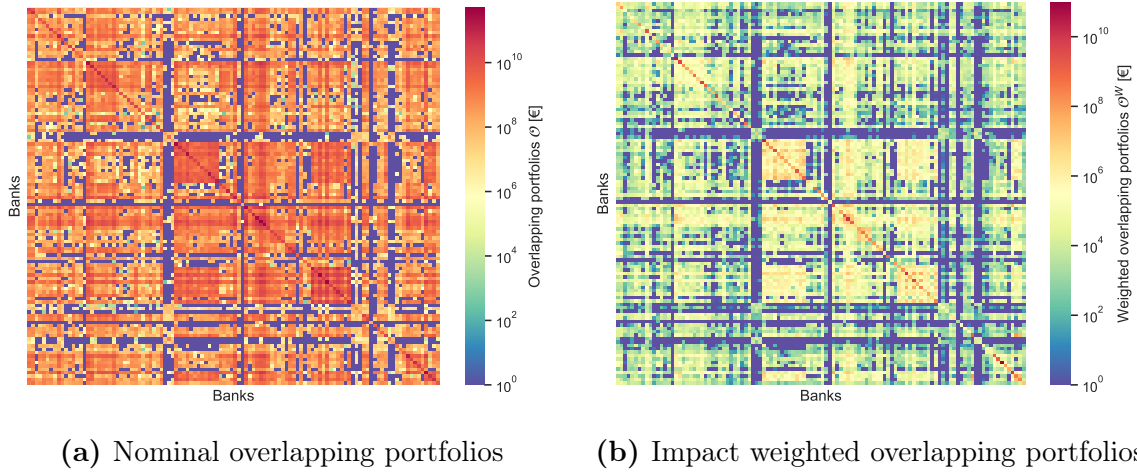


Figure 16: Weighting overlapping portfolios by security level price impacts reveals indirect contagion poses less risk than could otherwise inferred. Here, impact in (b) refers to quantile $q = 0.05$. Portfolios are those of the 126 largest euro area banking groups reporting in SHS-G, excluding short positions for simplicity. Banking groups on both axis are clustered by country of residence.

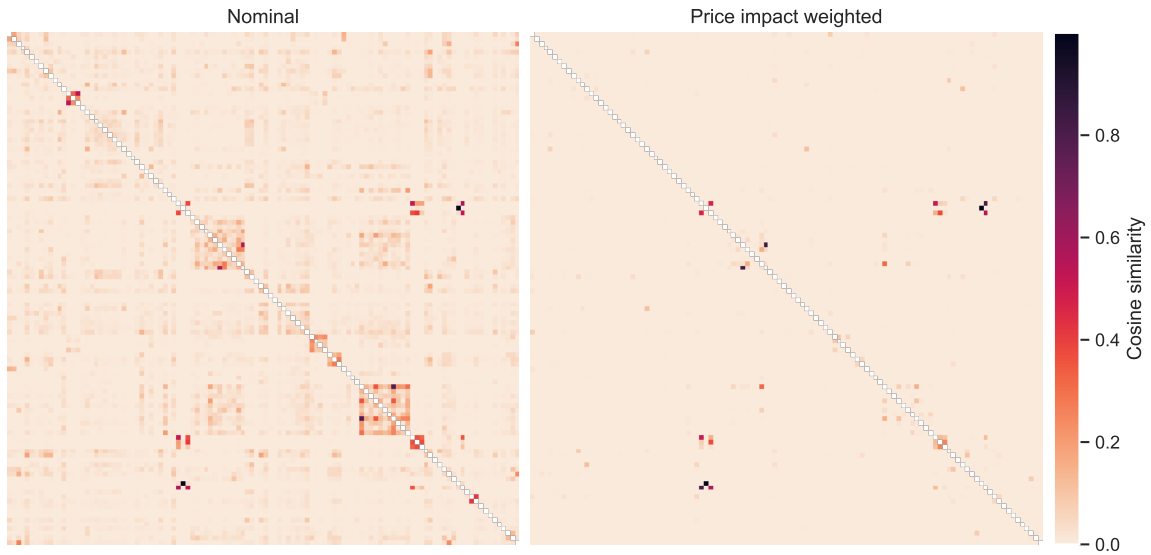


Figure 17: Nominal and price impact weighted ($q = 0.05$) cosine similarity matrices CS and CS^W . Banks on both axis are clustered by country of residence. Here portfolios do not account for short positions.

bonds, the results show major risk potential in small size non-financial corporation equity. Direct shocks from this category can reach up to 40%, which could lead to major cascade events when held in high concentrations among commonly held portfolios. The ability of large scale investment institutions to diversify portfolios, is therefore crucial to limit shock propagation from fire-sales.

The results presented in the section 5 show that taking into account heterogeneity across securities alleviates the risks shown by fire sale models. Due to the estimation approach, we were able to introduce derive an application of a price-at-risk measure as apposed to traditional average market price impacts to evaluate tail risk of possible market price movements as a consequence of several severity scenarios. These results furthermore show that system-level losses at the tail can be up to three times higher than average losses *ceteris paribus*.

