

# **The Magnitude and Cyclical Behavior of Financial Market Frictions**

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# The Modigliani–Miller Theorem

In a world of no financial market imperfections, capital structure is indeterminate and the aggregate mix of debt vs. equity is irrelevant for the evolution of the real economy.

The MM theorem has provided an important justification for abstracting from financing decisions in models of macroeconomic fluctuations:

- IS-LM framework
- Real business cycle models
- Canonical new-Keynesian model

## Financial Market Frictions: Theory

The seminal work of Bernanke & Gertler (1989) shows that financial factors can play an important role in the propagation of aggregate shocks—the *financial accelerator*.

- Debt-deflation during the Great Depression:  
Fisher (1933)
- Asymmetric information theory:  
Akerlof (1970), Stiglitz & Weiss (1981), etc.
- Micro-founded DGE models with financial market frictions:  
Carlstrom & Fuerst (1997), Kiyotaki & Moore (1997), etc.
- Open-economy research stimulated by the Asian crisis:  
Krugman (1999), Gertler, Gilchrist, & Natalucci (2003), etc.

## Financial Market Frictions: Evidence

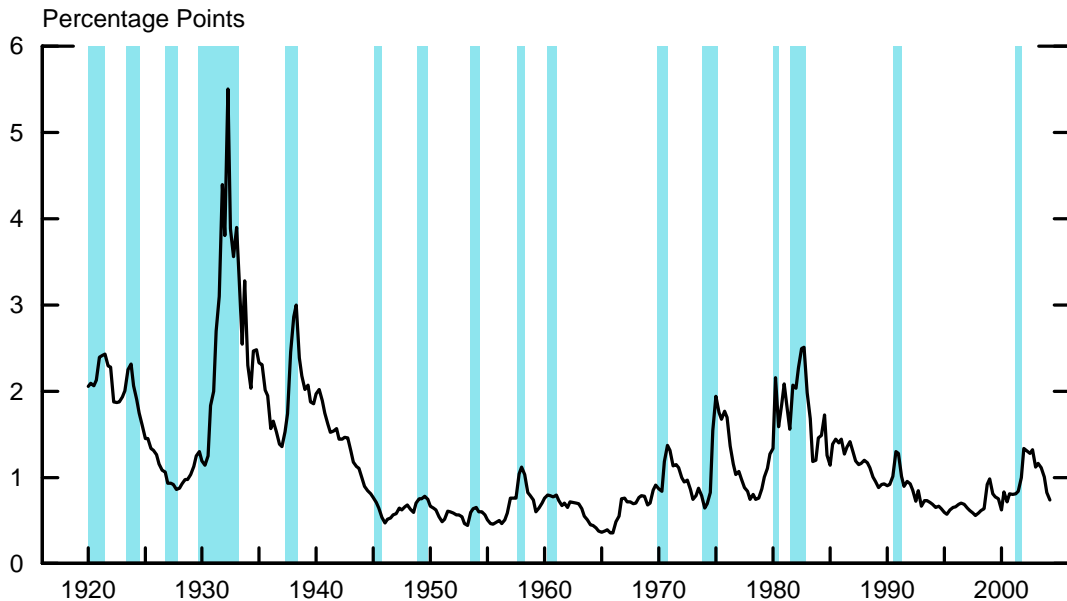
The quantitative significance of financial market frictions has remained elusive despite substantial empirical work:

- Reduced-form evidence linking balance-sheet variables to investment and employment:  
Gurley & Shaw (1960), etc.
- Structural analysis using proxies for capital market access (e.g., small vs. large firms, rated vs. non-rated firms):  
Fazzari et al. (1988), Gilchrist & Himmelberg (1995), etc.  
*Critique:* Cummins et al. (1994), Kaplan & Zingales (1997), etc.

## Outline

- Revisit microfoundations of BGG (1999) financial accelerator model.
- Analyze firm-level panel data linking credit spreads, market-based measures of default risk, and balance sheet variables during the 1997-2004 period.
- Directly estimate the magnitude of financial frictions.
- Examine the behavior of model-implied external finance premium (EFP).
- Directions for future research.

Figure 1: Historical Evolution of U.S. Corporate Credit Spreads



NOTES: The solid line depicts the difference in yields between the lowest-rated (Baa) and highest-rated (Aaa) investment-grade corporate bonds. The shaded vertical bars denote NBER-dated recessions.

## Overview of the BGG Framework

**Bankruptcy Costs:** Lender incurs costs proportional to total value of the firm if borrower defaults, giving rise to the *external finance premium*.

**Leverage-Spread Schedule:** Inverse relationship between the firm's net worth and its borrowing costs.

**Sources of Declining Net Worth:** Declines in equity prices and/or unexpected deflation.

**Amplification of Shocks:** Procyclical net worth leads to countercyclical external finance premium, enhancing swings in borrowing, investment, and output.

## Entrepreneur's Expected Return

**Total Return to Capital:**  $\omega_{i,t+1} R_{it}^k Q_t K_{it}$

$\omega$  = idiosyncratic productivity shock

$\omega \sim f(\omega | \theta_{it})$  with  $E[\omega_{it}] = 1$ , for all  $i$  and  $t$

$R_{it}^k$  = expected return to capital

$Q_t$  = price of capital

$Q_t K_{it} = B_{it} + N_{it}$  (value of firm = debt + net worth)

