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The Role of Debt and Equity Finance over the Business Cycle

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Abstract

Equity and debt issuance are procyclical for most listed US firms. The procyclicality of equity issuance decreases monotonically with firm size. At the aggregate level, however, the results are not conclusive because issuance is countercyclical for very large firms that, although few in number, have a large effect on the aggregate because of their enormous size.

To explain our empirical findings, we relax key assumptions of the standard model with credit market imperfections: linear technology, homogeneous firms, and debt as the only source of external finance. The cyclical behavior of equity is determined by the cyclical behavior of the shadow price of external funds. If firms use the standard debt contract then this shadow price, and thus equity, are procyclical. We calibrate the model to replicate key features of observed equity issuance. The model (i) generates a countercyclical default rate, (ii) magnifies shocks, and (iii) generates a stronger cyclical response for small firms. In contrast, the model without equity does the exact opposite.

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1 Introduction

Several predictions of dynamic stochastic general equilibrium (DSGE) models can be improved if they incorporate frictions in obtaining firm finance.[1] Nevertheless, existing models have shortcomings. First, in many models defaults do not take place in equilibrium, or if they do occur, then the default rate is procyclical.[2] Both predictions are counterfactual. Second, technology is often assumed to be linear, which implies that small firms are simply scaled down versions of large firms. Third, although these models are successful in generating a hump-shaped response for real activity (propagation), they are less successful in generating substantial magnification. In fact, the standard debt contract dampens shocks. Finally, debt is typically the only form of external finance. Fama and French (2005) and Frank and Goyal (2005) document, however, that firms frequently issue equity. Moreover, we document that equity finance is cyclical and that the strongest cyclical behavior is found for small firms.

The first contribution of this paper is to document the cyclical behavior of debt and equity issuance by firm size. There are a few studies that study the cyclical behavior of *aggregate* debt and equity finance, but these studies reach different conclusions regarding the cyclical behavior of the external financing sources.[3] Similarly, we find that conclusions for aggregate series depend on the particular definition of equity and debt issuance, the sample period, and the methodology used. In this paper, we use disaggregated data and a robust pattern emerges. Our findings can be summarized as follows.[4]

[1]See Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999).

[2]Modifications are possible to deal with this problem. For example, Bernanke, Gertler, and Gilchrist (1999) assume that aggregate productivity is not known when the contract is written. Dorofeenko, Lee, and Salyer (2006) generate a countercyclical default rate by letting idiosyncratic risk decrease with aggregate productivity.

[3]Choe, Masulis, and Nanda (1993) and Korajczyk and Levy (2003) find that equity issuance is procyclical, whereas Jermann and Quadrini (2006) find equity issuance (minus dividend payments) to be countercyclical. Choe, Masulis, and Nanda (1993) find debt issuance to be countercyclical, whereas Jermann and Quadrini (2006) find it to be procyclical. Korajczyk and Levy (2003) find book value leverage to be countercyclical. A more extensive discussion is given in the data appendix.

[4]In Covas and den Haan (2006), we show that the results are very similar when Canadian data are

- Debt and equity issuance are procyclical for the majority of firms.
- The procyclicality of equity issuance is decreasing with firm size.
- Debt and equity issuance are countercyclical for the top 1%. The opposite behavior for this small but quantitatively very important group explains the ambiguous results for aggregate data.[5]

Existing business cycle models typically assume that net worth can only increase through retained earnings and do not allow for equity issuance. The second contribution of this paper is to eliminate this deficiency by constructing a DSGE model with a firm problem that can explain the observed cyclical behavior of equity and debt finance and its relation to firm size. Firm behavior is size dependent, because we relax the standard assumption of linear technology.

The debt contract used is standard. That is, firms borrow through one-period debt contracts. Bankruptcy costs imply a premium on debt finance, but debt also has a tax advantage. The trade off between these two factors imply that there is an optimal amount of net worth (and leverage). Motivated by our empirical findings, we allow firms with lower levels of net worth to move closer to the optimal level by issuing equity. Equity issuance costs prevent firms, however, of moving directly to the optimal level. In particular, we follow Cooley and Quadrini (2001) and incorporate a reduced-form cost of issuing equity, which is assumed to be increasing in the amount of equity issued.

A priori it is not clear whether equity issuance will be pro or countercyclical. The procyclical need for additional funds is a reason for equity issuance to be procyclical. Equity issuance could also be countercyclical, however, because it is easier to borrow during good times. We show the latter effect is not present when the standard debt contract is used. Although it is easier to borrow during good times, the desire to expand also leads to an increase in the default rate, which in turn leads to an increase in the shadow price of an additional unit of net worth. Consequently, equity issuance is procyclical. This

used.

[5] The top 1% covers 18% of gross stock sales, 28% of sales, and 34% of assets in the Compustat data set.

effect is stronger for small firms, since the shadow price of additional funds is close to zero and not sensitive to business cycle conditions for large firms.

With only debt financing, the default rate increases with aggregate productivity and output is less volatile than in the frictionless model. In the prototype version of our model, equity issuance costs depend only on the amount of equity issued. Although equity issuance is costly, allowing firms to raise external equity almost completely gets rid of this dampening and the increase in the default rate caused by the debt contract. The prototype version of our model, however, cannot overturn the undesirable properties of the standard debt contract. With a countercyclical price of risk and countercyclical cost of issuing equity, the model can generate the procyclical behavior of debt and equity issuance that is observed in the data as well as its dependence on firm size. The model can also generate a counteryclical default rate. The more realistic behavior of firm financing implies substantial magnification of shocks and stronger cyclical responses for small firms.

The organization of this paper is as follows. In the next section, we document how the firms' financing sources move over the business cycle. In Section 3, we discuss the static version of our model, which is simple enough to derive some analytic results. In Section 4, we discuss the dynamic model and in Section 5 we document the properties of the model. The last section concludes.

2 Cyclical Properties of Financing Sources

This section documents how the firms financing sources vary over the business cycle.

2.1 Data set and methodology

Our data set consists of annual Compustat data from 1971 to 2004. To study the importance of firm size, we rank firms using last period's end-of-period asset value. We then construct J firm categories and look at the cyclical behavior of debt and equity for each group $j \in \{1, \ldots, J\}$. A firm group is defined by a lower and an upper percentile. Our firm groups are [0,25%], [0,50%], [0,75%], [0,99%], [90%,95%], [95%,99%], and [99%,100%]. The behavior of the very largest firms is different from that of the other firms. To understand which large firms behave differently, we consider several groups in the top decile.[6]

Table 1 provides a set of summary statistics for each of these groups. In particular, we find that smaller firms have lower leverage and exhibit higher asset growth. Smaller firms finance a much larger fraction of asset growth with equity, whereas larger firms finance a larger fraction with debt and retained earnings.[7]

In this section, we report results for sale of stock, change in (the book value of) equity,[8] gross issuance of long-term debt, change in liabilities, and retained earnings. This paper comes with an extensive data appendix in which we give more detailed information on the methodology used and the time series behavior of the series. The appendix also reports results for the net sale of stock[9] and the net issuance of long-term debt and considers different ways to construct variables for the different groups. The results are similar to those reported here.

Our measures for real activity are real GDP and the real value of the group's assets. We use two procedures to construct a cyclical measure for firm finance. In the *flow approach*, the period t observation is the amount of funds raised in period t divided by a trend value of the assets of the group considered. We do not divide by the actual (i.e., cyclical) asset value, because we would lose information by doing so. For example, an observed decrease in the ratio of equity *relative* to assets is consistent with a decrease as well as an increase in the amount of equity. One only knows for sure that other forms of financing increase by more.

According to the flow approach, the net change in equity for firms that are in group j

[6] Systematic and quantitatively important deviating cyclical behavior is found for the top 1%.

[7] These results are consistent with those reported in Frank and Goyal (2005).

[8] Change in equity is defined as in Fama and French (2005). As documented in the data appendix, we obtained very similar results with the alternative definition of Baker and Wurgler (2002).

[9] We prefer the gross series over the net sale of stock because, as pointed out by Fama and French (2001, 2005), firms often repurchase stock and then re-issued to the sellers of an acquisition, to employee stock ownership plans, and to executives who exercise their stock options. These re-issued stock do not show up as a sale of stock, since they do not lead to a cash flow. The repurchases, however, do show up. So although these transactions leave equity unchanged, they would cause a reduction in sales minus repurchases.

in period t would be equal to

$$F_t^E(j) = \frac{\sum_{i \in j_t} (E_{i,t}^{\$} - E_{i,t-1}^{\$})/p_t}{S_t^{T,A\$}(j)}, \tag{1}$$

where $E_{i,t}^{\$} - E_{i,t-1}^{\$}$ is the change in equity for firm i, $S_t^{T,A\$}(j)$ is the trend of the real asset value of firms in group j, and p_t is the producer price level in year t.[10] A disadvantage of the flow approach is that some series are quite volatile. In particular, the series frequently display sharp changes that are reversed in the next period. Therefore, we also construct a cyclical measure of firm finance that puts less emphasis on the high frequency movements of the data. This is the *level approach*, which we also illustrate with the change in equity. The initial value is set equal to

$$L_1^E(j) = \frac{\sum_{i \in j_1} E_{i,1}^{\$}}{p_1} \tag{2}$$

and subsequent values are defined using

$$L_t^E(j) = L_{t-1}^E(j) + \frac{\sum_{i \in j_t} \left(E_{i,t}^{\$} - E_{i,t-1}^{\$} \right)}{p_t}. \tag{3}$$

This variable is then logged and the cyclical component is obtained by applying the HP filter. If the composition of the groups would not change, then $L_t^E(j)$ would simply be (the deflated) aggregate equity of group j. More generally, $L_t^E(j)$ can be interpreted as the (deflated) equity level of a hypothetical firm for which the percentage change in equity is identical to the observed change in equity for group j.

We also consider a modified approach that corrects for possible changes in $L_t^E(j)$ caused by changes in the average firm size of group j. The results are similar to the results reported here and are only discussed in the data appendix.

2.2 Empirical results

In this section, we discuss

[10] Scaling by the trend asset value is not enough to render the $F_t^E(j)$ series stationary, presumably because of long-term shifts in firm financing. We remove the remaining trend using the HP filter, but very similar results are obtained when a linear trend is used.

- the correlation between real activity and equity as well as debt finance,
- the correlation between debt and equity finance, and
- the correlation between real activity and retained earnings.

2.2.1 Cyclical behavior of equity

Results for equity issuance are reported in Tables 2 and 3. Table 2 uses the level approach and Table 3 uses the flow approach. The top half of each table uses GDP as the real activity variable and the bottom half uses the book value of assets. Each panel reports results for two equity series, the sale of stock and the change in equity.

Correlation between equity finance and GDP. At the aggregate level, the coefficients are small and not even the sign is robust. For the sale of stock, the correlation coefficient is equal to 0.20 and -0.001 for the level and the flow approach, respectively. For the change in equity the corresponding coefficients are -0.07 and 0.07.[11]

Although, the cyclical behavior of aggregate equity depends on the particular definition of equity and methodology used, a robust pattern emerges at the disaggregate level. For both definitions and approaches equity behavior is procyclical for all firm groups considered that exclude the top 5%. For the level approach, several coefficients are significant at the 5% (or lower) level using a one-sided test. For the flow approach, less coefficients are significant.[12] The correlation coefficients are higher for the gross series than for the net, which makes sense since one can expect repurchases to be procyclical.

[11] Using data from the Flow of Funds Accounts of the Federal Reserve Board, Jermann and Quadrini (2006) find a positive correlation between GDP and aggregate net equity payouts as a fraction of GDP. Equity payouts are dividends *minus* net equity issuance. Their positive correlation, thus, corresponds to some of the negative correlation coefficients reported here for some equity measures. Moreover, with the measure of Jermann and Quadrini (2006), it is more likely to attain a countercyclical equity issuance, since it is net of dividend payments and is expressed as a fraction of GDP. See the data appendix for a further discussion.

[12] The lower significance is not surprising given the stronger emphasis on higher frequencies.

In contrast, the correlation of the top 1% is negative for both definitions and approaches. For the level approach, the significance levels (using a one-sided test) are 6.3% for the change in equity and less than 1% for the sale of stock. No robust picture emerges for the sign of the correlation for the group of firms in between the 95$^{\text{th}}$ and the 99$^{\text{th}}$ percentile. Although the top 1% consists of a very small number of firms, it is important for aggregate behavior, since the distribution of firm size has an extremely fat right tail.

The positive correlation coefficient for the different firm groups indicate that equity is procyclical, but does not indicate for which group equity issuance moves the most over the cycle. To shed some light on this question we plot the cyclical components. Figure 1 plots the cyclical component of the sale of stock (level approach) and GDP for several firm categories that all exclude the top 1%.[13] The following observations can be made. First, the positive comovement between equity issuance and real activity is clear.[14] Second, cyclical movements are stronger for smaller firms. The third observation is that the lead-lag structure seems to change over time. For example, equity issuance is slightly leading GDP in the second half of the eighties but is slightly lagging GDP in the second half of the nineties, both are periods in which important fluctuations occur. This means that the magnitude for the correlation coefficients may very well underestimate the extent to which equity issuance and GDP are correlated.

Correlation between equity finance and assets. The bottom row of panels in Tables 2 and 3 reports the comovement of equity issuance and assets. The asset variable for the level approach is constructed as in Equation (3) and the asset variable for the flow approach is constructed as in Equation (1). Correlation coefficients are now often very high and very significant. Even for the top 1%, do we find three of the four coefficients to be positive and significant. When assets are used as the real activity measure, there is

[13] Details on the time series behavior of the top 1% is given in the data appendix.

[14] There is one exception. In the early seventies, the cyclical components of equity and GDP move together and in particular they both decline during the oil crisis. When the cyclical component of GDP recovers, however, the equity components continue to decline until the recessions of the early eighties, after which they again move closely with GDP. As documented below, there is a large build up in the cyclical components of debt and liabilities during this decade of low real interest rates.

strong support for the hypothesis that equity issuance is more cyclical for smaller firms. For example, for the sale of stock, the correlation coefficients for the bottom 25% (99%) are equal to 0.91 (0.15) and 0.80 (0.47) for the flow and level approach, respectively, and coefficients are highly significant for the bottom 25% and not significant for the bottom 99%.

2.2.2 Cyclical behavior of debt

In this section, we look at the correlation of real activity with long-term debt issuance and the change in total liabilities. Tables 4 and 5 report the results for the level and the flow approach, respectively.

Correlation between debt finance and GDP. At the aggregate level, the correlation between debt and GDP is positive and significant at at least the 4% level (one-sided test) for both debt measures and for both the level and the flow approach. As with equity, the results with aggregate data hide heterogenous behavior across the different firm groups. In particular, whereas the correlation coefficients for the bottom 25%, bottom 50%, bottom 75%, and even the bottom 99% are positive and significant, the correlation coefficient for the top 1% is insignificant, small, and for the level approach even negative.