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Appendices

A.1 Quantile regression

This section summarizes the methodological background of quantile regressions, originally laid down by Koenker and Bassett Jr (1978). The main advantage of this method is that it is able to estimate the quantile (instead of the mean as OLS does) of a variable based on some explanatory variables using all observations in the sample. The probability of a random variable X not exceeding x is denoted by the cumulative distribution function $F(x) = \mathbf{P}(X \leq x)$. The q -quantile of X is defined by

$$F^{-1}(\tau) = \inf \{x : F(x) \geq \tau\} = \hat{x},$$

i.e. the number that X does not exceed with at least probability q . Let us call the loss function $\rho_q(u) = u \cdot (q - \mathbf{1}\{u < 0\})$ ⁸, the expectation of which is $\mathbf{E}(\rho_q(u))$. It can be shown that $\mathbf{E}(\rho_q(X - \hat{x}))$ is minimized in \hat{x} if and only if $F(\hat{x}) = q$. The minimization problem

$$\mathbf{E}(\rho_q(X - \hat{x})) \longrightarrow \min$$

reduces to a linear programming routine.

In a regression setup, the q -quantile of variable y is estimated by $\mathbf{x}'\boldsymbol{\beta}$ and is denoted as

$$\hat{y}^q = \mathbf{x}'\hat{\boldsymbol{\beta}}.$$

Koenker and Machado (1999) also define a goodness of fit measure for quantile regressions that we use in our analysis. Using the sum of loss functions for all observations of \mathbf{x}_i : $\hat{V}(q) = \sum_i \rho_q(y_i - \mathbf{x}_i'\hat{\boldsymbol{\beta}})$. When the model is restricted to an intercept without explanatory variables, let the same measure denote $\tilde{V}(q)$. The *pseudo* R^2 is then defined as an analog of R^2 :

$$R^1(q) = 1 - \frac{\hat{V}(q)}{\tilde{V}(q)}.$$

A.2 Convex hull

This method initially finds the convex hull of the set of points, after which the first few points are used to establish the upper boundary. This method draws a polygon around the cloud of points, which intuitively can be seen as a rubber band around a set of nails N , hammered into wood. The subset of nails that are in contact with the rubber band is the convex hull of set N . More formally, the convex hull is the smallest convex polygon containing the points. A polygon is defined as a combination of line segments or edges, joined end-to-end in a cycle, in which the end points of the edges are called vertices. The convexity of the polygon means that any line segment (\overline{pq}) between the two points p, q within the polygon also lies inside the polygon.

⁸ $\mathbf{1}\{A\}$ is the indicator function of event A .

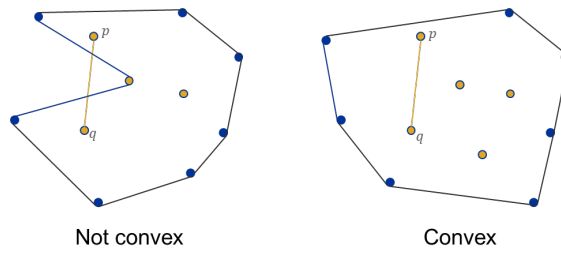


Figure A1: Convexity of a cloud of points

Lastly, the smallest polygon refers the any proper subset of the convex hull excludes at least one point in N . This implies that every vertex of the convex hull is a point of N .

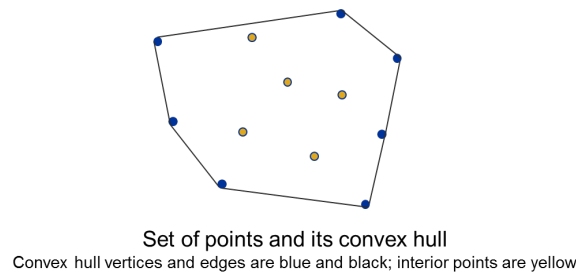


Figure A2: Complete convex hull

It can be noticed that N might have interior points that are not vertices of the convex hull.

One of the approaches to calculating the Convex Hull is the gift wrapping (Jarvis March) algorithm. This method consists of two stages. During the first stage, the algorithm finds the leftmost point s comparing the x - coordinate. Since, by definition, there are no points left of this point, it must be a convex hull vertex. This will become the starting point of set N for the convex hull. Starting from point s with a line straight up and turning it clockwise until it finds the first point, this will be the next convex hull vertex. This process continues for at each vertex until it reaches point s again figure A3.

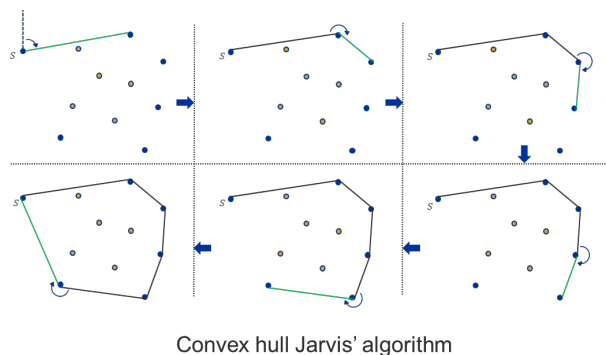


Figure A3: Cloud of points including their convex hull vertices (yellow and blue respectively), line segments connecting the convex hull vertices (black lines)

A.3 Contagion model of fire sales

This appendix is an excerpt of the fire sale mechanism for overlapping portfolios in a system of banks and funds, as introduced in Sydow et al. (2021). Banks and funds start proportionally selling tradable assets to close liquidity gaps. In this modelling block, banks are willing to sell only their non-eligible securities since they have access to central bank funding using their high quality liquid assets (HQLA). We assume that there is no endogenous price impact for HQLA.⁹ By contrast, funds sell all kinds of securities holdings as they do not have access to central bank funding. Thus, they do not discriminate between eligible and non-eligible types of assets and sell all of their securities holdings.