### **Informational Assumptions:**

$\{R_{it}^k, Q_t, \theta_{it}\}$  known to both lender and entrepreneur

$\omega_{i,t+1}$  not directly observed by lender

**Optimal Financial Arrangement** specifies loan amount  $B_{it}$  and contractual (gross) interest rate  $R_{it}^b$



## Entrepreneur's Expected Return (contd.)

- **No Default:** if  $\omega_{i,t+1} \geq \omega_{it}^*$ , entrepreneur pays  $R_{it}^b B_{it}$  to lender, where

$$R_{it}^b B_{it} = \omega_{it}^* R_{it}^k Q_t K_{it}$$

and keeps  $(\omega_{i,t+1} - \omega_{it}^*) R_{it}^k Q_t K_{it}$

- **Default:** if  $\omega_{i,t+1} < \omega_{it}^*$ , then entrepreneur gets 0

**Ex-ante Return:**  $\psi_{it} R_{it}^k Q_t K_{it}$ ,

$$\psi_{it} \equiv \psi(\omega_{it}^* | \theta_{it}) = \int_{\omega_{it}^*}^{\infty} (\omega - \omega_{it}^*) f(\omega | \theta_{it}) d\omega$$

## Lender's Expected Return

- **No Default:** loan is repaid and lender receives  $R_{it}^b B_{it} = \omega_{it}^* R_{it}^k Q_t K_{it}$
- **Default:** lender takes over the project and incurs *bankruptcy costs*, receiving

$$(1 - \mu_t) \omega_{i,t+1} R_{it}^k Q_t K_{it}, \quad 0 \leq \mu_t < 1$$

**Ex-ante Return:**  $\xi_{it} R_{it}^k Q_t K_{it}$ ,

$$\xi_{it} \equiv \xi(\omega_{it}^* | \mu_t, \theta_{it}) = (1 - \mu_t) \int_0^{\omega_{it}^*} \omega f(\omega | \theta_{it}) d\omega + \omega_{it}^* \int_{\omega_{it}^*}^{\infty} f(\omega | \theta_{it}) d\omega.$$

**Zero-Profit Condition:**  $\xi_{it} R_{it}^k Q_t K_{it} = R_t B_{it}$ .

## Optimal Debt Contract

Entrepreneur's expected return:  $\psi_{it} \left(1 + \frac{B_{it}}{N_{it}}\right) R_{it}^k N_{it}$

**Optimal Default Threshold:**

$$\begin{aligned} \max_{\omega_{it}^*} & \left[ \log(\psi_{it}) + \log \left(1 + \frac{B_{it}}{N_{it}}\right) \right] \\ \text{s.t.} & 1 + \frac{B_{it}}{N_{it}} = \frac{1}{1 - (1 + \rho_{it})\xi_{it}} \end{aligned}$$

where

$$\rho_{it} = \frac{R_{it}^k}{R_t} - 1 \quad (\text{external finance premium})$$

EFP is different from contractual credit spread  $\frac{R_{it}^b}{R_t} - 1$

## Optimal Debt Contract (contd.)

**First-Order Condition:** Optimal threshold  $\omega_{it}^*$  satisfies

$$\frac{\partial \log(\psi_{it})}{\partial \omega_{it}^*} + \frac{\partial \log(1 + B_{it}/N_{it})}{\partial \omega_{it}^*} = 0.$$

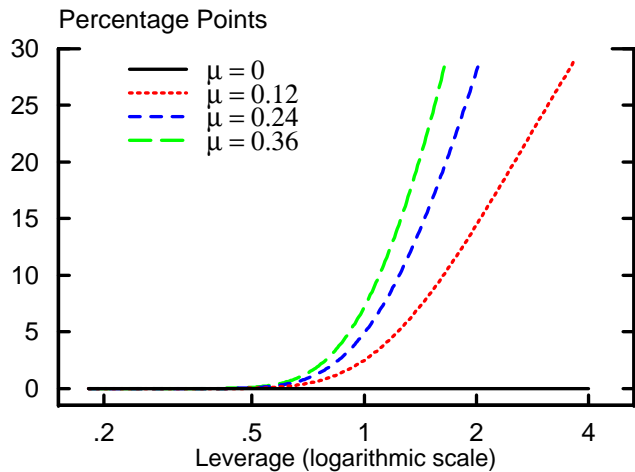
where leverage ratio  $B_{it}/N_{it}$  depends on  $\omega_{it}^*$  through lender's zero-profit condition.

**Interpretation:** marginal cost of reducing entrepreneur's share of total proceeds = marginal benefit of raising project's scale