Figures 2 and 3 plot the cyclical component of GDP together with the cyclical components of long-term debt issuance and the net-change in total liabilities, respectively. The level approach is used to construct the financing variables. It shows that the cyclical component for the bottom 25%, the bottom 50%, and the bottom 99% move together closely for both debt definitions. The graphs make clear that the issuance of long-term debt and the change in liabilities lag the cycle, which is also made clear by the higher correlation coefficients of the debt variables with lagged GDP.

The graph provides no reason to believe that changes in debt issuance over the business cycle are quantitatively more important for smaller firms. The one episode where a much sharper increase and subsequent decrease were observed for groups that exclude the larger firms is in the first half of the seventies. Here debt issuance lags output, however, so that debt is still increasing while GDP is already contracting.

Correlation between debt finance and assets. As with equity, the differences between the different firm categories are smaller when assets are used as the real activity variable. For long-term debt issuance, it still is the case that the correlation coefficients are smaller for the larger firms, but they are always positive, even for the top 1% (although not significant for the flow approach). Interestingly, a very uniform pattern of high and significant correlation coefficients is observed for the change in total liabilities. That is, the correlation coefficients are above 0.9 for both approaches, even for the top 1%.

2.2.3 Comovement of equity and debt

Table 6 reports the correlation between the gross equity and the gross debt measure, i.e. change in equity and change in liabilities, as well as the correlation between the net equity and the net debt measure, i.e. sale of stock and long-term debt issuance. The correlation coefficients are almost all positive for different firm categories, definitions, and approaches. Several coefficients are significant. The only negative contemporaneous coefficient is found for the [95%,99%] size category using the gross measures and the flow approach.

Above we showed that the cyclical behavior of equity and debt issues is quite different for firms in the top 1%. Nevertheless, the correlation of the two external financing sources for the top 1% has the same sign as the coefficients for the smaller firms, i.e., positive. Several coefficients for the top 1% are highly significant. This result and the fact that debt and equity for the top 1% are positively correlated with assets suggest that the main difference between small and large firms is the cyclical behavior of assets.[15].

Using the flow-of-funds data from the Federal Reserve Board, Jermann and Quadrini (2006) find that equity issuance is countercyclical, debt issuance is procyclical, and aggregate equity and aggregate debt are negatively correlated. For some measures, we also find equity issuance to be countercyclical at the aggregate level. The positive correlation

[15] In fact, the correlation coefficient (t-statistic) for the cyclical components of asset and GDP is between 0.39 (2.54) and 0.47 (3.59) for the firms in the bottom 25% and bottom 75%, respectively, while it is -0.02 (-0.08) for firms in the top 1%.

between equity and debt, however, is a robust finding when Compustat data are used.[16] This suggests that there is a difference between Compustat data and the flow-of-funds data used by Jermann and Quadrini (2006). The flow-of-funds series are net, so leveraged buyouts could be behind the negative correlation between equity issuance and debt issuance. Indeed, Baker and Wurgler (2000) argue that the merger waves in the 80s and 90s are quantitatively important for fluctuation in the flow-of-funds net equity and net debt series. A reduction in equity because of a leveraged buyout would not show up in our equity series.[17]

One could argue, however, that one should not clean the data for the effects of leveraged buyouts when trying to discover the cyclical behavior of debt and equity issuance. Although, leveraged buyouts did occur in concentrated waves, they did occur when economic conditions were very favorable, that is, one could argue that they are procyclical. Note, however, that although this question is important for the cyclicality of the aggregate series, it is not important for the cyclicality of the majority of firms since mainly the largest firms are affected by mergers.

2.2.4 Cyclical behavior of retained earnings

In Table 7, we report the cyclical behavior of retained earnings, profits, and dividends. We only report results for the flow approach.[18] Again there is a striking difference between the results for small and large firms. Whereas, retained earnings are procyclical and

[16] In the data appendix we consider alternative series and find one exception. Using the flow approach we find a negative correlation between net sale of stock and net long-term debt issues. As pointed out by Fama and French (2001), however, the net sale of stock measure does not deal correctly with reissues of stock. The measurement error works in the direction of making the series less procyclical.

[17] A reduction in equity obviously would not show up in the gross series. It also would not show up in the net series since a firm that disappears from the sample because of a merger is not used in the construction of the set of firm observations in that period. Finally, firms involved in major mergers (Compustat footnote code AB) are eliminated from the sample.

[18] The level approach takes the log of retained earnings. For firms in the group with the smallest firms retained earnings are persistently negative. This means that accumulated earnings at some point become negative and one cannot take the log anymore.

significant for large firms, they are countercyclical (but insignificant) for small firms. The countercyclicality for the bottom 25%, 50%, and 75% is due to firms in the bottom 25%. For firms between the 25^{th} and the 50^{th} percentile, the correlation is 0.20 with a t-statistic of 1.24. For firms between the 50^{th} and the 75^{th} percentile, the correlation is 0.29 and significant with a t-statistic of 2.56.

Retained earnings are equal to profits minus dividends. The cyclical behavior of profits mimics that of retained earnings, that is, countercyclical and insignificant for small firms but significantly procyclical for large firms.

When assets are used as the real activity measure, then both the countercyclical behavior of retained earnings and profits for small firms and the procyclical behavior of large firms become stronger. This suggests that expansions go together with lower profits for small firms, whereas this is not the case for large firms.

The correlation coefficients for dividends are typically positive and often significant. The correlation is stronger when GDP is used instead of assets, especially for firms in the bottom 25%. Thus, dividends typically increase during good times, but more so when good times are characterized as increases in overall activity then by increases in overall firm assets. This is to be expected, since the higher investments are likely to put pressure on dividends.

3 Static Model

In this section, we develop a one-period version of the model. The simplicity will be helpful in understanding the shortcomings of the standard debt contract such as dampening of shocks and procyclicality of the default rate. In this section we also make clear what determines the cyclical behavior of equity and why properties of the standard debt contract make equity issuance procyclical.

3.1 Debt contract

3.1.1 Description of firm financing problem

Technology is given by

$$\theta\omega k^{\alpha} + (1-\delta)k, \tag{4}$$

where k stands for the amount of capital, θ for the aggregate productivity shock (with $\theta > 0$), ω for the idiosyncratic productivity shock (with $\omega \geq 0$ and $\mathrm{E}(\omega) = 1$), and δ for the depreciation rate. The value of θ is known at the beginning of the period when the debt contract is written, but ω is only observed at the end of the period.

It is standard to assume that (i) agency problems are only present in the sector that produces investment commodities and (ii) that technology in this sector is linear, that is, $\alpha = 1$. The linearity assumption is convenient for computational reasons, since it means that agency costs do not depend on firm size and a representative firm can be used. Neither the underlying assumptions nor the result that firm size does not matter are appealing. Therefore, we use a standard nonlinear production function and agency problems are present in all sectors.[19]

The firm's net worth is equal to n and debt finance occurs through one-period contracts. That is, the borrower and lender agree on a debt amount, $(k-n)$, and a borrowing rate, r^b. The firm defaults if the resources in the firm are not enough to pay back the amount due. That is, the firm defaults if ω is less than the default threshold, $\overline{\omega}$, where $\overline{\omega}$ satisfies

$$\theta\overline{\omega}k^{\alpha} + (1-\delta)k = (1+r^b)(k-n). \tag{5}$$

If the firm defaults then the lender gets

$$\theta\omega k^{\alpha} + (1-\delta)k - \mu\theta k^{\alpha}, \tag{6}$$

where μ represent bankruptcy costs, which are assumed to be a fraction of expected

[19] Chari, Kehoe, and McGrattan (2006) show that financial frictions in the investment sector correspond to having "investment wedges" and they argue that these have played at best a minor role in several important economic downturns.

revenues.[20] In an economy with $\mu > 0$, default is inefficient and would not happen if the first-best solution could be implemented. Bankruptcy costs are assumed to be unavoidable, however, and the borrower and the lender cannot renegotiate the contract. The idea is that the situation in which firms do not have enough resources to pay the contractually agreed upon payments is like a distress state. It will involve, for example, loss of confidence, loss of sales, distress sales of assets, and loss of profits.[21]

Using (5), the firm's expected income can be written as

$$\theta k^\alpha F(\overline{\omega}) \text{ with } F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega d\Phi(\omega) - (1 - \Phi(\overline{\omega}))\overline{\omega}, \tag{7}$$

and the lender's expected revenues as

$$\theta k^\alpha G(\overline{\omega}) + (1-\delta)k \text{ with } G(\overline{\omega}) = 1 - F(\overline{\omega}) - \mu\Phi(\overline{\omega}), \tag{8}$$

where $\Phi(\omega)$ is the CDF of the idiosyncratic productivity shock, which we assume to be differentiable.

The values of $(k, \overline{\omega})$ are chosen to maximize the expected end-of-period firm income subject to the constraint that the lender must break even. Thus,

$$\begin{gathered} w(n;\theta) = \max_{k,\overline{\omega}} \min_{\zeta} \theta k^\alpha F(\overline{\omega}) + \zeta \left[\theta k^\alpha G(\overline{\omega}) + (1-\delta)k - (1+r)(k-n)\right] \\ \text{s.t. } \zeta \geq 0, \end{gathered} \tag{9}$$

where ζ is the Lagrange multiplier corresponding to the bank's break-even constraint. Rewriting the break-even condition for the bank gives

$$\frac{\theta k^\alpha G(\overline{\omega})}{\delta + r} = k - \frac{(1+r)n}{\delta + r}. \tag{10}$$

[20] The results in this section go through if bankruptcy costs are a fraction of actual output, $\theta\omega k^\alpha$, or a fraction of the interest payments.

[21] In the framework of Townsend (1979), bankruptcy costs are verification costs and debt is the optimal contract. It is not clear to us, however, that verification costs are large enough to induce quantitatively interesting agency problems. Indeed, Carlstrom and Fuerst (1997) include estimates for lost sales and lost profits and set μ equal to 0.25 in their calibration. Under this alternative interpretation of bankruptcy costs, debt would no longer be the optimal contract. Convenience and a long history of the use of debt financing, however, can explain the dominant role of debt finance.

This equation makes clear the role of the depreciation rate. A depreciation rate less than one allows the firm to leverage its net worth. That is, the lower the depreciation rate, the larger the share of available resources that is not subject to idiosyncratic risk. Because of this, the bank can lend out a positive amount, i.e., $k > n$, even if the firm always defaults, i.e., $\overline{\omega} = G(\overline{\omega}) = 0$.

For an interior solution, the optimal values for k and $\overline{\omega}$ satisfy the break-even condition of the bank (10) and the first-order condition

$$\frac{\alpha\theta k^{\alpha-1}F(\overline{\omega})}{\delta + r - \alpha\theta k^{\alpha-1}G(\overline{\omega})} = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})}. \tag{11}$$

The Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone and is always greater or equal to one. That is,

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1 - \mu\Phi'(\overline{\omega})/(1-\Phi(\overline{\omega}))} \geq 1. \tag{12}$$

3.1.2 Properties of the default rate

Assumption A

- *The maximization problem has an interior solution.*[22]
- *At the optimal value of $\overline{\omega}$ the CDF satisfies*

$$\frac{\partial\left(\Phi'(\overline{\omega})/(1-\Phi(\overline{\omega}))\right)}{\partial\overline{\omega}} > 0. \tag{13}$$

This inequality is a weak condition and is satisfied for numerous CDFs for any value of ω.[23] The following proposition characterizes the behavior of the default rate.

[22] This is not necessarily the case. For example, if aggregate productivity is low, depreciation is high, bankruptcy costs are high, and/or the CDF of ω has a lot of mass close to zero, then $k = n$ may be the optimal outcome.

[23] Such an assumption is standard in the literature. For example, Bernanke, Gertler, and Gilchrist (1999) assume that $\partial\left(\omega d\Phi(\omega)/(1-\Phi(\omega)\right)/\partial\omega > 0$, which would be the corresponding condition if bankruptcy costs are—as in Bernanke, Gertler, and Gilchrist (1999)—a fraction of actual (as opposed to expected) revenues.

Proposition 1 *Suppose that Assumption A holds. Then*

$$\begin{aligned}\frac{d\overline{\omega}}{dn} &= 0 \text{ when } \alpha = 1,\\ \frac{d\overline{\omega}}{dn} &< 0 \text{ when } \alpha < 1, \text{ and}\\ \frac{d\overline{\omega}}{d\theta} &> 0 \text{ when } n > 0.\end{aligned}$$

The proofs of the propositions are given in the appendix. The first two parts of the proposition say that an increase in the firm's net worth has no effect on the default rate when technology is linear, i.e., $\alpha = 1$, but reduces the default rate when technology exhibits diminishing returns, i.e., $\alpha < 1$. This is an interesting result since it makes clear that for the case considered in the literature, i.e., the case with $\alpha = 1$, an increase in net worth, which is the key variable of the net-worth channel, does not lead to a reduction in the default rate. The last part of the proposition says that an increase in aggregate productivity increases the default rate. That is, an increase in θ changes the firm's trade off between expansion (higher k) and less defaults in favor of expansion. More intuition is given in the appendix.

With $\alpha = 1$, an increase in θ, thus, leads to an increase in the default rate and any subsequent increase in net worth would leave have no effect on it. Consequently, with $\alpha = 1$ and without further modifications the dynamic version of the model generates a procyclical default rate, which is counterfactual. With $\alpha < 1$, the increase in n that follows an increase in θ has a downward effect on the default rate, but we never found this effect to be large enough to generate a countercyclical default rate in a model with only debt.

3.1.3 Dampening frictions

Cochrane (1994) argued that there are few external sources of randomness that are very volatile. The challenge for the literature is, thus, to build models in which small shocks can lead to substantial fluctuations. The debt contract has the unfortunate property that it dampens shocks. That is, the responses of real activity and capital in the model with the debt contract are actually less than the responses when there are no frictions in obtaining external finance. This is summarized in the following proposition. Let y be aggregate output and let y^{net} be aggregate output net of bankruptcy costs. Also, let $\widetilde{k}$ and $\widetilde{y}$ be the solution to capital and aggregate output in the model without frictions, respectively.

Proposition 2 *Suppose that $n > 0$ and Assumption A holds. Then*

$$\frac{d\ln k}{d\ln\theta} < \frac{d\ln\widetilde{k}}{d\ln\theta} = \frac{1}{1-\alpha}\,d\ln\theta,\ \textit{and} \tag{14}$$

$$\frac{d\ln y^{net}}{d\ln\theta} < \frac{d\ln y}{d\ln\theta} < \frac{d\ln\widetilde{y}}{d\ln\theta} = \frac{\alpha}{1-\alpha}\,d\ln\theta. \tag{15}$$

Important in understanding this proposition is that net worth, n, is fixed when aggregate productivity, θ, changes. For example, consider an enormous drop in θ. Now n is suddenly very large relative to θ, but this means that frictions no longer matter. The disappearance of the agency problem implies that the effect of the drop in θ is less. Key is, thus, that $n > 0$. The proof in the appendix makes clear that if $n = 0$,[24] that there is no such increase in n/θ when θ decreases and consequently the percentage change in capital and output is equal to that of the frictionless model if $n = 0$.