Let $h_t^{\text{trd}} = \{h_{i,\phi,t}^{\text{trd}}\}_{i,\phi}$ denote the portfolio matrix of tradable securities at time t , by market values, where i is the holder and ϕ is the security. Similarly, h_t^{red} is the portfolio matrix of redeemable holdings. Moreover, $\mathbf{p}_t = (p_{\phi,t})_{\phi}$ denotes the vector of prices of tradable securities.

We assume that fire sales are applied to cover liquidity shortfalls proportional to the share of tradable securities in the securities holdings portfolio.

Then, starting from time t , the fire sale algorithm proceeds as follows:

- (i) Determine the supply value $S_{\phi,t}$ that will be sold of each security (at the final prices). Based on the slicing hypothesis, the sale is done pro rata for the value of each security in the initial portfolio, meaning that entity-level and aggregated supply of ϕ are

$$s_{i,\phi} = \frac{h_{i,\phi}^{\text{trd}}}{\sum_{\varphi} (h_{i,\varphi}^{\text{trd}} + h_{i,\varphi}^{\text{red}})} g_i \quad \text{and} \quad S_{\phi} = \sum_i s_{i,\phi} \quad (18)$$

respectively, $s_{i,\phi}$ is the value that i wants to recover from ϕ . Note that the choice of the liquidation approach, here the slicing approach, may be a crucial driver for the magnitude of the shock transmission between sectors. For example, under the waterfall approach (selling the most liquid assets first), the magnitude of the shock transmission may be considerably reduced due to a reduced price impact.

- (ii) Determine the new vector of prices $\mathbf{p}_{t+1} = (p_{\phi,t+1})_{\phi}$ using the total amounts sold:

$$p_{\phi,t+1} = p_{\phi,t} (1 - B_{\phi} (1 - \exp(-S_{\phi} \lambda_{\phi} / B_{\phi}))) \quad (19)$$

and update the value of tradable portfolios

$$h_{t+1}^{\text{trd}} = h_t^{\text{trd}} \left(\frac{\mathbf{p}_{t+1}}{\mathbf{p}_t} \right)^{\top}. \quad (20)$$

⁹An alternative approach could be the reconstruction of HQLA from granular securities within our simulation, which would allow for a price impact on the amount of available HQLA. However, this not only means higher computational costs but also difficulties to assess at which point a bank would turn to the central bank to exchange specific assets for cash.

(iii) Find the new NAV vector $\mathbf{p}_{t+1}^* = (p_{i,t}^*)_{i \in \text{IF}}$ of funds, and update the value of redeemable portfolios:

$$h_{t+1}^{\text{red}} = h_t^{\text{red}} \cdot \left(\frac{\mathbf{p}_{t+1}^*}{\mathbf{p}_t^*} \right)^\top . \quad (21)$$

(iv) Update internal accounting variables of entities to reflect changes of portfolio values. Let i be a financial institution and ϕ a security. When we account for the change in REA, it is the change from $h_{i,\phi,t}$ to $h_{i,\phi,t+1}$ that matters. However, when we want to account for losses only and see the impact on the total capital we need to disentangle what is converted as cash from actual losses that stem from the decrease in prices.

Cash holdings are updated with the amounts received after the iteration has converged:

$$\mathbf{c}_{i,t+1} = \mathbf{c}_{i,t} + \sum_{\phi} s_{i,\phi} . \quad (22)$$

Note that S_{ϕ} increases due to the price declines and the iteration terminates thanks to the finite amount of assets that can be sold and the introduction of a lower boundary for security prices. Assuming that the whole residual liquidity need is recovered by i we have a change in capital due to the price impact given by $m_{i,t+1} = \sum_{\phi} (h_{i,\phi,t+1} - h_{i,\phi,t})$. More generally, using the change in prices we get

$$m_{i,t+1} = \sum_{\phi} \frac{h_{i,\phi,t}}{p_{\phi,t}} (p_{\phi,t+1} - p_{\phi,t}) . \quad (23)$$

A.4 Estimated security-level price impact parameters

Average of β_0	Sector and market cap.					
	FC			NFC		
	100b	10b	1b	100b	10b	1b
Quantiles						
0.05	-0.1495	-0.1608	-0.2315	-0.1521	-0.2163	-0.4843
0.1	-0.1083	-0.1116	-0.1410	-0.1084	-0.1513	-0.3058
0.15	-0.0825	-0.0825	-0.0966	-0.0843	-0.1151	-0.2139
0.2	-0.0644	-0.0626	-0.0696	-0.0657	-0.0887	-0.1562
0.25	-0.0505	-0.0479	-0.0500	-0.0513	-0.0683	-0.1163
0.3	-0.0399	-0.0359	-0.0357	-0.0387	-0.0512	-0.0845
0.35	-0.0302	-0.0256	-0.0241	-0.0274	-0.0361	-0.0585
0.4	-0.0209	-0.0163	-0.0147	-0.0172	-0.0222	-0.0359
0.45	-0.0116	-0.0078	-0.0059	-0.0078	-0.0091	-0.0159
0.5	-0.0025	0.0002	0.0024	0.0016	0.0027	0.0029