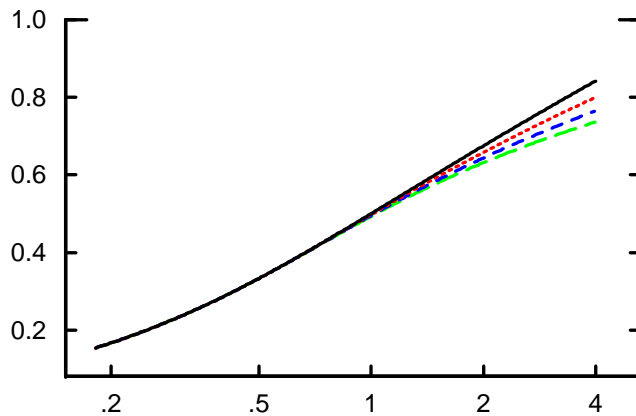
Figure 2

## Varying the Bankruptcy Cost Parameter

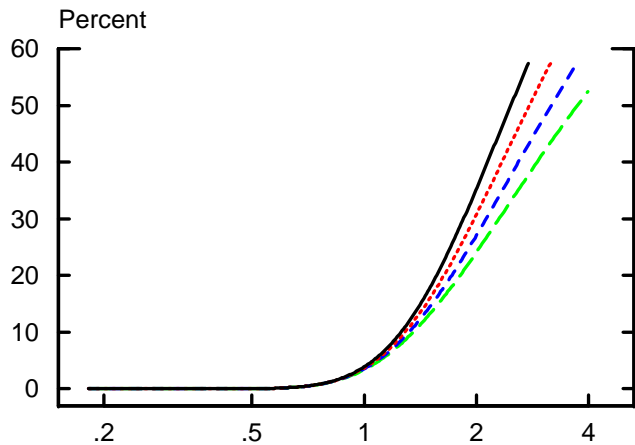
External Finance Premium



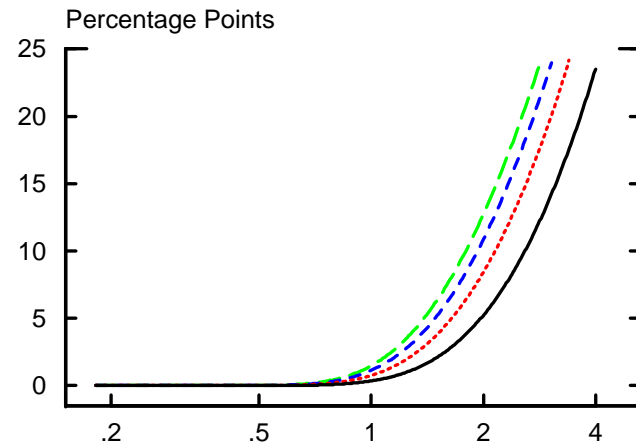
Default Productivity Threshold



Probability of Default



Credit Spread



## Data Description

We linked firm-level data from the following sources:

- Compustat: income and balance sheet variables (quarterly)
- Merrill Lynch: yields on outstanding publicly-traded corporate securities (daily)
- Moody's/KMV: expected default probabilities (monthly)

## Key Variables

**Leverage Ratio:** book value of long-term debt relative to market capitalization

**Credit Spreads:** corporate yield less estimated Treasury yield

- Treasury yield curve estimated daily (Svensson, 1994)
- tax adjustment (Cooper & Davydenko, 2002)

**Default Probabilities:** expected default frequencies (EDFs)

- option-theoretic approach to calculate a firm-specific distance to default (DD)
- actual defaults used to construct a statistical mapping from DD to EDF

Table 1: Summary Statistics

Variable	Minimum	Median	Maximum
Sales (\$ millions)	1.6	996	75,237
Mkt. Capitalization (\$ millions)	6.1	3,425	308,998
Leverage Ratio	0.02	0.43	15.4
Credit Spread <sup>a</sup> (p.p.)	0.07	1.50	30.87
No. of Issues Traded	1	2	65
Avg. Portfolio Maturity (years)	1	8	30
Share of Traded Debt <sup>b</sup> (%)	1	48	100
S&P Credit Rating	CC2	BBB2	AAA
Year-Ahead EDF (%)	0.02	0.33	19.9