3.1.4 Tax advantage and optimal leverage

Applying the envelope condition to (9) gives

$$\frac{\partial w(n;\theta)}{\partial n} = \zeta(\overline{\omega})(1+r). \tag{16}$$

Equation (12) implies that the Lagrange multiplier, $\zeta(\overline{\omega})$, is strictly bigger than 1 as long as defaults are non-zero. Consequently, adding a unit of net worth to the firm increases end-of-period firm value by more than $1+r$ and firms have the incentive to drive the use of debt down by building up net worth. That is, in the model described so far there is no benefit of debt to balance bankruptcy costs.

The trade-off theory of corporate finance argues that the deductibility of interest payments provides such a benefit and leads to a (non-zero) optimal leverage ratio.[25] In the full

[24] Because $\alpha < 1$ the solution is well defined even if $n = 0$.

[25] Graham (2000) finds that the tax benefits of debt are, on average, equivalent to 10 percent of the value of the firm.

dynamic model, we assume that taxes are a fraction of corporate profits. Here, after-tax cash on hand is simply a fixed fraction of before-tax cash on hand. The advantage of this less realistic way to model taxes is that the problem is almost unchanged, except that the objective of the firm and the Lagrange multiplier are multiplied by $(1-\tau)$. The expression for the value of an extra unit of net worth (16) is now equal to

$$\frac{\partial w(n;\theta)}{\partial n} = \zeta(\overline{\omega})(1+r) = \frac{(1-\tau)(1+r)}{1-\mu\Phi'(\overline{\omega})/(1-\Phi(\overline{\omega}))}. \tag{17}$$

When the level of net worth, n, is equal to the frictionless level of capital, $\widetilde{k}$, and $\tau > 0$, then $\overline{\omega} = 0$ and $\zeta < 1$. Thus, for $n = \widetilde{k}$, the internal rate of return is less than $1+r$. When $n = 0$, the internal rate of return exceeds $1 + r$ as long as the tax rate is not too high. Continuity then implies that there is a level of net worth, n^*, with $n^* < \widetilde{k}$, such that the internal rate of return is equal to $1 + r$.

If the owner could attract external equity and transact at the market rate r, then the firm's net worth would always be equal to n^*. He would attract equity when $n < n^*$, i.e., when the internal rate of return exceeds r, and he would take money out of the firm when $n > n^*$, i.e., when the internal rate of return is less than r. In other words, the optimal leverage ratio is equal to $(k^* - n^*)/k^*$, where k^* is the optimal level of capital corresponding to $n = n^*$.[26]

3.2 Equity contract

The standard assumption is that firms can increase net worth only through retained earnings. This clearly is not consistent with the data, since firms do raise external funds through equity issuance. In this section, we modify the problem and allow the firm to attract external equity.

[26] Business cycle models that incorporate frictions typically assume that the discount rate of the entrepreneur exceeds the market interest rate. This also accomplishes that at some point the entrepreneur prefers to take funds out of the firm. Incorporating the tax advantage allows us to do this without relying on such an assumption that is hard to verify.

3.2.1 Costs of issuing equity

We follow Cooley and Quadrini (2001) and use a reduced-form approach and assume that equity costs are increasing with the amount of equity raised. Whereas Cooley and Quadrini (2001) assume that the cost of issuing equity is linear, we assume that these costs are quadratic, that is, $\lambda(e) = \lambda_0 e^2$ for $e > 0$.[27] Because of these costs, firms' net worth does not jump instantaneously to the optimal level, n^*. Instead, for any level $n < n^*$ some equity will be issued to reduce the gap. Since there are no costs to issue dividends, a firm can reduce its level of net worth to n^*.

Equity issuance costs in our model are like underwriting fees and it does not matter whether the current or the new owners pay them. Alternatively, one could interpret the equity issuance costs as a reduced-form representation for other types of costs associated with convincing others to become co-owners such as adverse selection. The question arises whether such an adverse selection problem should not affect the debt problem. To some extent it probably should and a framework that analyzes the effect of different frictions on different types of contracts would be a worthwhile exercise.

3.2.2 Description of the equity issuance problem

At the beginning of the period, the firm chooses equity, e, and debt issuance, $k - n = k - (e + x)$. A lender that buys equity (debt) does not obtain any information that is helpful in alleviating the friction of the debt (equity) contract. Recall that $w(n;\theta)$ is the expected end-of-period value of a firm that has net worth equal to n. The equity issuance decision is represented by the following maximization problem.

$$\begin{aligned} v(x;\theta) &= \max_{e,s}\ (1-s)\tfrac{w(x+e;\theta)}{1+r} \\ \text{s.t.}\quad e &= s\left(\tfrac{w(x+e;\theta)}{1+r}\right) - \lambda(e), \end{aligned} \tag{18}$$

where s is the ownership fraction the providers of new equity obtain in exchange for e. In this specification, it is assumed that the equity issuance costs are paid by the outside

[27] This avoids a nondifferentiability when zero equity is being issued. Jermann and Quadrini (2006) also assume a quadratic cost of issuing equity. Hansen and Torregrosa (1992) and Altinkiliç and Hansen (2000) show that underwriting fees do indeed display increasing marginal costs.

investor, but this is irrelevant.[28]

The expected rate of return for equity providers is equal to

$$\frac{\alpha w(x+e,\theta)-(e+\lambda(e))}{e+\lambda(e)}=\frac{(1+r)\,(e+\lambda(e))-(e+\lambda(e))}{e+\lambda(e)}=r.$$

Consequently, providers of equity financing obtain the same expected rate of return as debt providers.

The first-order condition for the equity issuance problem is given by

$$\frac{1}{1+r}\frac{\partial w(x+e;\theta)}{\partial e}=1+\frac{\partial\lambda(e)}{\partial e}. \tag{19}$$

That is, the marginal cost of issuing one unit of equity, $1+\partial\lambda/\partial e$, has to equal the expected benefit. Since $\partial\lambda/\partial e$ is equal to zero at $e=0$, the firm will issue equity whenever $\partial w/\partial e>1+r$. Since $\partial\lambda/\partial e>0$ for $e>0$, however, the firm does not increase equity up to the point where $\partial w/\partial e=1+r$.

3.2.3 Cyclicality of equity issuance

In this section, we address the question of how equity issuance responds to an increase in aggregate productivity. Clearly, when aggregate productivity is high, the need for external finance increases. This suggests that equity issuance should increase during a boom. But since another form of finance is possible, it may also be the case there is a substitution out of equity into debt. The following proposition shows that the latter is not the case in our model.[29]

Proposition 3 *Suppose that Assumption A holds. Then*

$$\frac{de}{d\theta}>0 \text{ for } n>0. \tag{20}$$

[28]Both equation (18) and the problem in which issuance costs are paid by the firm correspond to maximizing $w(x+e;\theta)/(1+r)-e-\lambda(e)$ with respect to e.

[29]Levy and Hennessy (2006) develop a model in which equity is procyclical and debt is countercyclical, whereas Jermann and Quadrini (2006) develop a model in which equity is countercyclical and debt is procyclical.

That is, when aggregate productivity increases firms that issue equity will issue more and firms that issue dividends ($e < 0$) will issue less dividends and possibly even issue equity. The result is driven by the result of Proposition 1 that the shadow price of external funds and the default probability are increasing with aggregate productivity (for a given value of net worth, $n = x + e$). Even though the firm could obtain more debt financing without additional equity, the rise in the default rate increases the Lagrange multiplier of the bank's break-even condition and increases, thus, the need for additional equity. Empirical evidence for this channel is provided by Gomes, Yaron, and Zhang (2006) who show that the shadow cost of external funds exhibits strong cyclical variation.

4 Dynamic Model

In this section, we first discuss the prototype dynamic model, which is a straightforward modification of the static model. Next, we discuss the benchmark model, which includes two additional features to generate procyclical equity issuance.

4.1 Prototype dynamic model

4.1.1 Technology

In addition to making firms forward looking, the dynamic prototype model has some features that are not present in the static model. All are related to technology. The first is the specification of the law of motion for productivity. Second, we introduce two minor changes in technology that are helpful in letting the model match some key statistics, such as leverage and the fraction of firms that pay dividends. In particular, we introduce stochastic depreciation and a small fixed cost.

Productivity. The law of motion for aggregate productivity, θ_t, is given by

$$\ln(\theta_{t+1}) = \ln(\bar{\theta})(1-\rho) + \rho \ln(\theta_t) + \sigma_\varepsilon \varepsilon_{t+1}, \tag{21}$$

where ε_t is an i.i.d. random variable with a standard Normal distribution.

Stochastic depreciation. For typical depreciation rates, firms only default for very low realizations of the idiosyncratic shock, because undepreciated capital provides a safety buffer. This generates high leverage. An important reason behind observed defaults is that the value of firm assets has deteriorated over time, for example, because the technology has become out of date. To capture this idea, we introduce stochastic depreciation, which makes it possible to generate a higher default probability while keeping the *average* depreciation rate unchanged. In particular, depreciation depends on the same idiosyncratic shock that affects production and is equal to

$$\delta(\omega_t) = \delta_0 \exp(\delta_1 \omega_t). \tag{22}$$

Fixed costs. For realistic tax rates, the model does not generate a high enough fraction of firms that pay out dividends. We introduce a small fixed cost, η, so that the model can match the observed fraction of dividend payers. Given the importance of internal funds, it is important to match data on funds being taken out of the firm.

4.1.2 Debt and equity contract

At the beginning of the period, θ_t and the amount of cash on hand, x_t, are known. After θ_t is observed each firm makes the dividend/equity decision and at the same time issues bonds. In the dynamic version, firms take into account the continuation value of the firm and maximize the expected end-of-period value of the firm instead of end-of-period cash on hand. The debt contract is, thus, given by

$$w(n_t;\theta_t) = \max_{k_t,\overline{\omega}_t,r_t^b} \mathrm{E}\left[\int_{\overline{\omega}_t}^{\infty} v(x_{t+1};\theta_{t+1})d\Phi(\omega) + \int_0^{\overline{\omega}_t} v(0;\theta_{t+1})d\Phi(\omega)|\theta_t\right] \tag{23}$$

s.t.

$$x_{t+1} = \theta_t\omega_t k_t^\alpha + (1-\delta(\omega_t))k_t - (1+r_t^b)(k_t-n_t) - \tau[\theta_t\omega_t k_t^\alpha - \delta(\omega_t)k_t - r_t^b(k_t-n_t)],$$

$$0 = \theta_t\overline{\omega}_t k_t^\alpha + (1-\delta(\overline{\omega}_t))k_t - (1+r_t^b)(k_t-n_t) - \tau[\theta_t\overline{\omega}_t k_t^\alpha - \delta(\overline{\omega}_t)k_t - r_t^b(k_t-n_t)],$$

$$\int_0^{\overline{\omega}_t} [\theta_t\omega_t k_t^\alpha + (1-\delta(\omega_t))k_t - \mu k_t^\alpha]d\Phi(\omega) + (1-\Phi(\overline{\omega}_t))(1+r_t^b)(k_t-n_t) = (1+r)(k_t-n_t).$$

Note that taxes are a constant fraction of taxable income, which is defined as operating profits net of depreciation and interest expense. Firms default when cash on hand is negative.[30] A firm that defaults on its debt is replaced by a new firm that starts with zero cash on hand. We also analyzed the model under the assumption that firms default when $v(x_{t+1}; \theta_{t+1}) < 0$. Since $v(0; \theta_{t+1}) > 0$, this means that firms only default when cash on hand is *sufficiently* negative. The results are very similar, but it is more difficult to solve the model.

The specification of the equity contract is still given by Equation (18), but $w(\cdot)$ is now given by Equation (23).

4.1.3 Household

A risk neutral household decides how much to invest in equity and corporate debt. Debt and equity investments have an expected return equal to the household's discount rate r, which is constant in the prototype model. The firm pays more, of course, because of the frictions in obtaining external finance and this premium is not constant.

Our model has heterogeneous firms. Without a constant risk free rate, solving the model would require keeping track of the cross-sectional distribution of firms' net worth levels. We have made no attempt to try to solve such a model. Algorithms to solve models with heterogeneous households (and homogenous firms) have only recently been developed and adding a cross-sectional distribution for our already quite complex setting would be quite a challenge.[31] Moreover, generating realistic pricing kernels would require a lot more than just making the household risk averse.[32] In the next section, we consider a modification of the model in which the required rate of return on equity varies according to an exogenously specified process. This has the disadvantage that the model is not a

[30] This would be the correct default cut off if firms can default and restart a firm with zero initial funds. This is consistent with our assumption of replacing a bankrupt firm with a new firm with zero internal funds.

[31] See den Haan (1996, 1997), Krusell and Smith (1997), and Algan, Allais, and den Haan (2006).

[32] Boldrin, Christiano, and Fisher (2001) are quite successful in replicating key asset price properties, but they use preferences that display habit formation, investment that is subject to adjustment costs, multiple sectors, and costs to move resources across sectors.

general equilibrium model. But it has the advantage that the model remains tractable and generates cyclical properties for the required rates of return of risky assets consistent with the data.

4.2 Benchmark Model

In the prototype model discussed so far, equity issuance is cyclical for the same reason that $\partial e/\partial\theta > 0$ in the static model. That is, the desire to expand when θ increases leads to an increase in the default rate, which increases the value of additional funds in the firm. In this section, we describe the benchmark model, which modifies the prototype model in two aspects. Both modifications provide reasons for equity issuance to be procyclical in addition to the reason identified with the prototype model.

A countercyclical price of risk. The risk premium on risky investments varies countercyclically.[33] This means that the end-of-period value of the firm in (18) should be discounted at a lower rate during good times, which in turn leads to an additional increase in the amount of equity being issued. To capture the cyclical variation in the required rate of return, we assume that equity providers discount firms' future payoffs with

$$M_t = \frac{\theta_t^\gamma}{1+r}. \tag{24}$$

Countercyclical issuance costs. One reason behind the issuance cost $\lambda(e)$ is the concern that a firm has an incentive to issue equity when it has private information that the firm is overvalued by the market. According to Choe, Masulis, and Nanda (1993), this concern is countercyclical. The idea the following. Firm value is affected by idiosyncratic and aggregate factors. The concern that the firm is exploiting private information is most likely to be related to the idiosyncratic component. Consequently, if aggregate conditions improve then the idiosyncratic component becomes less important and reduces the concern of investors to buy overvalued equity. To capture this mechanism, we allow the equity

[33] For empirical evidence on the countercyclical price of risk see Fama and French (1989), Schwert (1989), and Perez-Quiros and Timmermann (2000).

issuance cost to vary with aggregate productivity and set

$$\lambda(e_t;\theta_t) = \lambda_0 \theta_t^{-\lambda_1} e^2. \tag{25}$$

4.3 Results for the prototype model

This section reports results for the prototype version in which equity issuance cost, $\lambda(e)$, and the discount factor for firms' payouts do not vary over the business cycle. The parameters used are identical to the calibrated parameter values of the benchmark model discussed below, except that $\lambda_1 = \gamma = 0$.