Table A1: Equity estimation results for β_0

Average of $\lambda \cdot 10^9$ s	Sector and market cap.					
	FC			NFC		
	100b	10b	1b	100b	10b	1b
Quantiles						
0.05	-5.455	-12.141	-147.344	-3.448	-43.020	-363.363
0.1	-3.691	-8.296	-99.568	-2.212	-27.959	-241.179
0.15	-2.624	-6.093	-68.094	-1.618	-19.841	-166.081
0.2	-1.858	-4.556	-48.751	-1.158	-15.529	-119.307
0.25	-1.395	-3.410	-33.725	-0.869	-11.986	-86.113
0.3	-1.058	-2.542	-22.730	-0.637	-8.260	-60.387
0.35	-0.797	-1.819	-15.776	-0.419	-4.962	-41.038
0.4	-0.510	-1.130	-9.999	-0.240	-2.705	-23.335
0.45	-0.252	-0.528	-4.532	-0.106	-1.021	-11.227
0.5	-0.028	-0.018	0.389	0.022	0.076	0.071

Table A2: Equity estimation results, λ s, multiplied by 10^9

Average of β_0	Country						
Maturity / quantiles	AT	BE	DE	ES	FR	IT	NL
2y							
0.05	-0.0030	-0.0022	-0.0024	-0.0029	-0.0039	-0.0077	-0.0014
0.1	-0.0018	-0.0012	-0.0014	-0.0017	-0.0026	-0.0046	-0.0008
0.15	-0.0014	-0.0008	-0.0010	-0.0012	-0.0018	-0.0032	-0.0006
0.2	-0.0010	-0.0006	-0.0007	-0.0009	-0.0011	-0.0023	-0.0004
0.25	-0.0007	-0.0004	-0.0005	-0.0006	-0.0007	-0.0016	-0.0003
0.3	-0.0005	-0.0002	-0.0004	-0.0004	-0.0004	-0.0011	-0.0003
0.35	-0.0003	-0.0001	-0.0003	-0.0003	-0.0001	-0.0008	-0.0002
0.4	-0.0001	0.0000	-0.0002	-0.0001	0.0002	-0.0005	-0.0001
0.45	0.0001	0.0001	-0.0001	0.0000	0.0004	-0.0002	-0.0001
0.5	0.0002	0.0002	-0.0001	0.0001	0.0006	0.0001	0.0000
5y							
0.05	-0.0045	-0.0061	-0.0053	-0.0093	-0.0063	-0.0201	-0.0050
0.1	-0.0031	-0.0035	-0.0033	-0.0054	-0.0041	-0.0122	-0.0036
0.15	-0.0024	-0.0024	-0.0023	-0.0038	-0.0029	-0.0087	-0.0028
0.2	-0.0018	-0.0018	-0.0017	-0.0027	-0.0022	-0.0063	-0.0023
0.25	-0.0014	-0.0014	-0.0012	-0.0020	-0.0017	-0.0046	-0.0018
0.3	-0.0010	-0.0010	-0.0009	-0.0015	-0.0012	-0.0034	-0.0014
0.35	-0.0007	-0.0007	-0.0006	-0.0010	-0.0009	-0.0023	-0.0011
0.4	-0.0005	-0.0004	-0.0004	-0.0006	-0.0005	-0.0014	-0.0008
0.45	-0.0002	-0.0001	-0.0002	-0.0002	-0.0002	-0.0007	-0.0006
0.5	0.0001	0.0002	0.0001	0.0002	0.0001	0.0001	-0.0003
10y							
0.05	-0.0154	-0.0099	-0.0100	-0.0121	-0.0126	-0.0213	-0.0102
0.1	-0.0100	-0.0067	-0.0062	-0.0070	-0.0075	-0.0138	-0.0070
0.15	-0.0075	-0.0050	-0.0044	-0.0049	-0.0053	-0.0098	-0.0052
0.2	-0.0057	-0.0039	-0.0033	-0.0036	-0.0040	-0.0073	-0.0041
0.25	-0.0043	-0.0031	-0.0024	-0.0027	-0.0031	-0.0054	-0.0032
0.3	-0.0031	-0.0022	-0.0018	-0.0020	-0.0022	-0.0040	-0.0024
0.35	-0.0023	-0.0016	-0.0012	-0.0014	-0.0015	-0.0028	-0.0016
0.4	-0.0015	-0.0009	-0.0007	-0.0009	-0.0009	-0.0017	-0.0010
0.45	-0.0005	-0.0003	-0.0002	-0.0003	-0.0002	-0.0007	-0.0004
0.5	0.0005	0.0003	0.0003	0.0001	0.0003	0.0002	0.0003