*Panel Dimensions*

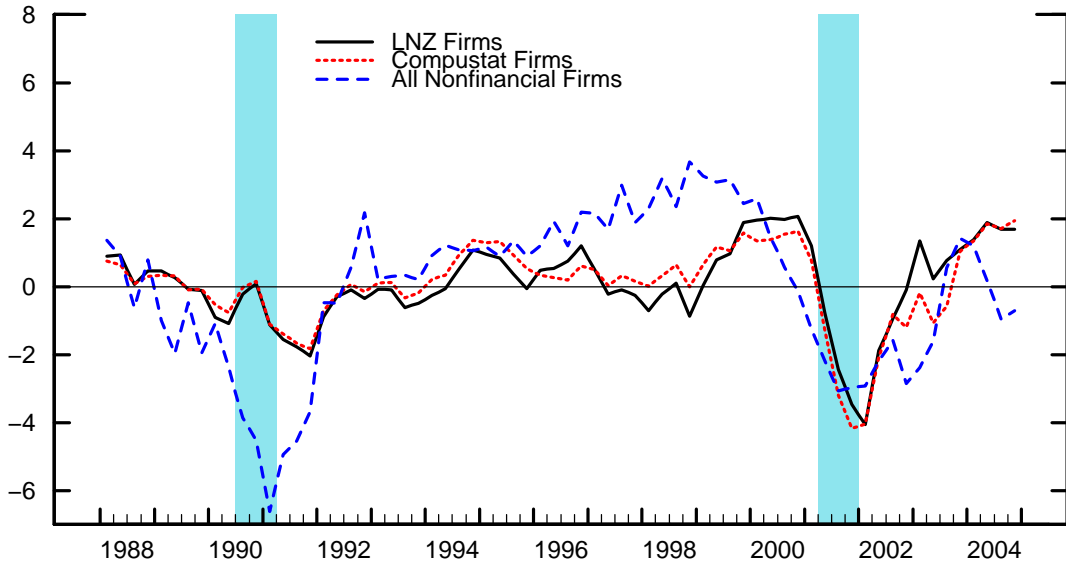
Observations = 14,451   Firms = 796

Min. Tenure = 4   Median Tenure = 16   Max. Tenure = 32



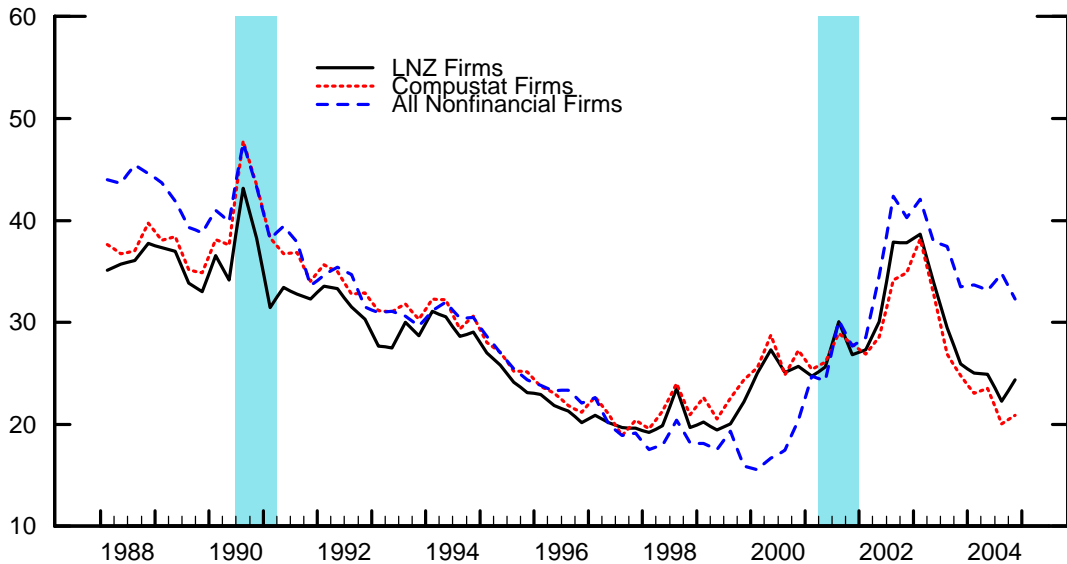
# Growth of Real Sales

Percent

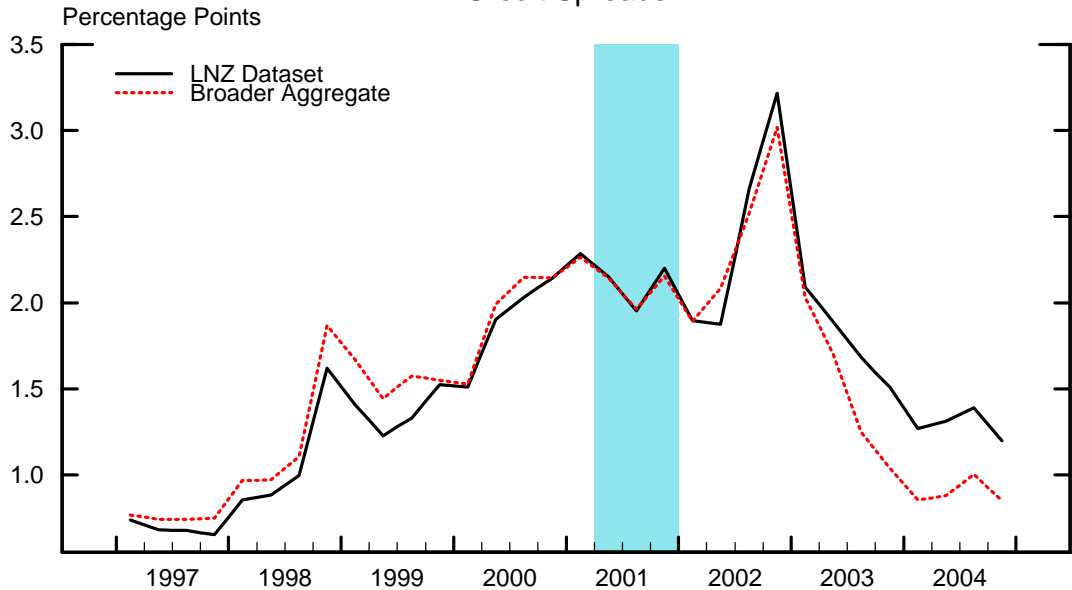


# Leverage

Percent



# Credit Spreads



# Expected Default Frequencies



## Estimation Methodology

Assumptions:

- Productivity shock:  $\log \omega_{it} \sim N(-0.5\sigma_{it}^2, \sigma_{it}^2)$  ( $\theta_{it} = \sigma_{it}$ )
- Bankruptcy costs:  $\mu_t$  common across firms but can vary over time.

For a given  $\mu_t$ , solve for  $\omega_{it}^*$  and  $\sigma_{it}$ :

$$\left[\frac{B}{N}\right]_{it} = -\frac{\psi'(\omega_{it}^* | \sigma_{it}) \xi(\omega_{it}^* | \mu_t, \sigma_{it})}{\psi(\omega_{it}^* | \sigma_{it}) \xi'(\omega_{it}^* | \mu_t, \sigma_{it})};$$
$$\text{EDF}_{it} = \Phi\left(\frac{\log \omega_{it}^* + 0.5\sigma_{it}^2}{\sigma_{it}}\right),$$

## Estimation Methodology (contd.)

Solutions  $\hat{\omega}_{it}^*$  and  $\hat{\sigma}_{it}$  yield model-implied EFP

$$1 + \hat{\rho}_{it} = \frac{\psi'(\hat{\omega}_{it}^* | \hat{\sigma}_{it})}{\psi'(\hat{\omega}_{it}^* | \hat{\sigma}_{it}) \xi(\hat{\omega}_{it}^* | \mu_t, \hat{\sigma}_{it}) - \psi(\hat{\omega}_{it}^* | \hat{\sigma}_{it}) \xi'(\hat{\omega}_{it}^* | \mu_t, \hat{\sigma}_{it})}$$

and model-implied contractual credit spread:

$$\left[ \frac{\widehat{R^b}}{R} \right]_{it} = \hat{\omega}_{it}^* \left( 1 + \left[ \frac{B}{N} \right]_{it}^{-1} \right) (1 + \hat{\rho}_{it}).$$

Difference between *actual* and *model-implied credit spreads*:

$$\left[ \frac{R^b}{R} \right]_{it} - \left[ \frac{\widehat{R^b}}{R} \right]_{it} = \mathbf{x}_{it}^\top \beta_t + \epsilon_{it}.$$

## Estimation Methodology (contd.)

$x_{it}$  = vector of firm characteristics including credit rating and industry fixed effects (i.e., proxies for risk, liquidity, and term premiums)