Our model generates firm heterogeneity but not as much as that observed in the data. In particular, behavior of the larger firms is much more homogeneous than that observed in the data. One reason is that dividend paying firms reduce their net worth to the same optimal level and are, thus, ex-ante identical. These firms account for roughly half the firms in our artificial sample, hence we can summarize our findings with just three size classes. The results for the bottom tercile (small firms) and the top tercile (large firms), thus, give a good idea of the heterogeneity in our model economy.

For a typical firm in the bottom tercile, financial frictions are quantitatively important, and additional equity issuance helps in reducing them. In contrast, for a firm in the top tercile financial frictions may still be present, but they are less important. In particular, the tax advantage of debt often outweighs the remaining bankruptcy costs and dividends are, thus, important for firms in this category. In none of our simulations do we generate the highly skewed distribution of firm sizes that is observed in the data.

Figure 4 shows how output and the default rate respond to a one-standard-deviation positive shock to aggregate productivity. In addition to the responses for the prototype model, it also shows the responses for the frictionless model and the model with only debt as external finance.

The model without equity issuance. In the "only debt" model, the default rate is highly procyclical for firms in the bottom tercile. Even for firms in the top tercile, there is a small increase in the default rate. The counterfactual movement of the default rate is more important for small firms because agency problems are more important when net worth is small. In the model without equity issuance, small firms are, thus, less cyclical than large firms, which is also counterfactual.

Dampening in the different models. When we look at the output responses then we see that the differences between the different models are most pronounced for small firms. For example, in the model without equity issuance, output increases by less than output in the frictionless version of the model. In particular, the first period response of output in the “only debt" model is 15.3% less than the response in the frictionless version. In the model with equity issuance, the response of output is still less, but the first-period response is now only 6.6% less than the response in the frictionless model.

The model with equity issuance. In the prototype model, equity issuance increases in response to a positive productivity shock and the subsequent increase in net worth ensures that there is no longer a sharp increase in the default rate of small firms. Recall that the nonlinearity in the production function plays a key role, because with a linear production function the increase in net worth would have had no effect on the default rate. The inflow of external equity causes the first-period response of output for small firms in the prototype model to exceed the response in the "only debt" model by 10.2%. For large firms, the model even generates a small decrease in the default rate. The reason is that with positive tax rates, firms take funds out of the firm even when x_t is less than the frictionless level of capital, $\widetilde{k}_t$, namely when $x_t > n_t^*$. Even large firms, thus, use debt and face some (small) probability of default. When aggregate productivity increases, large firms issue less dividends and the higher net worth levels correspond with lower default rates. The effect is very small, however, since agency problems are not very important for large firms.

The default rate does not go down in the prototype model, unless the firm is very large and $x_t > n_t^*$. The reason is that equity increases exactly because the desire to expand leads to an increase in the default rate.

4.4 Calibration of the benchmark model

The model period is one year, which is consistent with the empirical analysis. For the discount factor, $\beta = (1+r)^{-1}$, the tax rate, τ, the persistence of the aggregate shock, ρ, and the curvature parameter in the production function, α, we use values that are standard in the literature. Its values, together with a reference source, are given in the top panel of Table 8. Note that the value of α is equal to 0.70, which is higher than the value of α used in models with labor, but is standard in models without labor.[34]

The other parameters are chosen to match some key first and second-order moments that our model should satisfy. The parameter values and the moments we target are given in the bottom panel of Table 8. Although, the parameters determine the values of the moments simultaneously, we indicate in the discussion below which parameter is most influential for a particular moment. In the table, this parameter is listed in the same row as the corresponding moment. The set of targeted first-order moments are the following.

- The ratio of investment to assets, which is pinned down by the parameter that controls average depreciation, δ_0.
- The fraction of firms that pay dividends, which is pinned down by the fixed cost, η. Note that the fixed cost affects profitability and, thus, the rate of return on internal funds. The fixed cost is equal to 17.1% of average aggregate output.
- The default rate, which is pinned down by the bankruptcy cost, μ. Bankruptcy costs are 15% of average output, which is slightly above the 12% used in Bernanke, Gertler, and Gilchrist (1999) and quite a bit below the estimate used in Carlstrom and Fuerst (1997).
- The default premium and leverage, which are pinned down by the volatility of the idiosyncratic shock, σ_ω, and the parameter that controls the volatility of deprecia-

[34] See, for example, Cooper and Ejarque (2003). It is easy to show that a problem in which technology is given by $k^{\alpha_k} l^{\alpha_l}$ and the wage is constant, is equivalent to a problem in which technology is given by k^{α} with $\alpha = \alpha_k / (1 - \alpha_l)$. When the original production function satisfies diminishing returns, for example, because of a fixed factor, then $\alpha < 1$.

tion, δ_1. The higher σ_ω and δ_1 the less certainty exists about the amount of available funds within the firm and the higher the premium on debt finance.

- Change in equity to assets, which is pinned down by the parameter that controls the cost of issuing equity, λ_0.

The set of targeted second-order moments are the following.

- The volatility of aggregate asset growth, which is pinned down by the standard deviation of the innovation to productivity, σ_ε.

- The volatility of change in equity, which is pinned down by the parameter that controls the variation in the cost of issuing equity, λ_1. The parameters λ_0 and λ_1 imply an average cost of equity issuance equal to 4.4% of equity raised. Kim, Palia, and Saunders (2005) report an average underwriting seasoned equity offering spread of 5.1% in the period between 1970 and 2004.

- The volatility of retained earnings, which is pinned down by the parameter that controls the variations in the price of risk, γ. In our model, the standard deviation of the required rate of return is equal to 0.16 percentage points.

The volatility of *equity issuance* and the volatility of retained earnings are controlled by the two features that distinguish the benchmark from the prototype model, i.e., the countercyclical variation in the cost of issuing equity and a countercyclical price of risk. Both increase the response of equity issuance to a positive productivity shock for firms that already issue equity. They differ, however, in how they affect firms that issue dividends and, thus, differ in how they affect retained earnings. For a firm that does not issue equity, a reduction in the cost of issuing equity has no direct effect. It still affects the firm indirectly, because it may be hit by some bad shocks in the future in which case equity finance does become relevant again. Since the firm is forward looking it would take this into account. In contrast, an increase in the discount factor does have a direct effect on firms that issue dividends. For a firm that issues dividends it must be the case that

$$M_t \left. \frac{\partial w(x+e)}{\partial e} \right|_{e=0} = \frac{\theta_t^{\gamma}}{1+r} \left. \frac{\partial w(x+e)}{\partial e} \right|_{e=0} < 1 + \left. \frac{\partial \lambda(e)}{\partial e} \right|_{e=0} = 1. \tag{26}$$

But an increase in the discount factor increases the left-hand side of the inequality. Consequently, an increase in the discount factor implies that the cut-off value for x_t at which $e_t = 0$ increases, i.e., at which firms issue neither dividends nor equity. This means that firms that issue dividends will issue less and some of them will even start issuing equity.

4.5 Results for the benchmark model

Figure 5 plots the impulse response functions for output and the default rate when aggregate productivity is hit by a positive one-standard-deviation shock. It also plots the responses in the prototype model. The figure shows that the model can generate a countercyclical default rate and that shocks are strongly magnified. In particular, the first-period response of output for small firms in the benchmark model is 84% higher than the response in the prototype model. Also, for aggregate output there is a considerable amount of magnification; the first-period response of output in the benchmark model is 45% higher than the response in the prototype model. The increase in equity issuance not only has a direct effect on output by increasing the amount of net worth, it also increases the amount of debt the firm can borrow and it reduces the default rate. For small firms, the average default rate drops by 118 basis points in the first period and continues to drop until it is 162 basis points below the pre-shock value in the third period. Even at the aggregate level is the drop in the default rate substantial. It drops by 39 basis points in the first period and the total reduction is 56 basis points.

The top panel of Figure 6 plots the responses of debt and equity for small firms, for large firms, and for the aggregate. The bottom panel of Figure 6 plots the responses of net worth for the three firm categories and plots at the aggregate level dividends and retained earnings. First, consider the responses for large firms. In the first period, net worth for large firms increases. The main reason is that the reduction in the price of risk induces dividend paying firms to issue less dividends, although there also is a small increase in equity issuance. The increase in retained earnings leads to an increase in debt financing.

Small firms respond to the positive productivity shock by sharply increasing equity. Debt also increases in the first couple years after the shock, but it increases by less than equity. After some time the impulse response function even turns negative. Even though debt is monotonically increasing in the aggregate shock, it is—except at low net worth levels—decreasing in net worth. Initially, the direct effect of the increase in productivity dominates and debt increases. After some time, the shift in the cross section towards larger firms implies a (small) reduction in debt levels relative to the preshock levels.

Table 9 reports the cross correlations between equity issuance and GDP, debt issuance and GDP, and debt and equity issuance for simulated and actual data. The coefficients have the same sign as their empirical counterpart. That is, both equity and debt issuance are procyclical. Correlation coefficients, however, are higher for the model. This is not very surprising, since we have only one aggregate shock in the model. The increase in average firm size following the productivity shock also explains that at some point the response of retained earnings becomes slightly negative and the response of dividends becomes slightly positive.

Finally, we present in Figure 7 the counterpart of the observed cyclical equity component plotted in Figure 1 and the observed cyclical debt component plotted in Figure 2 using long-term debt issuance and in Figure 3 using the change in total liablities. The top panel gives a typical simulation of equity issues for the bottom 25%, the bottom 50%, and the bottom 99%. As in Figure 1, equity issuance displays much larger cyclical swings for smaller firms. The bottom panel of Figure 7 plots the cyclical behavior of debt issuance for the same size classes. As in the data, the differences in debt issuance over the cycle across firm categories are smaller than for equity. In the simulated data, however, the cyclical movements for debt issues by small firms are noticeably larger, whereas in the data that was only observed in the seventies.

5 Concluding comments

The importance of external finance for aggregate cyclical fluctuations has been generally accepted. This paper adds to the empirical part of this literature by documenting the cyclical behavior of debt and equity issuance. We analyze the comovement using cyclical measures that are common in the macro literature and, more importantly, we analyze the cyclical behavior for different firm size classes. By disaggregating the data a much more robust set of results emerges when aggregate data are used, since the latter are heavily influenced by a very small set of very large firms.

The empirical results document that equity issuance is procyclical for most firms and highlight the need to use models that allow for both debt and equity finance. Exemplary papers that study environments in which both debt and equity are issued are Dewatripont and Tirole (1994), Myers (2000), and Hart (2001). These frameworks are quite abstract, however, and it would not be easy to incorporate them into a DSGE model. The equity issuance problem in this paper is much simpler, but the simplicity has several advantages. First, it made it clear that an important factor that determines whether equity issuance is pro or countercyclical is the cyclicality of the shadow price of external funds. We showed that for the standard debt contract, the shadow price is procyclical, which in turn leads to procyclical equity issuance. Second, the simplicity of the equity contract made it possible to numerically solve the model with heterogeneous firms. The numerical results document the quantitative importance of equity issuance for cyclical fluctuations and the ability of a calibrated model to replicate the cross-sectional findings of debt and equity issuance.

The simplicity also has its disadvantages. For example, using i.i.d. idiosyncratic shocks is very helpful in keeping the numerical analysis tractable. With persistent idiosyncratic shocks, however, a much richer cross-sectional distribution could be generated. In particular, one would have young firms with high idiosyncratic shocks, i.e., firms with still a low book value because of equity issuance costs but a high firm value. Such a model can explain the empirical finding that some firms with a large firm value issue equity.

Another simplification is that the model only allows for one-period debt contracts. With multi-period debt contracts, there is an additional reason why equity is procyclical. Equity issuance is a wealth transfer from the equity providers to the holders of long-term debt, since the additional equity reduces the probability of default. But this effect is likely to be less important during a boom since the probability of default is (should be) smaller.

Another weakness of the model that a richer framework might overcome is that leverage is decreasing in firm size, whereas in the data it is increasing. Note that a model in which leverage increases with firm size is likely to have a stronger net worth channel. If idiosyncratic risk is decreasing with firm size, then the model might generate leverage that is increasing with firm size.

A Proofs of propositions

Preliminaries. Before we give the proofs of the propositions, we give the formulas for the derivatives and present a lemma.

The first and second derivative of $F(\overline{\omega})$ are given by

$$\begin{aligned} F'(\overline{\omega}) &= -(1-\Phi(\overline{\omega})) \leq 0 \text{ and} \\ F''(\overline{\omega}) &= \Phi'(\overline{\omega}) \geq 0. \end{aligned}$$

The first and second derivatives of $G(\overline{\omega})$ are given by

$$\begin{aligned} G'(\overline{\omega}) &= -F'(\overline{\omega}) - \mu\Phi'(\overline{\omega}) \text{ and} \\ G''(\overline{\omega}) &= -F''(\overline{\omega}) - \mu\Phi''(\overline{\omega}). \end{aligned}$$

The sign of the *two* derivatives of $G(\overline{\omega})$ is not pinned down. For example, there are two opposing effects of an increase of $\overline{\omega}$ on $G(\overline{\omega})$. First, an increase in $\overline{\omega}$ reduces $F'(\overline{\omega})$, i.e. the share that goes to the *borrower*. This corresponds to an increase in lending rates and, thus, an increase in revenues from firms that do not default. Second, an increase in $\overline{\omega}$ implies an increase in bankruptcy costs. At the optimal value for $\overline{\omega}$, however, we know that $G'(\overline{\omega}) \leq 0$. If not then the bank could increase its own and firm profits by reducing $\overline{\omega}$. We summarize this result in the following lemma

Lemma 4 *At the optimal value of $\overline{\omega}$, $G'(\overline{\omega}) \leq 0$.*

To make the algebra a little bit less tedious we set without loss of generality $\delta = 1$ and $r = 0$.

Intuition for proposition 1. Both an increase in k and a reduction in $\overline{\omega}$ lead to an increase in firm profits and both lead to a reduction in bank profits at least around optimal choices for k and $\overline{\omega}$.[35] To satisfy the bank's break-even condition, the firm, thus, faces a trade off between a higher capital stock and a lower default rate.

If $\alpha = 1$, then the problem is linear and an increase in n simply means that the scale of the problem increases. Consequently, an increase in n does not affect the default rate but simply leads to a proportional increase *in* k. When $\alpha < 1$, the decreasing returns imply that an increase in k is not as attractive anymore and the firm will substitute part of the increase in k for a reduction in $\overline{\omega}$ when n increases.