Table A3: Bond estimation results for GOV, β_0

Average of $\lambda \cdot 10^9$ s	Country						
Maturity / quantiles	AT	BE	DE	ES	FR	IT	NL
2y							
0.050	-0.030	-0.026	-0.041	-0.457	-0.181	-0.399	-0.013
0.100	-0.018	-0.015	-0.024	-0.254	-0.133	-0.237	-0.008
0.150	-0.014	-0.010	-0.016	-0.167	-0.089	-0.152	-0.006
0.200	-0.010	-0.007	-0.012	-0.110	-0.051	-0.099	0.004
0.250	-0.007	-0.004	-0.009	-0.069	-0.034	-0.063	-0.003
0.300	-0.005	-0.003	-0.006	-0.039	-0.017	-0.036	-0.002
0.350	-0.003	-0.001	-0.005	-0.012	0.002	-0.022	-0.001
0.400	-0.001	0.000	-0.003	0.012	0.008	-0.012	-0.001
0.450	0.001	0.002	-0.002	0.037	0.016	-0.003	-0.000
0.500	0.002	0.003	-0.000	0.063	0.023	0.008	0.001
5y							
0.050	-0.063	-0.103	-0.107	-1.256	-0.127	-0.420	-0.050
0.100	-0.044	-0.056	-0.066	-0.709	-0.081	-0.261	-0.036
0.150	-0.034	-0.037	-0.046	-0.499	-0.058	-0.185	-0.028
0.200	-0.025	-0.029	-0.034	-0.348	-0.044	-0.133	-0.023
0.250	-0.020	-0.019	-0.024	-0.237	-0.034	-0.095	-0.018
0.300	-0.014	-0.014	-0.017	-0.150	-0.024	-0.071	-0.014
0.350	-0.010	-0.009	-0.011	-0.078	-0.016	-0.047	-0.011
0.400	-0.006	-0.005	-0.006	-0.016	-0.009	-0.025	-0.008
0.450	-0.003	-0.001	-0.001	0.036	-0.002	-0.008	-0.006
0.500	0.002	0.003	0.004	0.102	0.005	0.010	-0.003
10y							
0.050	-0.201	-0.134	-0.230	-2.171	-0.362	-0.811	-0.102
0.100	-0.132	-0.089	-0.142	-1.291	-0.228	-0.491	-0.070
0.150	-0.101	-0.066	-0.102	-0.875	-0.161	-0.337	-0.052
0.200	-0.079	-0.051	-0.076	-0.616	-0.119	-0.246	-0.041
0.250	-0.059	-0.040	-0.057	-0.442	-0.090	-0.181	-0.032
0.300	-0.044	-0.029	-0.041	-0.307	-0.066	-0.131	-0.024
0.350	-0.033	-0.020	-0.028	-0.198	-0.045	-0.090	-0.016
0.400	-0.022	-0.012	-0.016	-0.090	-0.026	-0.052	-0.010
0.450	-0.010	-0.003	-0.004	-0.010	-0.006	-0.017	-0.004
0.500	0.003	0.004	0.007	0.080	0.012	0.014	0.003

Table A4: Bond estimation results for GOV, λ s, multiplied by 10^9

Average of β_0	Country						
Maturity / quantiles	AT	BE	DE	ES	FR	IT	NL
2y							
0.05	-0.0032	-0.0063	-0.0043	-0.0069	-0.0055	-0.0098	-0.0052
0.1	-0.0019	-0.0037	-0.0023	-0.0033	-0.0030	-0.0049	-0.0028
0.15	-0.0013	-0.0025	-0.0015	-0.0019	-0.0019	-0.0030	-0.0018
0.2	-0.0010	-0.0018	-0.0011	-0.0012	-0.0014	-0.0020	-0.0012
0.25	-0.0007	-0.0013	-0.0008	-0.0008	-0.0010	-0.0014	-0.0009
0.3	-0.0005	-0.0010	-0.0006	-0.0005	-0.0007	-0.0010	-0.0006
0.35	-0.0004	-0.0007	-0.0004	-0.0003	-0.0004	-0.0007	-0.0004
0.4	-0.0002	-0.0005	-0.0003	-0.0001	-0.0002	-0.0004	-0.0002
0.45	-0.0001	-0.0002	-0.0001	0.0001	0.0000	-0.0003	0.0000
0.5	0.0000	0.0000	0.0000	0.0003	0.0002	-0.0001	0.0001
5y							
0.05	-0.0061	-0.0097	-0.0091	-0.0129	-0.0100	-0.0154	-0.0098
0.1	-0.0036	-0.0054	-0.0052	-0.0072	-0.0058	-0.0091	-0.0055
0.15	-0.0026	-0.0034	-0.0034	-0.0048	-0.0039	-0.0061	-0.0037
0.2	-0.0019	-0.0025	-0.0025	-0.0035	-0.0029	-0.0043	-0.0026
0.25	-0.0014	-0.0018	-0.0019	-0.0026	-0.0021	-0.0032	-0.0020
0.3	-0.0011	-0.0013	-0.0014	-0.0019	-0.0016	-0.0024	-0.0014
0.35	-0.0008	-0.0009	-0.0010	-0.0013	-0.0011	-0.0017	-0.0010
0.4	-0.0005	-0.0006	-0.0006	-0.0008	-0.0007	-0.0010	-0.0007
0.45	-0.0002	-0.0002	-0.0003	-0.0004	-0.0003	-0.0006	-0.0003
0.5	0.0000	0.0001	0.0000	0.0001	0.0000	-0.0001	0.0000
10y							
0.05	-0.0130	-0.0183	-0.0154	-0.0196	-0.0187	-0.0323	-0.0216
0.1	-0.0077	-0.0106	-0.0090	-0.0117	-0.0109	-0.0188	-0.0125
0.15	-0.0055	-0.0069	-0.0064	-0.0082	-0.0078	-0.0103	-0.0086
0.2	-0.0041	-0.0054	-0.0047	-0.0060	-0.0058	-0.0073	-0.0061
0.25	-0.0031	-0.0041	-0.0035	-0.0043	-0.0044	-0.0053	-0.0045
0.3	-0.0023	-0.0031	-0.0026	-0.0032	-0.0034	-0.0038	-0.0033
0.35	-0.0016	-0.0023	-0.0019	-0.0023	-0.0024	-0.0025	-0.0024
0.4	-0.0011	-0.0016	-0.0013	-0.0015	-0.0016	-0.0015	-0.0016
0.45	-0.0005	-0.0009	-0.0007	-0.0007	-0.0008	-0.0008	-0.0008
0.5	0.0000	-0.0003	0.0000	0.0000	-0.0001	-0.0002	0.0000