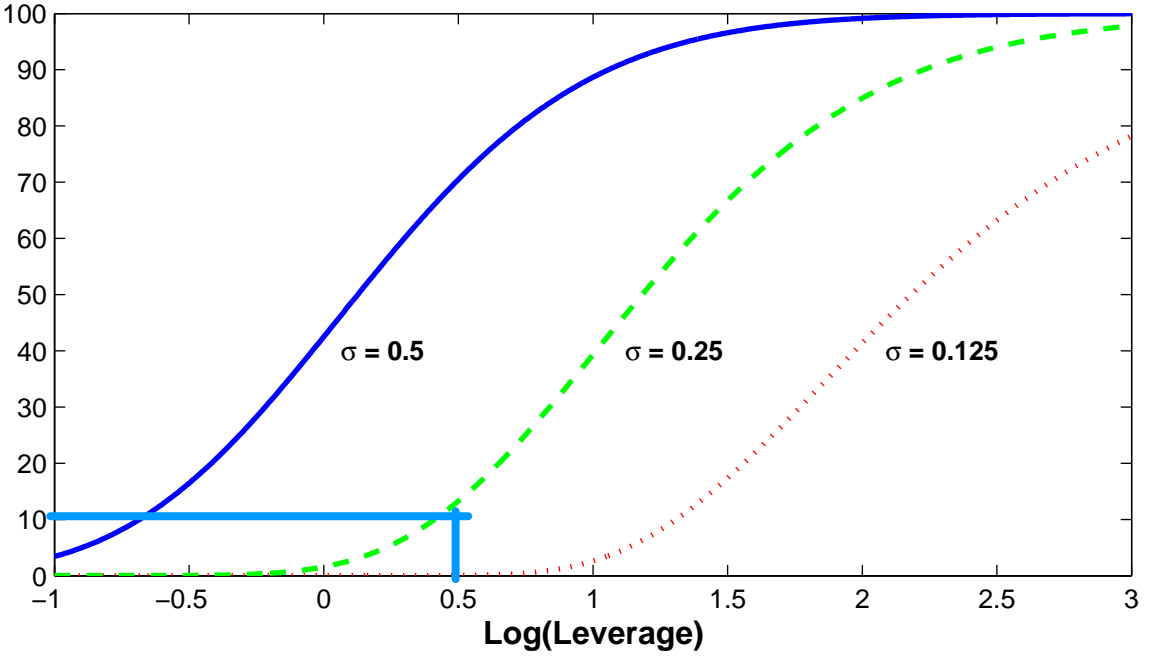
$\epsilon_{it}$  = stochastic disturbance with  $E[\epsilon_{it}] = 0$ ,  $E[\epsilon_{it}^2] = \nu_{it}^2$ , and  $E[\epsilon_{it}\epsilon_{jt}] = 0$

**Objective:** Choose  $\mu_t$  to minimize  $\sum_{i=1}^N \epsilon_{it}^2$

# Estimation Methodology

(annual rates in percent)

## Default Probability



## Credit Spread

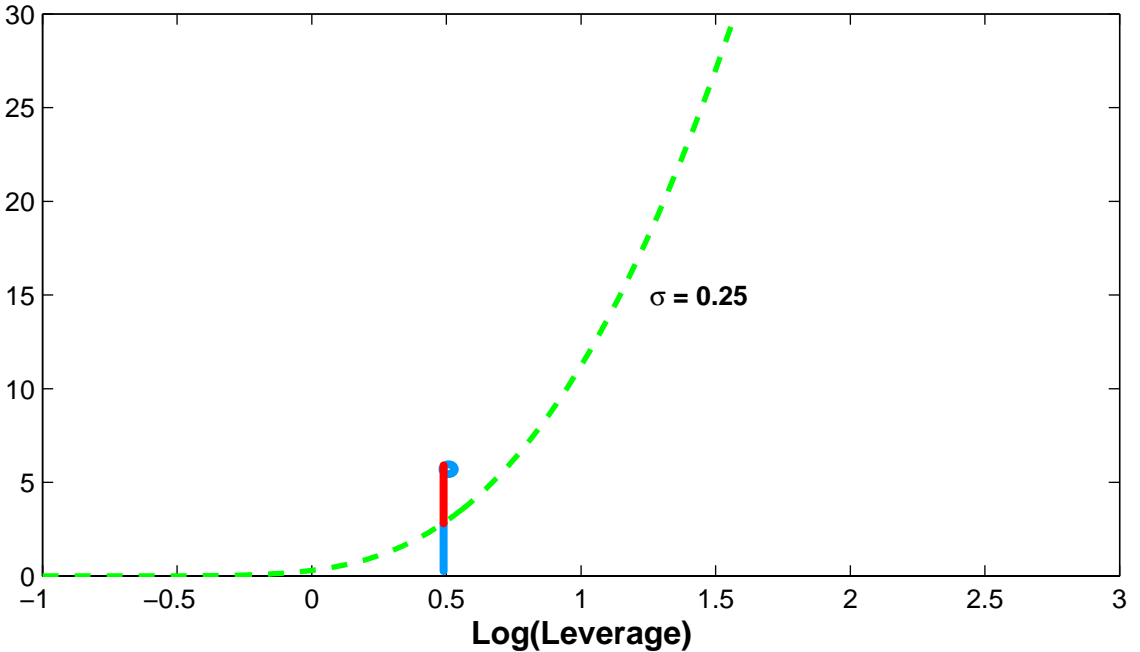
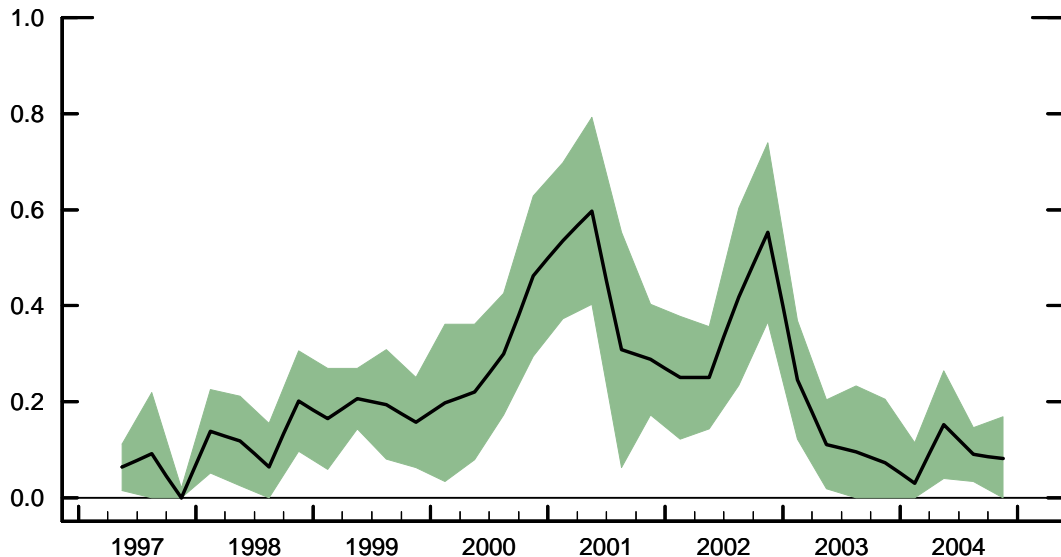