Now consider what happens if aggregate productivity increases. For the firm, the relative benefit of a higher capital stock versus a lower default rate does not change.[36] An increase in θ means, however, that the break-even condition for the bank becomes steeper, that is, because the bank's revenues in case of default increase, capital becomes cheaper relative to $\overline{\omega}$. In other words, when aggregate productivity is high then this is a good time for the firm to expand even when it goes together with a higher default rate. In itself this may not be an implausible or undesirable outcome, but it would be if *it leads to procyclical default rates*, which is counterfactual. With $\alpha = 1$ that would indeed happen. With $\alpha < 1$ an increase in net worth reduces the default rate. Consequently, it is possible that subsequent increases in net worth through retained earnings (that would occur in the dynamic version of the model) would compensate for the upward pressure on the default rate caused by the increase in aggregate productivity. In our numerical experiments, however, we find that the direct effect of the increase in aggregate productivity is substantially stronger.

Proof of proposition 1. The result that $d\overline{\omega}/dn = 0$ when $\alpha = 1$ follows directly from the first-order condition (11). Now consider the case when $\alpha < 1$. Rewriting the

[35] At very low levels of k, the marginal product of capital is very high and bank profits may be increasing in k. Such low levels of k are clearly not optimal since an increase in k would then improve both firm and bank profits.

[36] That is, the iso-profit curve does not depend on aggregate productivity.

first-order condition gives

$$
\begin{aligned}
\frac{1}{\alpha\theta k^{\alpha-1}} &= -\frac{G'(\overline{\omega})}{F'(\overline{\omega})}F(\overline{\omega}) + G(\overline{\omega}) && (27) \\
&= \left(1 - \frac{\mu\Phi'(\overline{\omega})}{(1-\Phi(\overline{\omega}))}\right) F(\overline{\omega}) + G(\overline{\omega}) && (28)
\end{aligned}
$$

Assumption A together with Lemma 4 imply that the right-hand side is decreasing with $\overline{\omega}$. Suppose to the contrary that $d\overline{\omega}/dn > 0$. Then (28) implies that an increase in net worth must lead to a decrease in capital. But an increase in $\overline{\omega}$ and a decrease in k reduces expected firm profits and this can never be optimal because the old combination of $\overline{\omega}$ and k are still feasible when n increases. Similarly, $d\overline{\omega}/dn = 0$ is not optimal. According to Equation (28), it implies that $dk/dn = 0$, but the zero-profit condition of the bank makes an increase in k feasible. Consequently, $d\overline{\omega}/dn < 0$.

We now show that $\partial\overline{\omega}/\partial\theta > 0$. By combining Equations (10) and (11) one obtains the following expression that does not depend on θ.

$$
-\frac{G'(\overline{\omega})}{F'(\overline{\omega})}F(\overline{\omega}) = \left(\frac{1}{\alpha(1-\frac{n}{k})} - 1\right) G(\overline{\omega}).
$$

The regularity condition in Assumption A implies that the left-hand side is decreasing in $\overline{\omega}$ and according to Lemma 4 we know that the right-hand side is *decreasing* in $\overline{\omega}$. An increase in k lowers the right-hand side. Consequently, an increase in k has to go together with a *decrease* in $\overline{\omega}$. Clearly, a decrease in $\overline{\omega}$ and k would not be optimal because this would reduce profits while the old combination remains feasible if θ increases. ■

Proof of proposition 2. Let $\widetilde{k}$ be the solution of capital when there are no frictions. This capital stock is given by

$$
\widetilde{k} = \left(\frac{1}{\alpha\theta}\right)^{1/(\alpha-1)} \tag{29}
$$

This gives

$$
\frac{d\widetilde{k}}{\widetilde{k}} = \frac{1}{1-\alpha}\frac{d\theta}{\theta}
$$

From the break-even condition of the bank we get

$$k^{\alpha}G\left(\overline{\omega}\right)d\theta+\theta\alpha k^{\alpha-1}G\left(\overline{\omega}\right)dk+\theta k^{\alpha}G'\left(\overline{\omega}\right)d\overline{\omega}=dk. \tag{30}$$

Using the break-even condition, this can be written as

$$\frac{k-n}{\theta}d\theta+\alpha\frac{k-n}{k}dk+\frac{k-n}{G\left(\overline{\omega}\right)}G'\left(\overline{\omega}\right)d\overline{\omega}=dk \text{ or} \tag{31}$$

$$\frac{d\theta}{\theta}+\alpha\frac{dk}{k}+\frac{G'\left(\overline{\omega}\right)}{G\left(\overline{\omega}\right)}d\overline{\omega}=\frac{k}{k-n}\frac{dk}{k} \text{ or} \tag{32}$$

$$\frac{dk}{k}=\frac{\frac{d\theta}{\theta}+\frac{G'(\overline{\omega})}{G(\overline{\omega})}d\overline{\omega}}{\frac{k}{k-n}-\alpha} \tag{33}$$

First consider the case when n is equal to zero. The denominator is then equal to the denominator in the expression for the case without frictions. From proposition 1, we know that $\overline{\omega}$ does not respond to a change in aggregate productivity, i.e., $d\overline{\omega}=0$. Consequently, the percentage change in capital in the model with frictions is equal to the percentage change in the model without frictions. When $n>0$, there are two factors that push in opposite directions. The denominator is now larger than $1-\alpha$ which dampens the increase in capital relative to the increase in the frictionless model. The increase in $\overline{\omega}$, however, implies an increase in $G(\overline{\omega})$, which makes capital more responsive relative to the increase in the frictionless model We will now show that the first effect dominates. The first-order conditions are given by

$$\zeta(\overline{\omega}) = \frac{\alpha\theta k^{\alpha-1}F(\overline{\omega})}{1-\alpha\theta k^{\alpha-1}G(\overline{\omega})} \tag{34}$$

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})}=\frac{1}{1-\mu\Phi'(\overline{\omega})/(1-\Phi\left(\overline{\omega}\right))} \tag{35}$$

Let

$$X(k,\overline{\omega})=\alpha\theta k^{\alpha-1}. \tag{36}$$

From (34) we get

$$
\begin{aligned}
FdX + XF'd\overline{\omega} &= \zeta' d\overline{\omega} - X\zeta G' d\overline{\omega} - XG\zeta' d\overline{\omega} - \zeta G dX \\
(F + \zeta G)dX &= (1 - XG)\zeta' d\overline{\omega} + X(1 - \Phi - \zeta(1 - \Phi - \mu\Phi'))d\overline{\omega} \\
&= (1 - XG)\zeta' d\overline{\omega} + 0
\end{aligned}
$$

Assumption A and Equation (35) imply that $\zeta(\overline{\omega})$ is increasing in $\overline{\omega}$. Now, suppose to the contrary that in the model with frictions the percentage increase in capital is bigger than the increase in the model without frictions. Then it must be the case that $X(k, \overline{\omega})$ is decreasing, since $\alpha\theta k^{\alpha-1}$ is constant in the model with frictions. But this means that the right-hand side is negative and the left-hand side is positive, which cannot be true. ■

Proof of proposition 3. Key in proving this proposition is the first-order condition of the equity-issuance problem, Equation (19). Since equity issuance costs do not to depend on aggregate productivity, equity issuance de(in)creases in response to an increase in aggregate productivity, θ, when $\partial w/\partial e$ de(in)creases with θ. The marginal value of an extra unit of equity in the firm, $\partial w/\partial e$, is equal to $\zeta(\overline{\omega})(1 + r)$. From Equation (12) we know that the Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone. Moreover, the regularity condition in Assumption A, guarantees that $\zeta(\overline{\omega})$ is increasing in $\overline{\omega}$, which means that the marginal value of an extra unit of equity, $\partial w/\partial e$, is increasing in $\overline{\omega}$. Since $\overline{\omega}$ is increasing with aggregate productivity, $\partial w/\partial e$ is increasing in aggregate productivity, which means that equity issuance is increasing. Thus, an increase in θ increases the default rate, which increase the value of an extra unit of net worth in the firm, $\partial w/\partial e$, which increases equity issuance.■

B Data appendix

Output and deflator. Real GDP is defined as real gross domestic product, chained 2000 billions of dollars. The source is U.S. Department of Commerce, Bureau of Economic Analysis. The PPI is the producer price index for industrial commodities. The source is U.S. Department of Labor, Bureau of Labor Statistics.

Compustat. The Compustat data set consists of annual data from 1971 to 2004. It includes firms listed on the three U.S. exchanges (NYSE, AMEX, and Nasdaq) with a non-foreign incorporation code. We exclude financial firms (SIC codes 6000-6999), utilities (4900-4949) and firms involved in major mergers (Compustat footnote code AB) from the whole sample. We also exclude firms with a missing value for the book value of assets and firm-years that violate the accounting identity by more than 10% of the book value of assets. Finally, we eliminate the firms most affected by the accounting change in 1988, namely GM, GE, Ford and Chrysler (see Bernanke, Campbell, and Whited, 1990, for details). Assets, A, is the book value of assets (Compustat data item 6). Net change in total liabilities, ΔL, is the change in Compustat data item 181 between period t and $t-1$. Retained earnings, ΔRE, is the change in the balance sheet item for (accumulated) retained earnings (36). Change in the book value of equity, ΔE, equals the change in stockholders' equity (216) minus retained earnings. Sale of stock, ΔS, equals sale of common and preferred stock (108) and ΔD, equals issuance of long-term debt (111). Leverage, $\frac{L}{A}$, equals liabilities (181) divided by assets. Dividends equals dividends per share by ex-date (26) multiplied by the number of common shares outstanding (25). Operating income equals operating income before depreciation (13). Investment equals capital expenditures (30) plus advertising (45) plus research and development (46) plus acquisitions (129).

Default rate. The annual default rate is from Moody's (mnemonic USMDDAIW in Datastream) and it is for all corporate bonds in the US.

Robustness & extensions. This paper comes with an extensive data appendix. It can be downloaded from `http://www.bankofcanada.ca/ec/fcovas/cyclical.pdf`. It contains results for (i) additional definitions of equity and debt issuance, (ii) alternative data sources for aggregate debt and equity issues,[37], and (iii) an alternative methodology to adjust for possible cyclical changes in the composition of the firm groups. The results are robust to these alternatives.

The data appendix also provides some additional information on the time series be-

[37] Namely, the Federal Reserve Bulletin and the Flow of Funds.

havior of the top 1% and it documents that the default rate is countercyclical. Finally, the appendix also discusses the literature in more detail.

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Table 1: Summary statistics for different size classes

Size classes	**# of firms**	**% Assets**	$\mathbf{\frac{L}{A}}$	$\mathbf{\frac{\Delta A}{A}}$	$\mathbf{\frac{\Delta L}{\Delta A}}$	$\mathbf{\frac{\Delta E}{\Delta A}}$	$\mathbf{\frac{\Delta RE}{\Delta A}}$	$\mathbf{\frac{\Delta S}{\Delta A}}$	$\mathbf{\frac{\Delta D}{\Delta A}}$
$[0, 25\%]$	715	0.006	0.410	0.307	0.348	0.637	0.014	0.526	0.287
$[0, 50\%]$	1415	0.026	0.448	0.214	0.417	0.471	0.111	0.366	0.471
$[0, 75\%]$	2118	0.089	0.498	0.164	0.487	0.328	0.188	0.248	0.631
$[0, 99\%]$	2807	0.657	0.579	0.112	0.589	0.165	0.253	0.146	0.705
$[90\%, 95\%]$	144	0.132	0.586	0.109	0.611	0.129	0.263	0.122	0.717
$[95\%, 99\%]$	117	0.301	0.603	0.092	0.626	0.104	0.279	0.112	0.695
$[99\%, 100\%]$	29	0.343	0.601	0.079	0.630	0.091	0.284	0.116	0.531
All firms	2836	1	0.587	0.101	0.600	0.144	0.261	0.138	0.659

Notes: The data set consists of annual Compustat data from 1971 to 2004. Leverage, $\frac{L}{A}$, equals liabilities divided by assets. Asset growth, $\frac{\Delta A}{A}$, equals the change in the book value of assets from period $t-1$ to t divided by the current value of assets. Change in liabilities, ΔL equals the change in the book value of total liabilities. Change in equity, ΔE, equals the change in stockholders' equity minus retained earnings. Retained earnings, ΔRE, is the change in the balance sheet item for retained earnings. Sale of stock, ΔS, equals sale of common and preferred stock and, ΔD, is issuance of long-term debt. For further details on the data series used, see the data appendix.

Table 2: Cyclical behavior of equity issuance: level approach

Size classes	**Sale of stock and**			**Change in equity and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.02	0.24	**0.29**	0.03	0.26	**0.26**
	(-0.05)	(1.02)	(2.16)	(0.07)	(1.16)	(2.25)
$[0, 50\%]$	0.10	**0.33**	**0.31**	0.16	**0.32**	**0.23**
	(0.29)	(1.78)	(2.32)	(0.45)	(1.89)	(1.79)
$[0, 75\%]$	0.18	**0.35**	**0.30**	0.21	**0.28**	0.15
	(0.63)	(1.91)	(1.84)	(0.67)	(1.81)	(1.06)
$[0, 99\%]$	0.22	**0.36**	**0.33**	0.12	0.12	0.02
	(0.71)	(1.78)	(1.82)	(0.36)	(0.67)	(0.12)
$[90\%, 95\%]$	**0.42**	**0.45**	0.21	0.23	0.10	-0.12
	(2.59)	(5.45)	(1.61)	(0.75)	(0.62)	(-0.79)
$[95\%, 99\%]$	-0.03	0.12	**0.28**	-0.06	-0.09	-0.09
	(-0.07)	(0.49)	(2.48)	(-0.19)	(-0.47)	(-0.53)
$[99\%, 100\%]$	-0.26	**-0.43**	**-0.44**	-0.10	-0.36	**-0.42**
	(-0.93)	(-2.54)	(-3.94)	(-0.26)	(-1.53)	(-4.14)
All firms	0.12	0.20	0.16	0.04	-0.07	-0.15
	(0.34)	(0.83)	(0.93)	(0.12)	(-0.28)	(-1.17)

Size classes	**Sale of stock and**			**Change in equity and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.37**	**0.80**	**0.73**	**0.40**	**0.83**	**0.73**
	(4.31)	(7.95)	(7.67)	(3.95)	(9.30)	(5.52)
$[0, 50\%]$	**0.37**	**0.78**	**0.69**	**0.45**	**0.82**	**0.64**
	(2.81)	(7.28)	(4.16)	(3.09)	(9.73)	(2.44)
$[0, 75\%]$	**0.40**	**0.76**	**0.65**	**0.51**	**0.78**	0.55
	(2.59)	(7.04)	(2.86)	(2.82)	(9.45)	(1.57)
$[0, 99\%]$	0.23	0.47	**0.45**	0.47	**0.50**	0.23
	(0.69)	(1.39)	(1.89)	(1.19)	(2.28)	(1.32)
$[90\%, 95\%]$	0.24	**0.45**	**0.40**	0.45	**0.41**	0.10
	(1.11)	(3.10)	(2.88)	(1.54)	(2.39)	(0.59)
$[95\%, 99\%]$	0.12	0.25	0.20	0.43	0.37	-0.01
	(0.43)	(0.52)	(0.54)	(1.24)	(1.24)	(-0.06)
$[99\%, 100\%]$	**0.69**	**0.24**	**-0.23**	**0.80**	**0.62**	**0.05**
	(6.43)	(3.25)	(-2.50)	(8.76)	(4.66)	(0.37)
All firms	0.29	0.39	0.23	**0.59**	**0.51**	0.11
	(0.82)	(1.24)	(0.91)	(2.36)	(2.53)	(0.82)

Notes: All series are logged and HP filtered. For further details see the text and the data appendix. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997) and t-statistics are in parenthesis. The correlation coefficients statistically different from zero at the 5% significance level are highlighted in bold.