Table A5: Bond estimation results for FC, β_0

Average of $\lambda \cdot 10^9$ s	Country						
Maturity / quantiles	AT	BE	DE	ES	FR	IT	NL
2y							
0.050	-0.261	-0.232	-0.514	-0.421	-0.448	-3.172	-1.123
0.100	-0.140	-0.110	-0.252	-0.206	-0.210	-1.558	-0.652
0.150	-0.090	-0.073	-0.162	-0.112	-0.122	-0.882	-0.377
0.200	-0.063	-0.047	-0.107	-0.068	-0.081	-0.496	-0.242
0.250	-0.043	-0.034	-0.074	-0.044	-0.053	-0.276	-0.150
0.300	-0.031	-0.025	-0.049	-0.026	-0.028	-0.152	-0.098
0.350	-0.021	-0.017	-0.030	-0.012	-0.011	-0.070	-0.058
0.400	-0.011	-0.011	-0.016	0.002	0.004	-0.034	-0.026
0.450	-0.003	-0.005	-0.004	0.015	0.017	-0.014	0.003
0.500	0.006	0.000	0.009	0.028	0.030	0.007	0.028
5y							
0.050	-0.279	-0.212	-0.868	-1.042	-0.739	-4.478	-2.934
0.100	-0.159	-0.124	-0.489	-0.552	-0.387	-2.770	-1.521
0.150	-0.107	-0.085	-0.335	-0.358	-0.244	-1.910	-0.963
0.200	-0.068	-0.061	-0.236	-0.254	-0.169	-1.176	-0.651
0.250	-0.049	-0.046	-0.175	-0.189	-0.111	-0.930	-0.457
0.300	-0.036	-0.034	-0.122	-0.134	-0.074	-0.752	-0.330
0.350	-0.025	-0.025	-0.086	-0.085	-0.049	-0.584	-0.231
0.400	-0.015	-0.017	-0.051	-0.049	-0.031	-0.270	-0.145
0.450	-0.004	-0.008	-0.018	-0.013	-0.013	-0.132	-0.065
0.500	0.005	-0.000	0.009	0.019	0.004	0.006	0.007
10y							
0.050	-0.542	-0.359	-3.122	-1.478	-1.254	-2.841	-4.917
0.100	-0.293	-0.206	-2.088	-0.864	-0.613	-1.640	-2.501
0.150	-0.204	-0.133	-1.561	-0.612	-0.388	-1.124	-1.646
0.200	-0.144	-0.102	-1.206	-0.462	-0.275	-0.700	-1.140
0.250	-0.110	-0.075	-0.949	-0.332	-0.207	-0.505	-0.818
0.300	-0.081	-0.057	-0.736	-0.247	-0.154	-0.374	-0.587
0.350	-0.058	-0.043	-0.542	-0.181	-0.106	-0.267	-0.399
0.400	-0.039	-0.025	-0.360	-0.110	-0.068	-0.173	-0.247
0.450	-0.020	-0.013	-0.189	-0.046	-0.030	-0.092	-0.102
0.500	-0.003	0.003	0.004	0.009	0.003	-0.012	0.035