Figure 5: Bankruptcy Cost Parameter Estimates



NOTES: The solid line denotes the time-specific estimate of the bankruptcy cost parameter  $\mu_t$ . The shaded region represents the 95 percent confidence interval, computed using White's (1980) heteroscedasticity-consistent asymptotic covariance matrix.

Figure 6: Cross-Sectional Distribution of the External Finance Premium

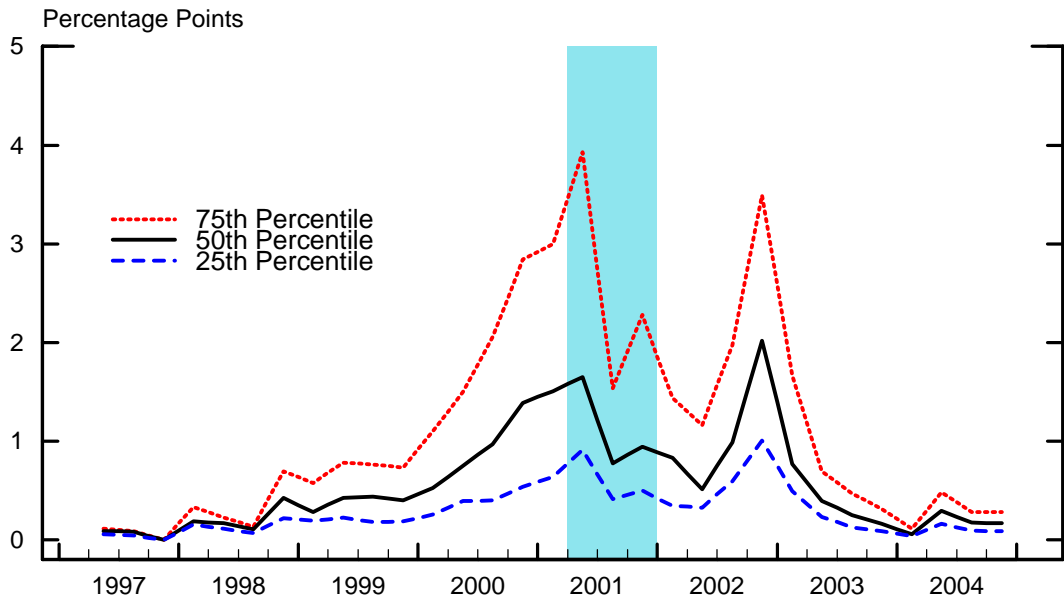


Figure 7: Time Variation in Idiosyncratic Shock Volatility

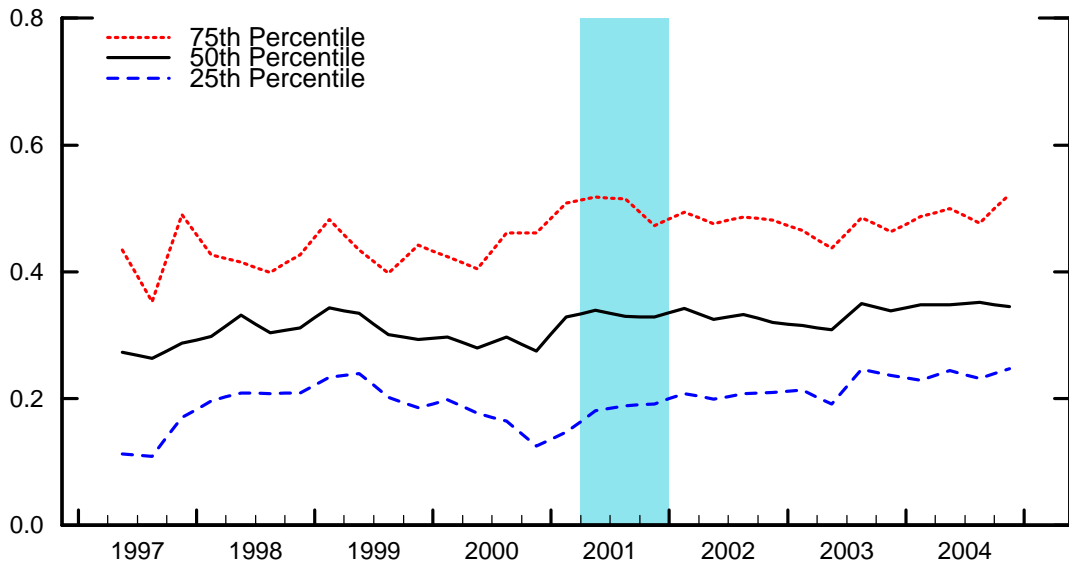


Figure 8: The External Finance Premium under the Counterfactual Scenario

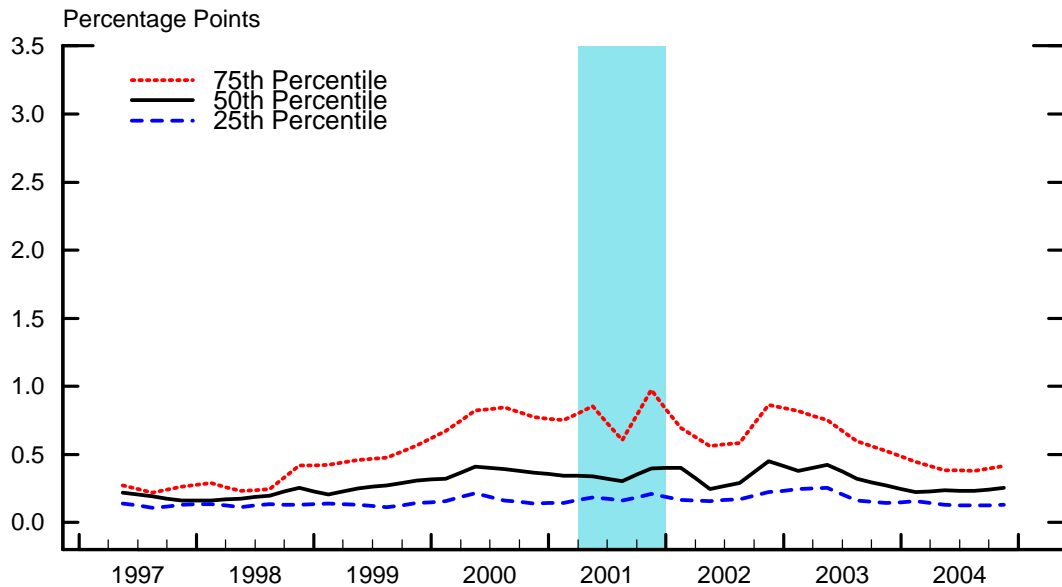
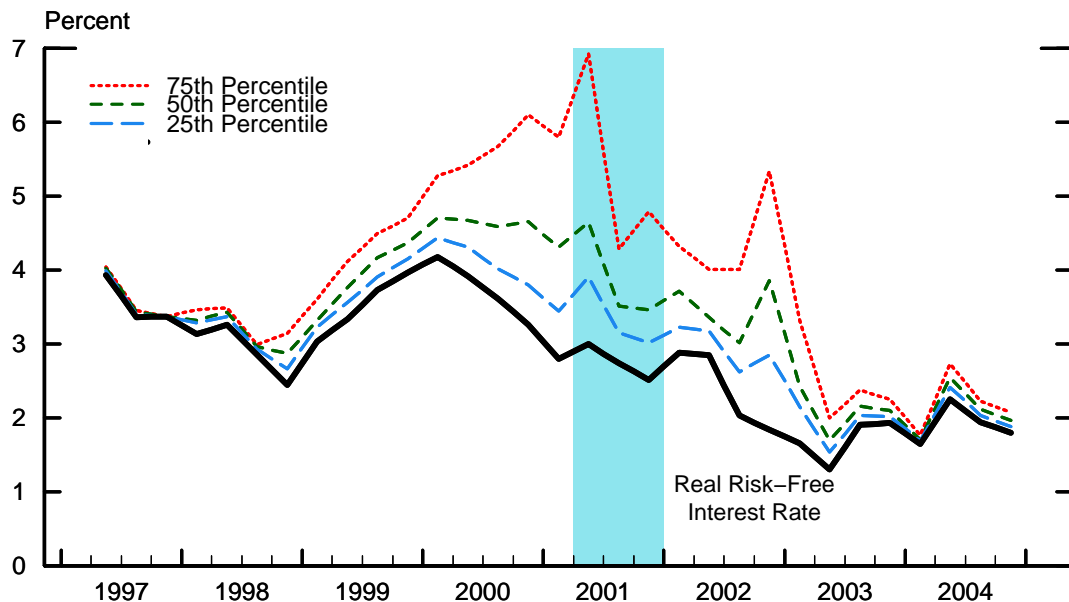


Figure 9: Benchmark Results for the Cost of External Finance



NOTES: The solid line denotes the risk-free real interest rate, that is, the 10-year nominal Treasury yield less expected inflation as measured by the Philadelphia Fed's Survey of Professional Forecasters. The other three lines denote the specified sales-weighted percentiles for the cost of external finance, that is, the risk-free rate plus the model-implied external finance premium.

Table 2: Goodness-of-Fit Comparison

Sample Period	Benchmark Model	No Frictions ( $\mu = 0$ )	No Fixed Effects
1997Q2 – 2000Q2	0.76	0.73	0.35
2000Q3 – 2002Q4	0.73	0.64	0.54
2003Q1 – 2004Q4	0.72	0.71	0.33

NOTES: Entries in the table denote the average of the period-specific adjusted  $R^2$  for the benchmark empirical specification, the alternative specification with no financial market frictions, and for the alternative specification with no rating-specific or industry-specific fixed effects.