Table 3: Cyclical behavior of equity issuance: flow approach

Size classes	Sale of stock and			Change in equity and		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.11	0.13	0.20	-0.03	0.19	0.20
	(-0.42)	(0.50)	(1.20)	(-0.13)	(0.75)	(1.43)
$[0, 50\%]$	-0.10	0.15	**0.22**	0.03	0.23	**0.20**
	(-0.43)	(0.63)	(1.66)	(0.15)	(1.17)	(2.08)
$[0, 75\%]$	-0.12	0.13	**0.24**	0.07	0.22	**0.17**
	(-0.58)	(0.56)	(1.88)	(0.35)	(1.18)	(2.04)
$[0, 99\%]$	-0.21	0.05	0.30	0.06	0.15	0.10
	(-1.20)	(0.22)	(1.35)	(0.28)	(0.63)	(0.93)
$[90\%, 95\%]$	-0.07	**0.31**	**0.31**	0.18	**0.25**	**0.09**
	(-0.47)	(2.56)	(3.18)	(1.05)	(1.79)	(1.90)
$[95\%, 99\%]$	**-0.28**	-0.29	0.08	0.04	-0.06	-0.14
	(-1.81)	(-1.10)	(0.30)	(0.18)	(-0.22)	(-0.83)
$[99\%, 100\%]$	0.08	-0.13	-0.23	**0.32**	-0.08	**-0.23**
	(0.46)	(-0.90)	(-0.76)	(4.07)	(-0.51)	(-1.83)
All firms	-0.14	-0.00	0.17	0.17	0.07	-0.01
	(-0.74)	(-0.00)	(0.58)	(1.03)	(0.30)	(-0.08)
Size classes	**Sale of stock and**			**Change in equity and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.22**	**0.91**	**0.35**	**0.31**	**0.91**	**0.28**
	(5.96)	(13.66)	(6.38)	(7.96)	(14.91)	(4.85)
$[0, 50\%]$	**0.16**	**0.81**	**0.28**	**0.26**	**0.80**	**0.15**
	(1.72)	(6.39)	(5.04)	(3.63)	(8.02)	(3.04)
$[0, 75\%]$	0.07	**0.65**	**0.33**	0.21	**0.63**	0.12
	(0.38)	(3.90)	(3.38)	(1.13)	(4.69)	(1.49)
$[0, 99\%]$	-0.13	0.15	**0.34**	0.15	0.23	-0.06
	(-0.41)	(0.54)	(2.81)	(0.48)	(0.84)	(-0.61)
$[90\%, 95\%]$	-0.11	**0.35**	**0.29**	0.03	**0.28**	**-0.18**
	(-0.39)	(2.87)	(2.76)	(0.14)	(2.10)	(-1.66)
$[95\%, 99\%]$	-0.08	-0.18	0.09	**0.31**	0.16	**-0.28**
	(-0.93)	(-0.71)	(0.32)	(2.03)	(0.51)	(-3.63)
$[99\%, 100\%]$	0.33	-0.03	**-0.24**	**0.36**	**0.48**	**-0.39**
	(1.26)	(-0.16)	(-1.96)	(2.38)	(3.06)	(-5.30)
All firms	-0.01	0.07	0.08	0.28	0.28	**-0.25**
	(-0.04)	(0.21)	(0.41)	(1.02)	(1.15)	(-2.69)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details see the text and the data appendix. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997) and t-statistics are in parenthesis. The correlation coefficients statistically different from zero at the 5% significance level are highlighted in bold.

Table 4: Cyclical behavior of debt issuance: level approach

Size classes	LT debt issues and			Change in liabilities and		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	**0.27**	**0.30**	0.11	**0.49**	**0.44**	0.10
	(3.01)	(3.94)	(0.87)	(4.86)	(3.53)	(0.57)
$[0, 50\%]$	**0.29**	**0.30**	0.08	**0.62**	**0.49**	0.04
	(3.45)	(4.12)	(0.63)	(8.06)	(3.77)	(0.18)
$[0, 75\%]$	**0.38**	**0.35**	0.08	**0.69**	**0.52**	-0.00
	(5.08)	(4.03)	(0.65)	(9.28)	(3.60)	(-0.01)
$[0, 99\%]$	**0.50**	**0.31**	0.06	**0.84**	**0.43**	-0.15
	(3.84)	(2.07)	(0.50)	(21.86)	(3.04)	(-0.81)
$[90\%, 95\%]$	**0.51**	**0.36**	0.13	**0.81**	**0.50**	-0.04
	(3.38)	(2.16)	(1.30)	(20.53)	(4.53)	(-0.27)
$[95\%, 99\%]$	**0.47**	**0.19**	0.02	**0.78**	**0.26**	**-0.24**
	(2.34)	(1.28)	(0.17)	(12.48)	(1.65)	(-2.35)
$[99\%, 100\%]$	-0.05	-0.13	**-0.26**	**0.35**	-0.05	**-0.52**
	(-0.23)	(-0.82)	(-1.91)	(3.60)	(-0.44)	(-5.97)
All firms	**0.41**	**0.23**	-0.02	**0.71**	**0.26**	**-0.33**
	(3.36)	(1.77)	(-0.14)	(10.52)	(2.11)	(-2.43)
Size classes	**LT debt issues and**			**Change in liabilities and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.44**	**0.77**	**0.57**	**0.64**	**0.90**	**0.63**
	(3.82)	(7.68)	(7.61)	(7.06)	(21.44)	(13.30)
$[0, 50\%]$	**0.40**	**0.71**	**0.58**	**0.67**	**0.92**	**0.67**
	(3.81)	(6.40)	(7.71)	(6.86)	(33.65)	(13.28)
$[0, 75\%]$	**0.42**	**0.69**	**0.58**	**0.69**	**0.94**	**0.68**
	(5.14)	(8.67)	(11.70)	(8.33)	(67.39)	(9.25)
$[0, 99\%]$	**0.47**	**0.60**	**0.54**	**0.68**	**0.93**	**0.65**
	(5.06)	(7.74)	(5.25)	(9.77)	(61.14)	(9.28)
$[90\%, 95\%]$	**0.54**	**0.67**	**0.57**	**0.70**	**0.94**	**0.69**
	(4.99)	(7.68)	(6.49)	(8.64)	(59.81)	(12.98)
$[95\%, 99\%]$	**0.40**	**0.49**	**0.45**	**0.59**	**0.90**	**0.61**
	(2.56)	(4.31)	(3.69)	(8.93)	(38.83)	(4.19)
$[99\%, 100\%]$	**0.29**	**0.26**	0.11	**0.70**	**0.94**	**0.62**
	(2.93)	(2.02)	(0.49)	(11.18)	(78.84)	(10.04)
All firms	**0.35**	**0.55**	**0.54**	**0.68**	**0.93**	**0.67**
	(4.06)	(8.93)	(7.47)	(12.12)	(65.71)	(10.14)

Notes: All series are logged and HP filtered. For further details see the text and the data appendix. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997) and t-statistics are in parenthesis. The correlation coefficients statistically different from zero at the 5% significance level are highlighted in bold.

Table 5: Cyclical behavior of debt issuance: flow approach

Size classes	**LT debt issues and**			**Change in liabilities and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	0.10	**0.45**	0.29	0.19	**0.56**	0.27
	(0.48)	(6.57)	(1.16)	(1.13)	(6.54)	(0.96)
$[0, 50\%]$	0.17	**0.53**	**0.30**	**0.21**	**0.62**	**0.24**
	(1.11)	(4.74)	(2.40)	(2.13)	(12.09)	(1.80)
$[0, 75\%]$	0.24	**0.59**	**0.40**	**0.25**	**0.69**	**0.27**
	(1.59)	(6.62)	(2.90)	(3.56)	(18.31)	(1.98)
$[0, 99\%]$	**0.52**	**0.44**	0.36	**0.54**	**0.74**	0.24
	(5.75)	(1.91)	(1.09)	(7.21)	(11.53)	(0.88)
$[90\%, 95\%]$	**0.40**	**0.39**	0.36	**0.44**	**0.74**	0.35
	(5.21)	(1.78)	(1.24)	(5.09)	(29.00)	(1.20)
$[95\%, 99\%]$	**0.47**	0.20	0.21	**0.66**	**0.61**	0.11
	(3.81)	(0.59)	(0.81)	(9.53)	(4.14)	(0.34)
$[99\%, 100\%]$	0.18	0.02	-0.13	**0.57**	**0.56**	0.02
	(1.15)	(0.12)	(-1.58)	(10.70)	(9.40)	(0.10)
All firms	**0.52**	**0.40**	0.29	**0.60**	**0.73**	0.16
	(6.05)	(1.97)	(1.01)	(12.29)	(10.60)	(0.67)

Size classes	**LT debt issues and**			**Change in liabilities and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.23**	**0.41**	0.26	**0.26**	**0.63**	**0.31**
	(1.93)	(2.70)	(1.62)	(2.59)	(13.29)	(1.89)
$[0, 50\%]$	**0.34**	**0.53**	**0.25**	**0.29**	**0.76**	**0.24**
	(4.83)	(2.92)	(3.03)	(3.34)	(11.16)	(1.95)
$[0, 75\%]$	**0.33**	**0.65**	**0.33**	**0.32**	**0.88**	**0.25**
	(4.81)	(5.93)	(3.94)	(4.21)	(20.44)	(2.04)
$[0, 99\%]$	**0.59**	**0.39**	0.26	**0.39**	**0.94**	**0.34**
	(7.99)	(3.57)	(1.29)	(5.94)	(33.45)	(2.37)
$[90\%, 95\%]$	**0.60**	**0.39**	0.19	**0.35**	**0.94**	**0.32**
	(6.57)	(2.55)	(1.13)	(3.81)	(36.70)	(2.07)
$[95\%, 99\%]$	**0.56**	**0.24**	0.05	**0.43**	**0.90**	**0.38**
	(4.92)	(1.95)	(0.30)	(13.90)	(31.19)	(3.18)
$[99\%, 100\%]$	**0.30**	0.05	-0.06	0.15	**0.94**	**0.18**
	(2.31)	(0.70)	(-0.48)	(1.57)	(91.58)	(1.71)
All firms	**0.52**	**0.35**	0.28	**0.31**	**0.94**	**0.32**
	(8.93)	(3.10)	(1.53)	(4.29)	(34.89)	(2.02)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details see the text and the data appendix. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997) and t-statistics are in parenthesis. The correlation coefficients statistically different from zero at the 5% significance level are highlighted in bold.

Table 6: Comovement between debt and equity

	Level Approach						Flow Approach					
Size classes	**Sale of stock and LT debt issues**			**Change in equity and change in liab.**			**Sale of stock and LT debt issues**			**Change in equity and change in liab.**		
	ΔD_{t-1}	ΔD_t	ΔD_{t+1}	ΔL_{t-1}	ΔL_t	ΔL_{t+1}	ΔD_{t-1}	ΔD_t	ΔD_{t+1}	ΔL_{t-1}	ΔL_t	ΔL_{t+1}
$[0, 25\%]$	0.10	**0.39**	**0.52**	0.20	**0.56**	**0.64**	0.23	0.05	0.12	0.28	**0.28**	0.12
	(1.15)	(4.46)	(4.78)	(1.41)	(4.78)	(6.26)	(0.97)	(0.85)	(1.16)	(1.37)	(3.14)	(1.41)
$[0, 50\%]$	0.11	**0.39**	**0.49**	**0.31**	**0.58**	**0.57**	0.08	0.05	**0.23**	0.15	**0.23**	0.10
	(1.24)	(3.81)	(4.33)	(1.80)	(7.08)	(3.47)	(0.49)	(0.86)	(3.29)	(0.61)	(2.54)	(0.98)
$[0, 75\%]$	0.08	**0.40**	**0.51**	**0.39**	**0.56**	**0.46**	0.05	**0.18**	**0.30**	0.12	**0.22**	0.07
	(0.71)	(3.30)	(4.22)	(1.83)	(5.60)	(2.22)	(0.24)	(2.12)	(3.15)	(0.48)	(2.22)	(0.65)
$[0, 99\%]$	-0.02	0.25	**0.35**	0.22	0.25	0.13	0.29	0.19	0.11	0.09	-0.01	-0.00
	(-0.06)	(0.75)	(3.79)	(0.48)	(0.74)	(0.93)	(1.60)	(1.13)	(0.86)	(0.35)	(-0.06)	(-0.02)
$[90\%, 95\%]$	0.27	**0.40**	**0.53**	0.30	0.27	0.06	0.23	0.24	**0.23**	0.09	0.13	-0.08
	(1.57)	(3.58)	(3.19)	(0.91)	(1.13)	(0.77)	(1.09)	(1.57)	(1.78)	(0.45)	(1.53)	(-1.06)
$[95\%, 99\%]$	-0.09	0.00	-0.02	0.12	0.07	-0.14	**0.37**	0.19	-0.24	0.22	-0.07	-0.20
	(-0.17)	(0.00)	(-0.10)	(0.25)	(0.15)	(-0.53)	(2.11)	(1.04)	(-0.77)	(1.61)	(-0.26)	(-1.48)
$[99\%, 100\%]$	0.13	**0.26**	-0.02	**0.69**	**0.55**	0.10	**0.20**	**0.58**	0.06	**0.34**	**0.34**	**-0.32**
	(1.10)	(1.96)	(-0.13)	(4.71)	(2.94)	(0.71)	(2.84)	(11.06)	(0.54)	(2.45)	(2.51)	(-4.96)
All firms	-0.08	0.14	0.11	0.37	0.29	0.01	0.26	0.21	0.00	0.20	0.07	**-0.18**
	(-0.19)	(0.34)	(0.62)	(1.05)	(0.89)	(0.06)	(1.43)	(1.30)	(0.01)	(0.83)	(0.33)	(-1.71)

Notes: For the level approach all series are logged and HP filtered. For the flow approach real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details see the text and the data appendix. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997) and t-statistics are in parenthesis. The correlation coefficients statistically different from zero at the 5% significance level are highlighted in bold.