Table A6: Bond estimation results for FC, λ s, multiplied by 10^9

Average of β_0	Country						
Maturity / quantiles	AT	BE	DE	ES	FR	IT	NL
2y							
0.05	-0.0026	-0.0034	-0.0063	-0.0104	-0.0108	-0.0116	-0.0037
0.1	-0.0016	-0.0022	-0.0032	-0.0047	-0.0044	-0.0059	-0.0024
0.15	-0.0011	-0.0016	-0.0020	-0.0028	-0.0025	-0.0038	-0.0016
0.2	-0.0008	-0.0012	-0.0014	-0.0018	-0.0017	-0.0026	-0.0011
0.25	-0.0006	-0.0009	-0.0011	-0.0014	-0.0012	-0.0018	-0.0007
0.3	-0.0004	-0.0007	-0.0008	-0.0010	-0.0008	-0.0012	-0.0005
0.35	-0.0003	-0.0006	-0.0006	-0.0007	-0.0005	-0.0008	-0.0003
0.4	-0.0001	-0.0005	-0.0003	-0.0005	-0.0003	-0.0005	-0.0002
0.45	0.0000	-0.0003	-0.0001	-0.0003	0.0000	-0.0002	-0.0001
0.5	0.0001	-0.0002	0.0000	-0.0001	0.0002	0.0000	0.0000
5y							
0.05	-0.0126	-0.0057	-0.0074	-0.0220	-0.0077	-0.0207	-0.0063
0.1	-0.0079	-0.0034	-0.0042	-0.0118	-0.0042	-0.0110	-0.0034
0.15	-0.0057	-0.0024	-0.0029	-0.0076	-0.0026	-0.0072	-0.0020
0.2	-0.0040	-0.0017	-0.0020	-0.0057	-0.0019	-0.0050	-0.0014
0.25	-0.0029	-0.0012	-0.0015	-0.0042	-0.0014	-0.0036	-0.0010
0.3	-0.0022	-0.0009	-0.0011	-0.0029	-0.0010	-0.0025	-0.0007
0.35	-0.0015	-0.0006	-0.0007	-0.0019	-0.0007	-0.0017	-0.0005
0.4	-0.0010	-0.0004	-0.0004	-0.0011	-0.0004	-0.0010	-0.0003
0.45	-0.0006	-0.0002	-0.0002	-0.0003	-0.0002	-0.0003	-0.0002
0.5	-0.0002	0.0000	0.0000	0.0004	0.0000	0.0003	0.0000
10y							
0.05	-0.0291	-0.0075	-0.0102	-0.0299	-0.0154	-0.0327	-0.0096
0.1	-0.0168	-0.0043	-0.0061	-0.0180	-0.0084	-0.0175	-0.0054
0.15	-0.0117	-0.0030	-0.0043	-0.0135	-0.0058	-0.0119	-0.0036
0.2	-0.0091	-0.0022	-0.0031	-0.0087	-0.0042	-0.0088	-0.0026
0.25	-0.0066	-0.0016	-0.0023	-0.0069	-0.0031	-0.0064	-0.0020
0.3	-0.0050	-0.0012	-0.0018	-0.0056	-0.0023	-0.0047	-0.0014
0.35	-0.0037	-0.0009	-0.0013	-0.0035	-0.0016	-0.0033	-0.0010
0.4	-0.0024	-0.0006	-0.0009	-0.0027	-0.0010	-0.0022	-0.0007
0.45	-0.0012	-0.0003	-0.0005	-0.0015	-0.0004	-0.0011	-0.0003
0.5	-0.0003	0.0000	-0.0001	-0.0007	0.0001	-0.0001	0.0000

Table A7: Bond estimation results for NFC, β_0

Average of $\lambda \cdot 10^9$ s Maturity / quantiles	Country						
	AT	BE	DE	ES	FR	IT	NL
2y							
0.05	-1.087	-3.378	-3.858	-0.435	-2.414	-5.599	-3.691
0.1	-0.637	-2.234	-2.064	-0.161	-1.086	-2.662	-2.397
0.15	-0.438	-1.551	-1.291	-0.066	-0.650	-1.644	-1.592
0.2	-0.317	-1.223	-0.914	-0.024	-0.447	-1.105	-1.119
0.25	-0.232	-0.924	-0.689	-0.016	-0.315	-0.817	-0.713
0.3	-0.171	-0.740	-0.515	-0.011	-0.223	-0.572	-0.480
0.35	-0.121	-0.593	-0.367	-0.007	-0.138	-0.375	-0.321
0.4	-0.078	-0.464	-0.227	-0.004	-0.074	-0.258	-0.215
0.45	-0.029	-0.327	-0.127	-0.002	-0.013	-0.133	-0.107
0.5	0.006	-0.235	-0.034	-0.001	0.039	-0.040	-0.010
5y							
0.05	-2.597	-5.673	-5.929	-9.162	-4.453	-6.652	-6.258
0.1	-1.443	-3.400	-3.178	-3.797	-2.452	-3.627	-3.393
0.15	-1.034	-2.394	-2.075	-1.934	-1.533	-2.523	-2.040
0.2	-0.767	-1.742	-1.465	-1.483	-1.078	-1.790	-1.426
0.25	-0.569	-1.247	-1.087	-1.042	-0.764	-1.302	-1.035
0.3	-0.415	-0.899	-0.785	-0.692	-0.526	-0.934	-0.745
0.35	-0.272	-0.647	-0.549	-0.480	-0.356	-0.631	-0.529
0.4	-0.167	-0.406	-0.346	-0.318	-0.214	-0.374	-0.342
0.45	-0.078	-0.204	-0.172	-0.243	-0.074	-0.151	-0.162
0.5	0.001	-0.013	-0.014	-0.168	0.063	0.050	-0.002
10y							
0.050	-2.770	-8.007	-4.344	-8.744	-6.576	-14.026	-7.429
0.100	-1.517	-4.394	-2.502	-3.884	-3.419	-6.607	-4.303
0.150	-1.173	-2.932	-1.846	-2.510	-2.220	-4.464	-2.926
0.200	-0.886	-2.101	-1.246	-1.614	-1.575	-3.217	-2.125
0.250	-0.640	-1.563	-0.921	-1.190	-1.161	-2.341	-1.595
0.300	-0.446	-1.133	-0.687	-0.750	-0.862	-1.711	-1.169
0.350	-0.322	-0.817	-0.535	-0.479	-0.582	-1.231	-0.831
0.400	-0.193	-0.537	-0.347	-0.314	-0.363	-0.812	-0.538
0.450	-0.077	-0.263	-0.201	-0.129	-0.155	-0.388	-0.256
0.500	0.018	-0.003	-0.039	0.086	0.032	-0.048	0.025