Table 3: Fixed Effects and Bankruptcy Costs

Sample Period	Benchmark Model	No Fixed Effects
1997Q2 – 2000Q2	0.15	0.48
2000Q3 – 2002Q4	0.40	0.73
2003Q1 – 2004Q4	0.11	0.45

NOTES: Entries in the table denote the average of the period-specific bankruptcy cost parameter  $\mu_t$  for the benchmark empirical specification and for the alternative specification with no rating-specific or industry-specific fixed effects.

Figure 10: Model-Implied vs. Observed Recovery Rates

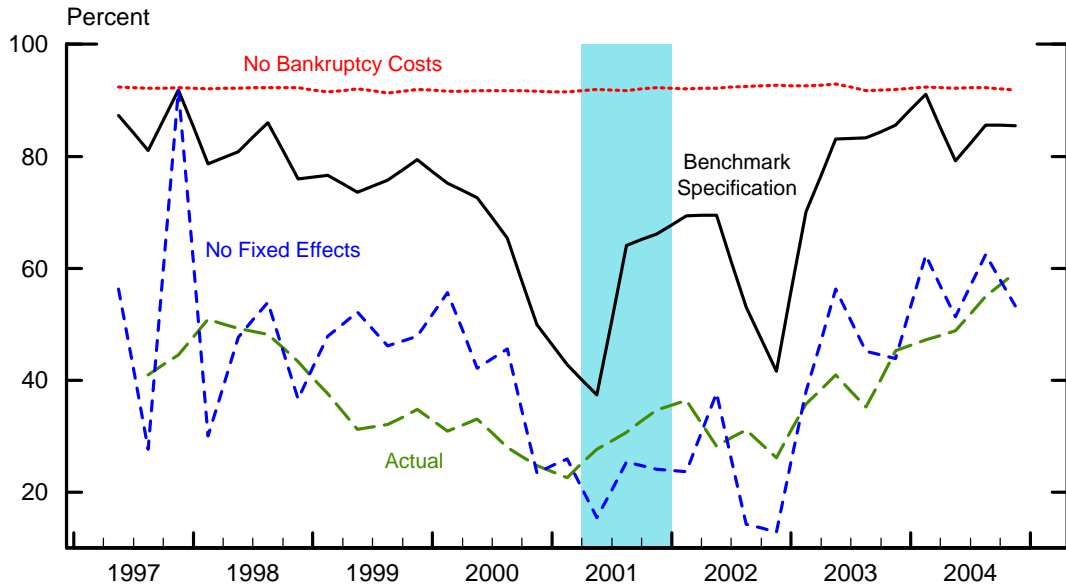
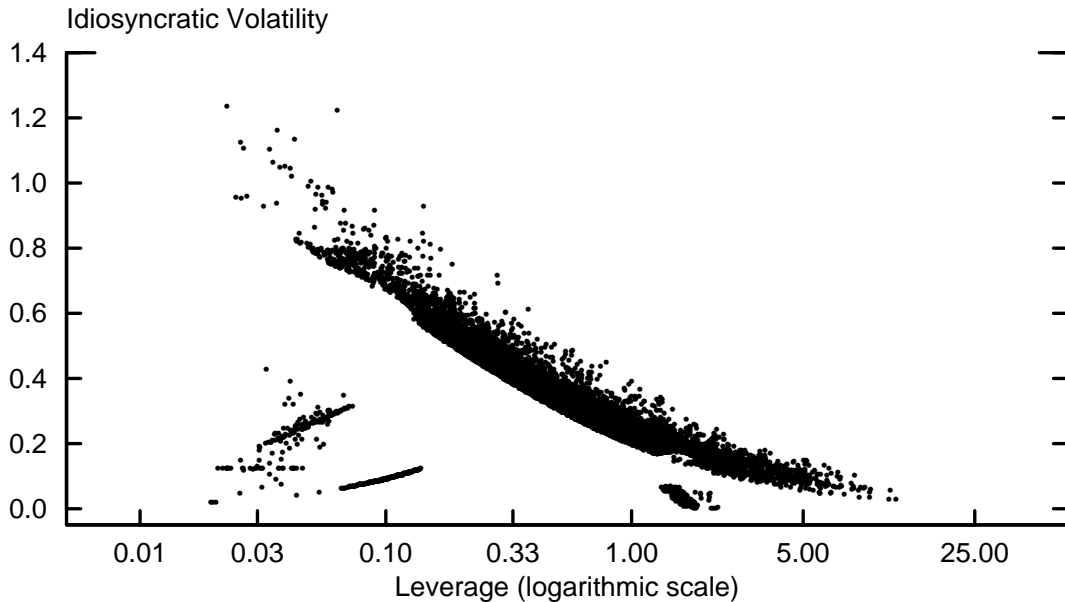




Figure 11: The Implied Relation between Leverage and Idiosyncratic Volatility



## Summary of Results

We obtain precise time-specific estimates of the structural parameter measuring the extent of financial market frictions, rejecting the null hypothesis of frictionless capital markets.

Model-implied EFP exhibits a strong cyclical pattern:

- During the 1997–99 period, the median firm faced a small premium when raising external funds in credit markets.
- In 2000, the EFP increased more than 100 bps. and remained elevated until early 2003.
- The high EFP largely offset the decline in real risk-free rate associated with the easing of monetary policy.

## Directions for Future Research

Address limitations of existing framework:

- allow for *cross-sectional* heterogeneity in  $\mu$  (e.g., firm size, credit quality, etc.)
- allow for non-Gaussian distribution of idiosyncratic productivity shock  $\omega$
- allow for multi-period debt contracts

## Directions for Future Research (contd.)

Investigate the link between model-implied external finance premium and capital expenditures.

Examine macroeconomic implications of financial market frictions in light of micro evidence (e.g., the relationship between corporate leverage and financial fragility).

Use perturbation methods to obtain 2nd-order approximation of the DGE model around steady state and characterize optimal monetary policy.

Extend the analysis to open economies.