Table 7: Cyclical behavior of retained earnings, profits and dividends: flow approach

Size classes	Retained earnings and			Profits and			Dividends and		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.15	-0.17	**-0.25**	-0.11	-0.17	**-0.31**	**0.59**	**0.47**	-0.11
	(-1.02)	(-0.59)	(-2.17)	(-0.60)	(-0.62)	(-3.06)	(5.95)	(3.58)	(-0.56)
$[0, 50\%]$	-0.18	0.03	-0.02	-0.17	0.01	-0.08	**0.31**	-0.03	-0.21
	(-0.73)	(0.10)	(-0.17)	(-0.61)	(0.03)	(-0.94)	(3.51)	(-0.10)	(-1.49)
$[0, 75\%]$	-0.16	0.18	0.08	-0.15	0.24	**0.13**	**0.36**	0.28	0.05
	(-0.69)	(0.69)	(1.29)	(-0.55)	(0.91)	(2.85)	(3.10)	(1.26)	(0.30)
$[0, 99\%]$	0.09	**0.46**	**0.17**	0.08	**0.58**	**0.27**	**0.13**	**0.28**	**0.38**
	(0.41)	(3.41)	(2.18)	(0.39)	(4.91)	(2.84)	(2.01)	(3.27)	(7.01)
$[90\%, 95\%]$	0.03	**0.39**	0.12	0.10	**0.55**	**0.29**	**0.17**	**0.22**	**0.29**
	(0.11)	(2.05)	(1.45)	(0.42)	(4.03)	(3.53)	(3.88)	(3.81)	(5.33)
$[95\%, 99\%]$	0.16	**0.48**	**0.18**	0.10	**0.60**	**0.27**	-0.10	**0.19**	**0.45**
	(1.23)	(7.61)	(2.33)	(0.90)	(6.26)	(2.63)	(-1.18)	(2.10)	(5.24)
$[99\%, 100\%]$	0.33	**0.38**	0.10	0.17	**0.39**	0.07	0.09	0.19	0.23
	(1.05)	(4.79)	(0.38)	(0.88)	(4.01)	(0.36)	(0.52)	(1.39)	(1.10)
All firms	0.22	**0.46**	0.14	0.12	**0.53**	**0.20**	0.14	**0.30**	**0.39**
	(0.80)	(4.04)	(1.19)	(0.59)	(5.12)	(1.73)	(1.23)	(3.03)	(5.62)

Size classes	Retained earnings and			Profits and			Dividends and		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	-0.30	**-0.60**	**-0.26**	-0.26	**-0.57**	**-0.30**	0.04	0.05	**-0.23**
	(-1.03)	(-2.22)	(-5.49)	(-0.75)	(-1.92)	(-6.15)	(0.29)	(0.19)	(-3.60)
$[0, 50\%]$	**-0.37**	-0.20	0.12	-0.34	-0.17	0.11	0.06	0.06	**0.13**
	(-1.68)	(-0.66)	(1.20)	(-1.19)	(-0.58)	(1.31)	(0.55)	(0.26)	(3.12)
$[0, 75\%]$	-0.22	0.10	**0.26**	-0.24	0.21	**0.34**	0.01	0.12	0.16
	(-1.18)	(0.29)	(1.86)	(-0.93)	(0.62)	(2.69)	(0.08)	(0.44)	(0.72)
$[0, 99\%]$	0.02	**0.71**	**0.37**	-0.02	**0.77**	**0.48**	-0.00	**0.19**	**0.39**
	(0.12)	(13.94)	(7.98)	(-0.13)	(10.35)	(9.90)	(-0.01)	(2.36)	(4.96)
$[90\%, 95\%]$	-0.00	**0.60**	**0.24**	0.03	**0.71**	**0.44**	0.15	**0.17**	**0.34**
	(-0.01)	(4.65)	(4.91)	(0.21)	(8.68)	(12.58)	(1.47)	(2.19)	(7.96)
$[95\%, 99\%]$	0.09	**0.77**	**0.53**	-0.01	**0.77**	**0.59**	**-0.14**	0.09	**0.41**
	(0.58)	(30.97)	(11.14)	(-0.05)	(15.72)	(8.02)	(-2.04)	(0.89)	(5.27)
$[99\%, 100\%]$	-0.00	**0.71**	**0.39**	0.03	**0.65**	**0.29**	-0.02	0.21	**0.34**
	(-0.04)	(10.73)	(5.58)	(0.33)	(5.74)	(5.46)	(-0.07)	(1.62)	(4.11)
All firms	0.06	**0.75**	**0.43**	-0.02	**0.77**	**0.47**	-0.12	**0.20**	**0.48**
	(0.32)	(13.04)	(6.61)	(-0.16)	(9.63)	(11.73)	(-0.62)	(2.17)	(9.86)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details see the text and the data appendix. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997) and t-statistics are in parenthesis. The correlation coefficients statistically different from zero at the 5% significance level are highlighted in bold.

Table 8: Calibration

Parameter		Source
β	1.022^{-1}	Zhang (2005)
α	0.70	Cooper and Ejarque (2003)
τ	0.296	Graham (2000)
ρ	0.95^4	Cooley and Hansen (1995)

Parameter		Moment	Data	Model
σ_ϵ	0.0074	Volatility of asset growth	0.039	0.037
σ_ω	0.31	Default premium	119bp	105bp
δ_0	0.082	Investment to assets	0.133	0.134
δ_1	-2.72	Leverage	0.587	0.532
η	0.0975	Fraction of dividend payers	0.469	0.429
μ	0.15	Default rate	0.022	0.020
λ_0	0.30	Change in equity to assets	0.015	0.011
λ_1	125	Volatility of change in equity	0.254	0.221
γ	0.138	Volatility of retained earnings	0.342	0.397

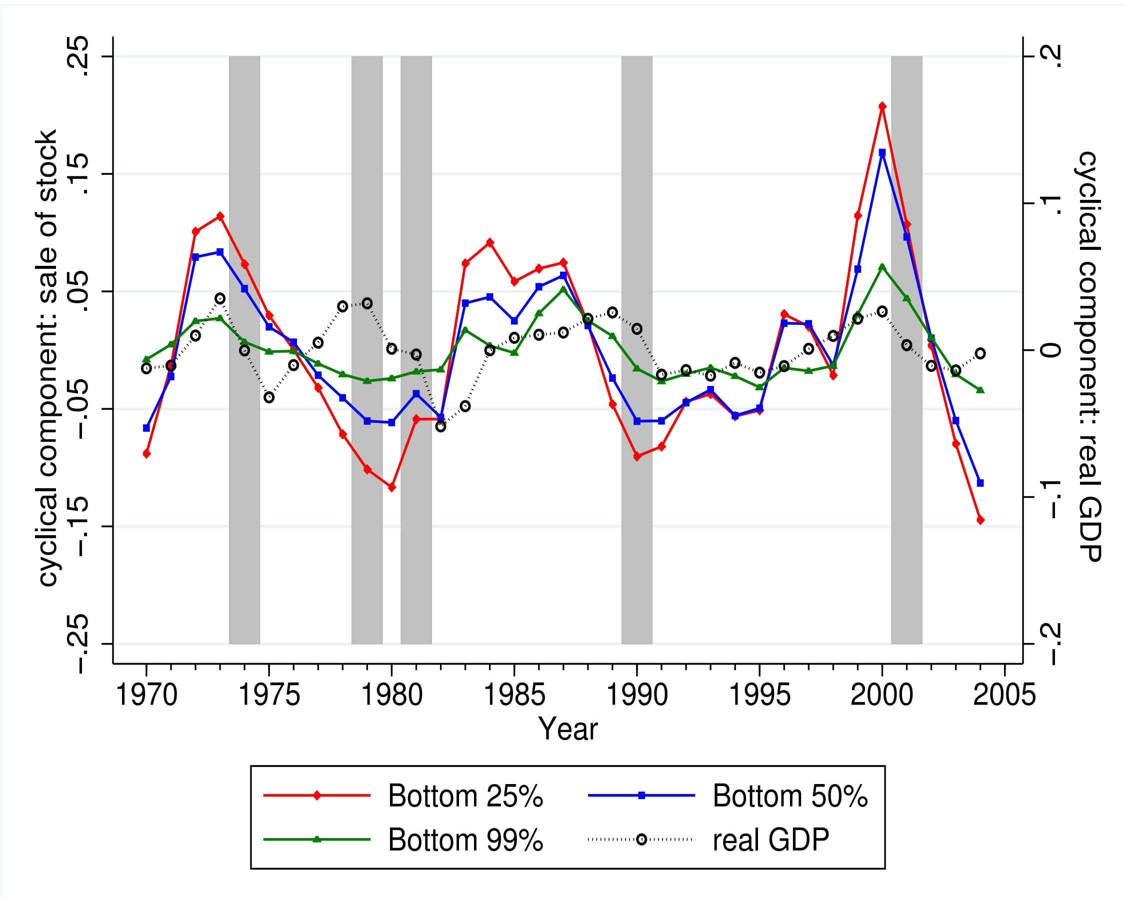
Notes on the model: The parameter β is the discount factor, α the curvature of technology, τ the tax rate and ρ is the persistence of the aggregate shock. The parameter σ_ϵ is the standard deviation of the aggregate shock, σ_ω is the standard deviation of the idiosyncratic shock, δ_0 is the depreciation rate, and δ_1 the stochastic depreciation parameter. The parameter η is the fixed cost, μ is the bankruptcy cost, and λ_0 the equity issuance cost. Finally, λ_1 controls the time-varying cost of equity and γ the variability of the firm's discount factor. The moments in the model are obtained by simulating an economy with 5000 firms for 5000 periods and discarding the first 500 observations. **Notes on the data:** Asset growth is the growth rate of the book value of assets. The default premium is the estimated default spread on corporate bonds taken from Longstaff, Mithal, and Neis (2005). Investment includes capital expenditures, advertising, research and development and acquisitions. Leverage equals liabilities divided by the book value of assets. Dividends is dividends per share by ex-date multiplied by the number of common shares outstanding. Change in equity equals the change in stockholders' equity minus retained earnings. The default rate is the average of annual default rates for all corporate bonds. Finally, retained earnings is the change in the balance sheet item for (accumulated) retained earnings. The volatility of asset growth, change in equity and change in liabilities are from the flow approach. The latter are expressed as a fraction of the volatility of asset growth. The sample period is from 1971 until 2004, except for the default rate series, which is from the period between 1986 until 2004. For further details on the data series used, see the data appendix.

Table 9: Cyclical behavior of debt and equity in the model

Size classes	**Data**			**Model**		
	Equity issues and GDP					
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
Bottom tercile	-0.04	0.19	0.19	0.12	0.79	0.43
Top tercile	0.19	0.001	-0.10	-0.03	0.28	0.15
All firms	0.17	0.07	-0.01	0.09	0.75	0.41
	Debt issues and GDP					
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
Bottom tercile	0.20	0.61	0.24	-0.10	0.69	0.48
Top tercile	0.60	0.70	0.12	0.04	0.23	0.15
All firms	0.60	0.73	0.16	-0.09	0.66	0.45
	Debt and Equity issues					
	E_{t-1}	E_t	E_{t+1}	E_{t-1}	E_t	E_{t+1}
Bottom tercile	0.10	0.21	0.14	-0.02	0.65	0.40
Top tercile	0.24	0.04	-0.27	-0.15	-0.07	0.15
All firms	0.20	0.07	-0.18	-0.04	0.55	-0.38

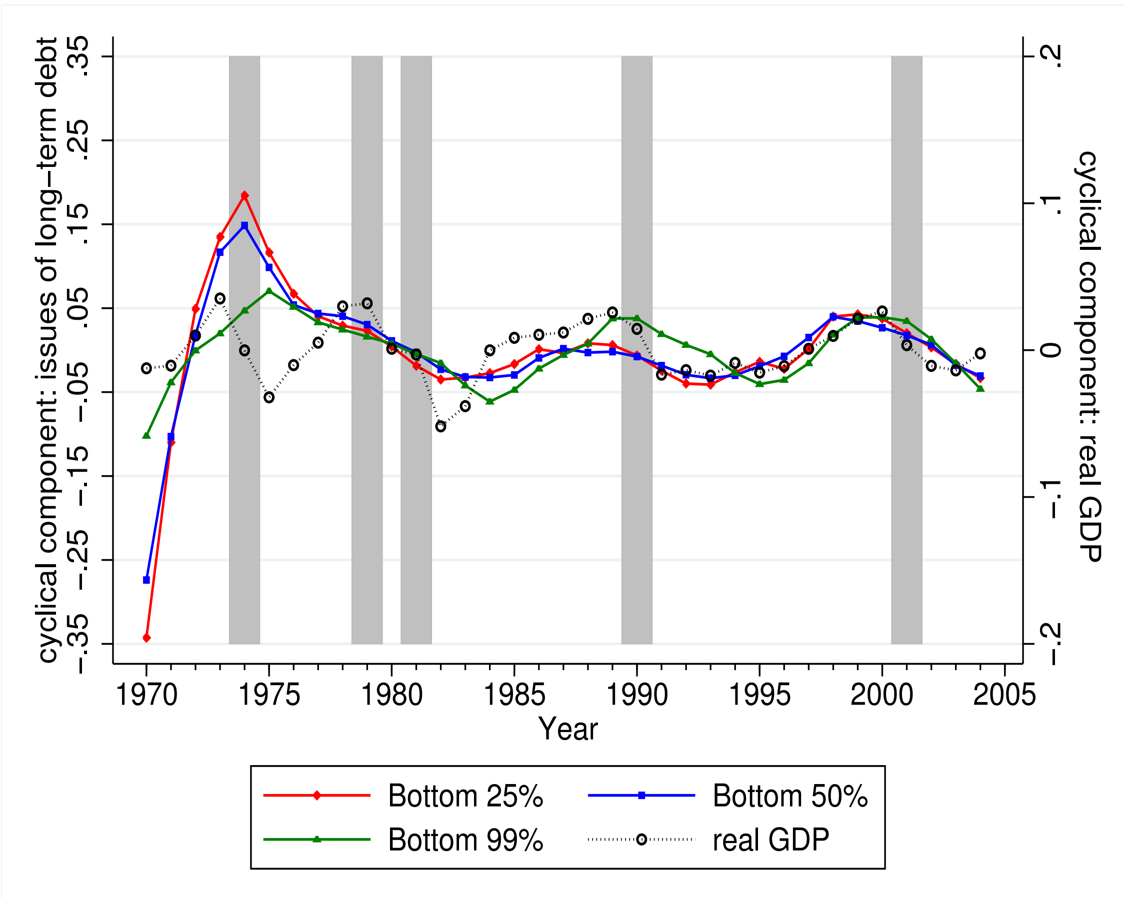
Notes: For the data the series selected are change in equity and change in liabilities following the flow approach. For the model we looked at the average of equity, e_t and debt, $(k_t - n_t)$ for the three different size classes.