Table A8: Bond estimation results for NFC, λ s, multiplied by 10^9

A.5 Results by credit ratings

Credit rating	Investment grade
AAA	Prime
AA+	
AA	High grade
AA-	
A+	Upper medium grade
A	
A-	
BBB+	Lower medium grade
BBB	
BBB-	
BB+	Non-investment grade (junk)
BB	
BB-	
B+	
B	
BB-	
B+	
B	
B-	
CCC+	
CCC	
CCC-	
CC	
C	
D	

Source: Standard & Poor's

Table A9: Standard & Poor's bond credit rating

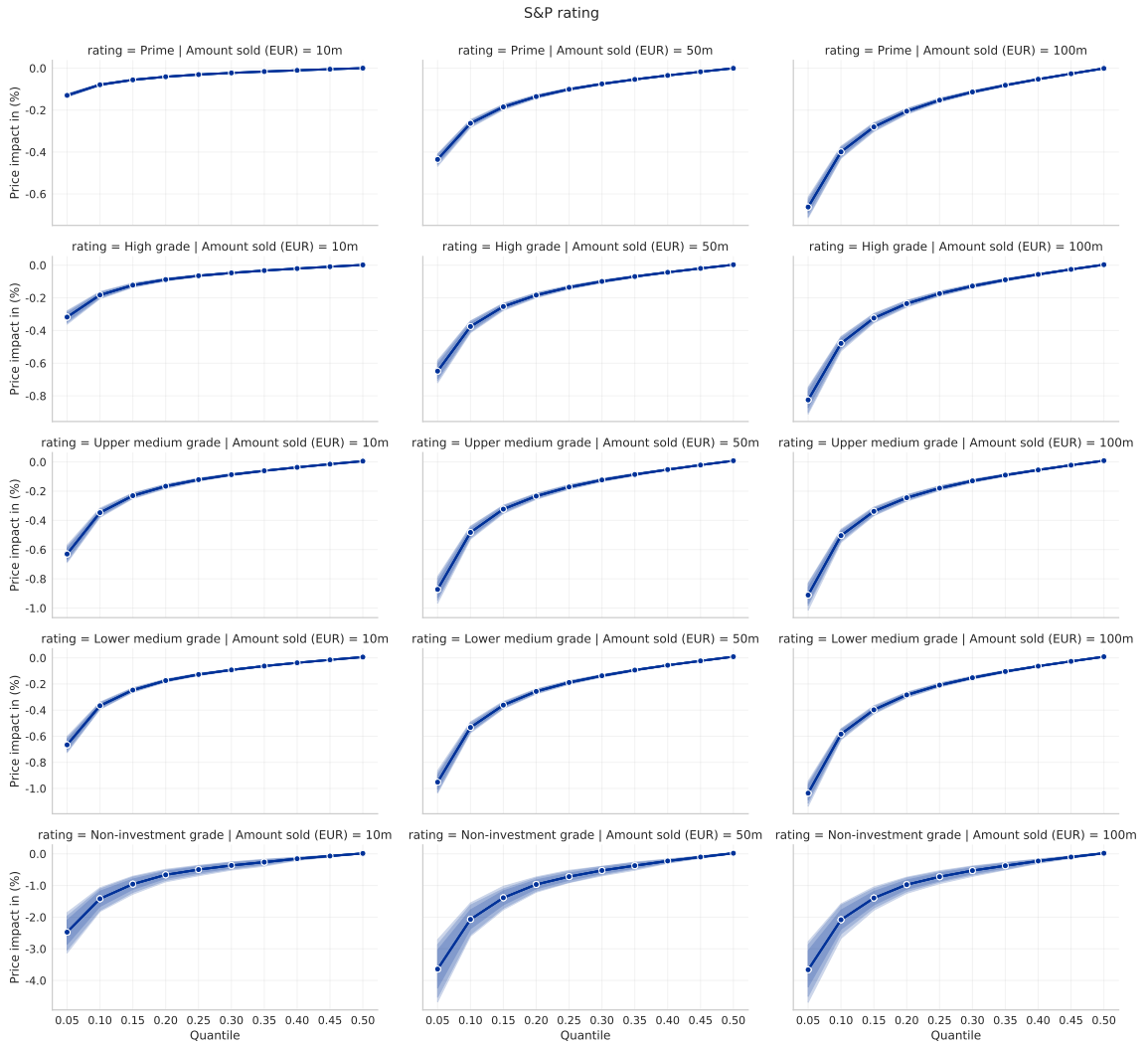


Figure A4: Bond price impacts for selected amounts sold by credit rating.

Credit grade	Maturity	Number of bonds	Outstanding amount (EUR Bln)
Prime	2y	320	551
	5y	451	697
	10y	358	590
Higher grade	2y	181	522
	5y	222	684
	10y	170	699
Upper medium grade	2y	154	149
	5y	192	188
	10y	152	180
Lower medium grade	2y	144	597
	5y	241	809
	10y	198	807
Non-investment grade	2y	8	7
	5y	25	25
	10y	20	26
Total		2,836	6,529

Table A10: Bond sample size by credit grade and maturity (2020Q4)