Figure 1: Cyclical behavior of sale of stock for different size classes



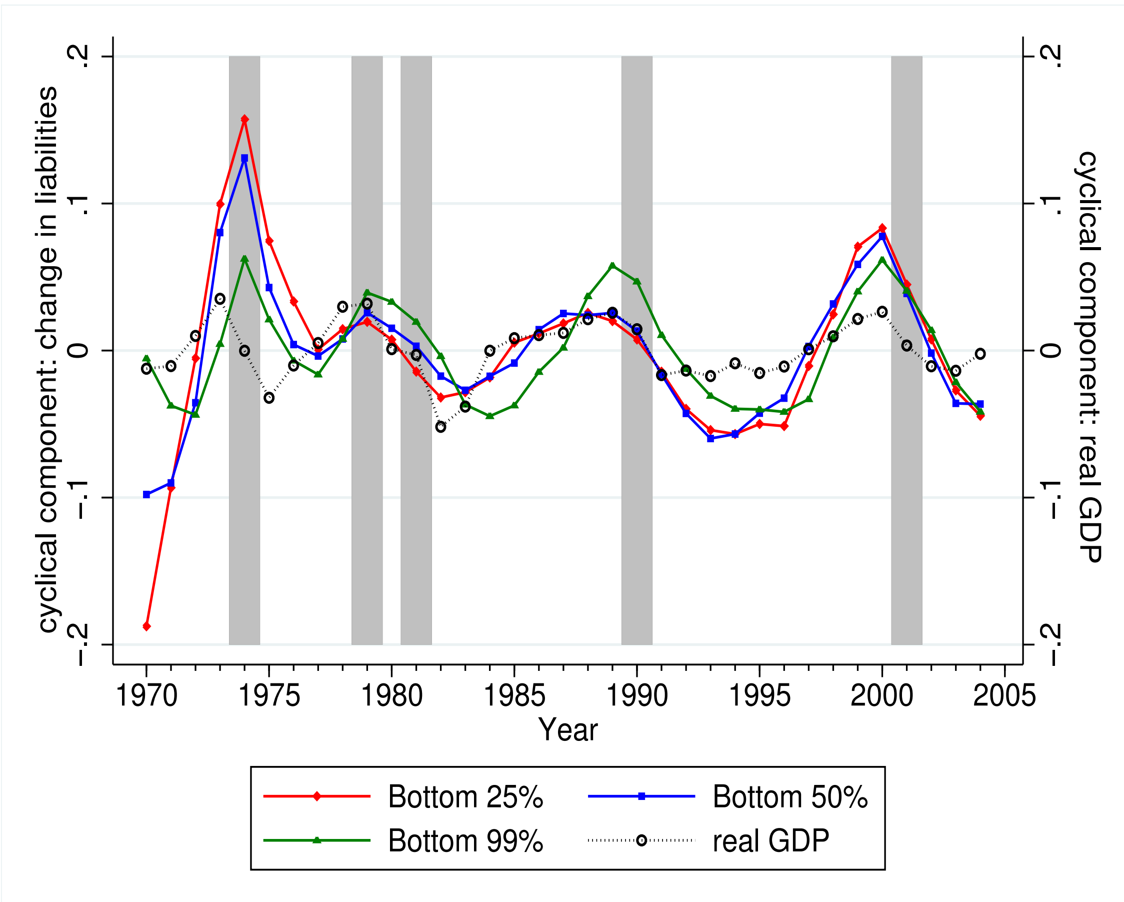
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 2: Cyclical behavior of issuance of long-term debt for different size classes



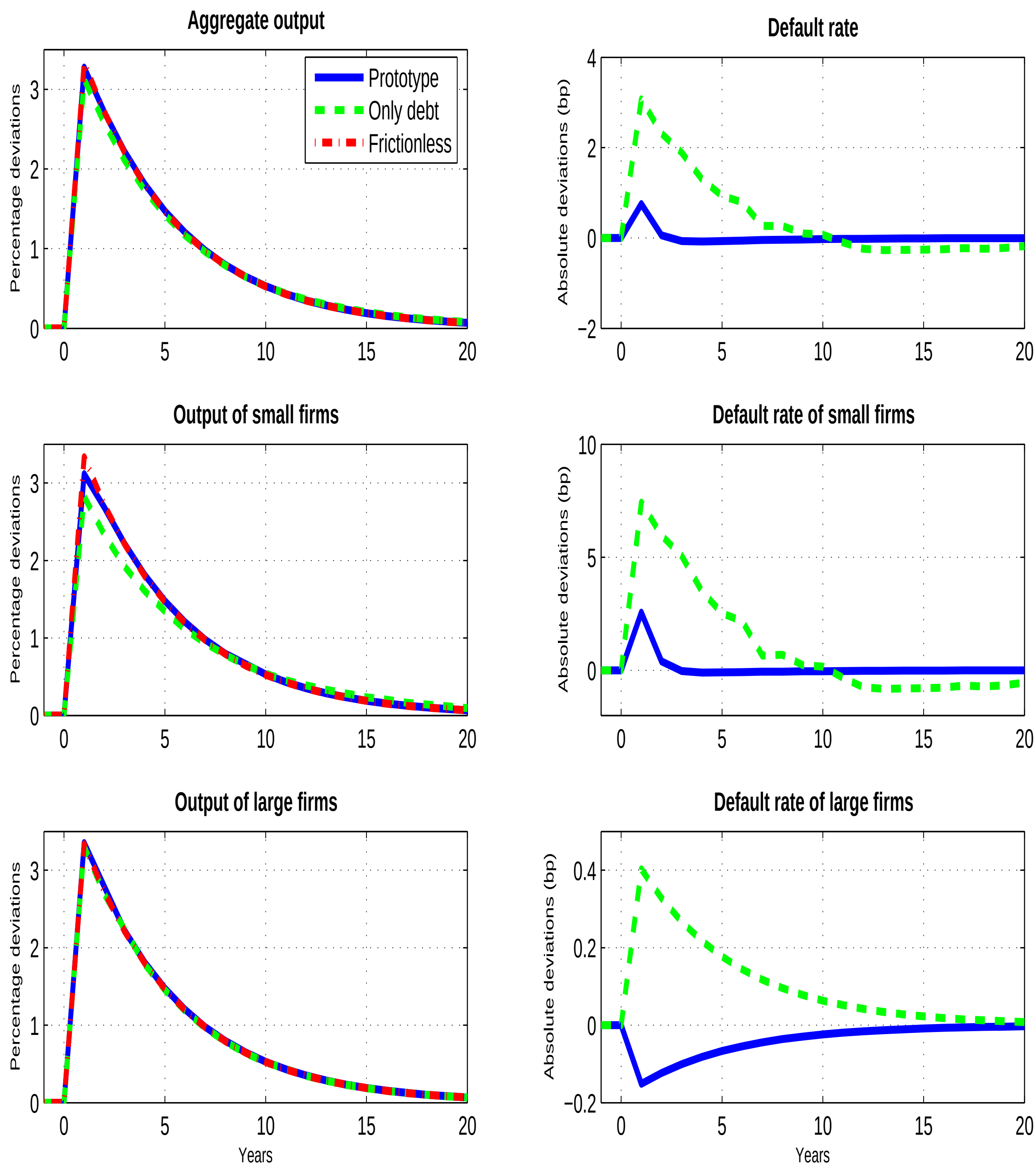
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 3: Cyclical behavior of change in liabilities for different size classes



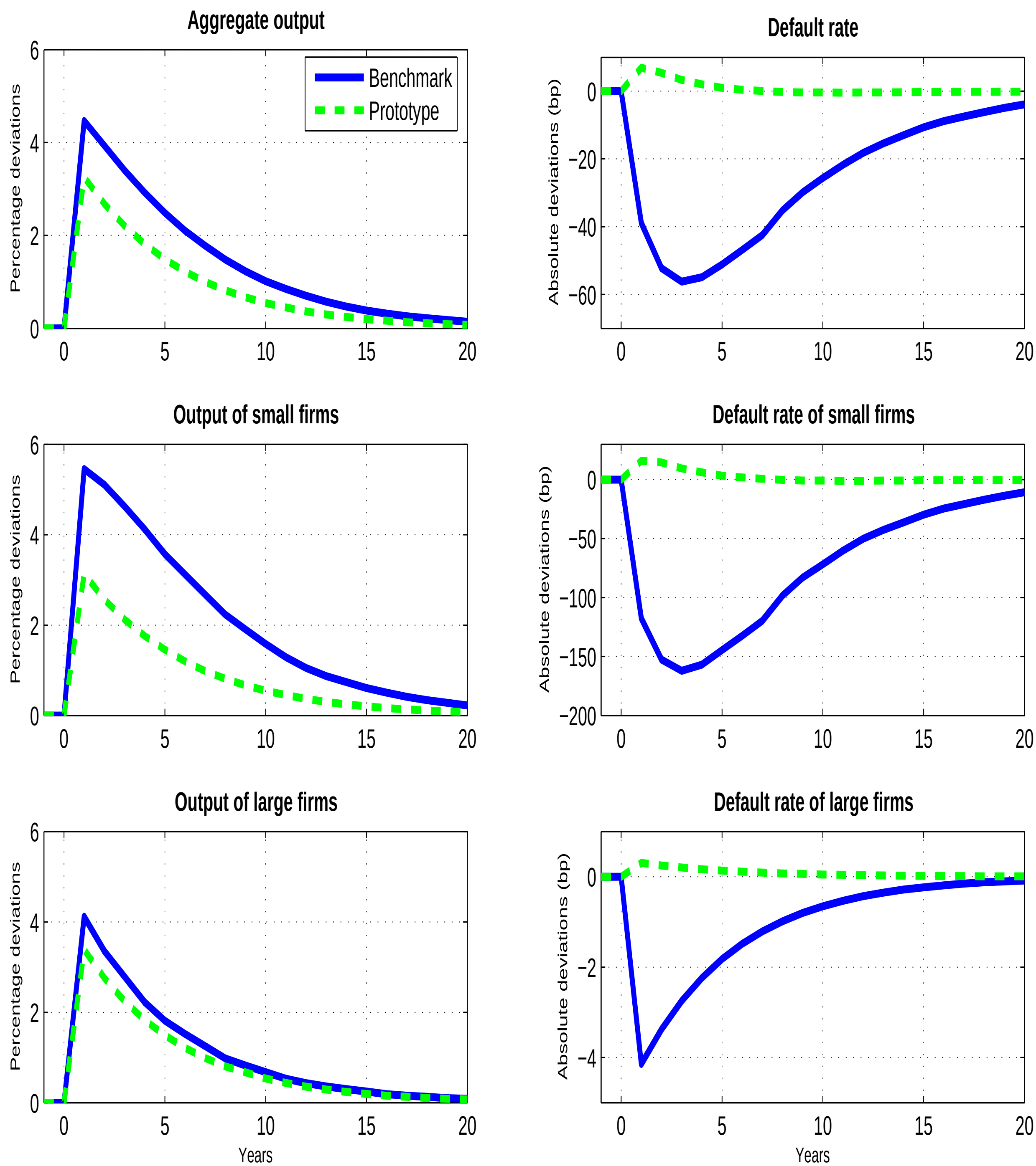
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details see the text and the data appendix.

Figure 4: Responses of output and the default rate to positive shock



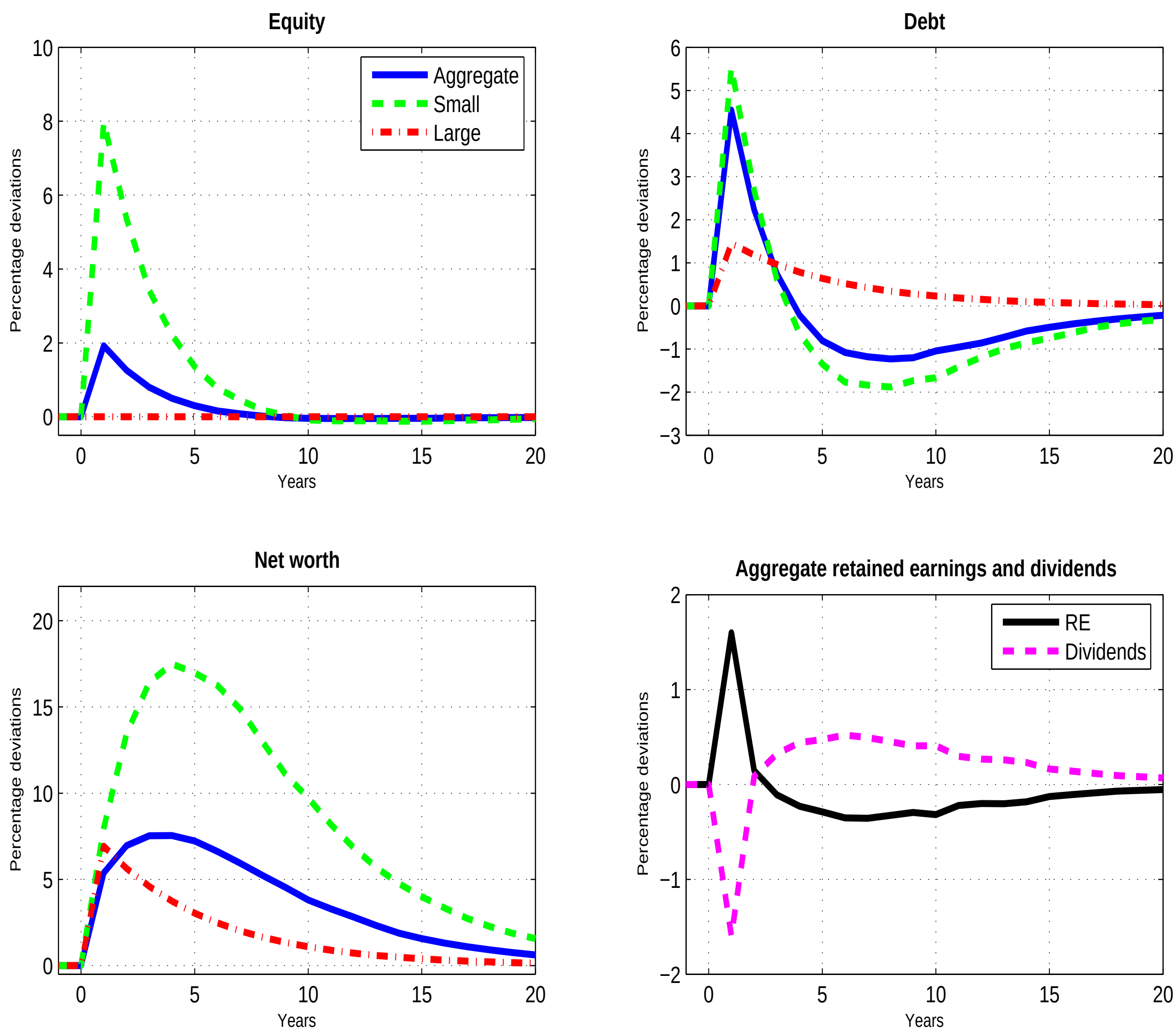
Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 5: Responses of output and the default rate to a positive shock



Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 6: Responses of debt, equity, net worth, retained earnings and dividends to a positive shock



Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 7: Cyclical behavior of financing sources in the model