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**Credit and the Natural Rate of Interest**  
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# Credit and the Natural Rate of Interest\*

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## Abstract

We analyze the role of the natural rate of interest as a tool to stabilize inflation in a sticky price model with transactions frictions and nominal debt contracts. Contrary to the case where transactions and credit frictions are absent, a definition of the natural rate as the real rate of return arising in the flexible-price equilibrium is not useful for monetary policy because it is not independent of the nominal interest rate. We show that, if the spread between the own return on money and the nominal interest rate is constant over time, there is a definition of natural rate which: (i) is independent of monetary policy, and (ii) delivers price stability if used as the intercept of an interest rate rule. Our definition is based on the idea of switching off all nominal frictions, namely price stickiness and the nominal denomination of firms' debt. This result, however, does not hold if the spread cannot be held constant. We also find that the dynamic behaviour of the natural rate in response to several shocks is significantly different from what would be observed in the absence of credit frictions.

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# 1 Introduction

The Wicksellian notion of natural rate of interest has recently attracted renewed attention. Woodford (2003) argues that the natural rate, defined as the equilibrium real rate of return in a model with flexible prices, represents a useful summary statistics of inflationary pressures in the sticky-price version of the same model. A key property of the natural rate is to be monetary policy invariant, since it can be expressed as a function of exogenous stochastic processes (and, possibly, of endogenous state variables). In the basic New-Keynesian sticky-price model, price stability can be implemented at all times through a Taylor-type policy rule where the policy instrument reacts proportionally to movements in the natural rate of interest.

One interesting property of this type of rule is that it generally approximates the optimal monetary policy. Moreover, whether or not optimal, an interest rate rule ensuring that price stability is maintained at all times represents a useful benchmark for policy analysis, because of the overriding emphasis placed on price stability by most central banks.

In Woodford (2003), the policy instrument coincides with the return on a one-period risk-free bond, which is also the only interest rate (with the same maturity) affecting aggregate demand. However, other interest rates can contribute to determine aggregate demand. Loan rates play a particular role, especially in countries where bank credit is the predominant source of financing for firms. Wicksell attributed an important role to banks and to the loan rate (or "money rate"), recognizing that "modern forms of credit [...] almost always imply the mediation of some bank or professional money-lender" (Wicksell, 1906, p. 214).

In this paper, we analyse how the existence of credit frictions alters the relevant notion of natural rate of interest and its role for the conduct of monetary policy. We address this question in a sticky price economy with transactions frictions and agency costs.

In our model, firms producing investment goods need to borrow in order to finance production. Credit takes the form of optimal debt contracts, as in the costly state verification setup of Carlstrom and Fuerst (2001) or Bernanke, Gertler and Gilchrist (1999). However, we depart from these standard settings in assuming that credit is denominated in nominal, rather than real, terms. This assumption, adopted in a different context in Christiano et al. (2003), is more consistent with the nominal nature of bank deposits and loans.

Our paper is also related to the evidence on the so-called cost channel, i.e. the fact that firms' marginal costs depend on the nominal interest rate (see e.g. Barth and Ramey

(2001)). The implications of the cost channel for optimal monetary policy have been analyzed by Ravenna and Walsh (2006), while Christiano, Eichenbaum and Evans (2005) incorporate the cost channel in a macro model estimated on aggregate US data. In those models, however, there are no agency costs and firms do not pay a premium over the policy rate when borrowing funds. Because of the presence of nominal debt contracts for firms in the investment sector, our model nests the real credit frictions set-up of Carlstrom and Fuerst (2001) or Bernanke, Gertler and Gilchrist (1999) and the cost channel effects emphasized by Ravenna and Walsh (2006) and Christiano, Eichenbaum and Evans (2005) in the transmission of monetary policy.

Since in our model external finance takes the form of nominal debt, the amount of liquidity available for firms' financing is affected by households' decisions on how to split nominal wealth between deposits at the financial intermediary and other nominal assets. We assume that nominal money balances are held by households because they yield liquidity services, according to the timing convention adopted by Lucas and Stokey (1987). One consequence of the presence of liquidity services is that it introduces real-balance effects in the economy.

The nominal denomination of firms' loans generates real effects of both surprise and expected inflation. While ensuring households' deposits completely from any idiosyncratic shock to firms, nominal debt contracts do not provide full insurance against aggregate inflationary shocks. A surprise increase in inflation has an impact on households' wealth through a reduction in the real value of nominal deposits. The real effects of expected inflation arise instead through the impact of the latter on the nominal interest rate. A higher nominal interest rate increases directly the cost of nominal debt. In order to cover higher financing costs when expected inflation is positive, firms in the investment sector must increase the real price of capital (whereas no adjustment would be needed in the presence of real debt contracts). This induces a reduction in the real wage and rental rates and in aggregate output. It is through this channel that expected inflation has real effects also under flexible prices.

The presence of transactions frictions and nominal debt contract have important consequences for the definition of the natural rate of interest. Contrary to the standard case analyzed in Woodford (2003, Chp. 2, section 1), the real interest rate is not independent of monetary policy in the flexible-price equilibrium. We show that, if the spread between the own return on money and the nominal interest rate is constant over time, there is a definition of natural interest rate which: (i) is independent of monetary policy, and (ii) delivers price stability if used as the intercept of an interest rate rule. Our definition of natural equilibrium is based on

the idea of switching off all nominal frictions, namely price stickiness and the nominal denomination of firms' debt. Once these frictions are eliminated, full dichotomy between real and nominal variables of the model is restored, and the real natural interest rate is independent of monetary policy. Moreover, a monetary policy that reacts proportionally to movements in the natural interest rate is able to manage aggregate demand in such a way to ensure that no price pressures ever arise in equilibrium. This result, however, does not hold if the spread cannot be held constant, e.g. if base money were not remunerated. In this case, inflation cannot be stabilized using the natural rate as an intercept of the Taylor rule, although in our calibrated model deviations from price stability are small.

In a numerical analysis of our model, we also assess the relevance of credit frictions for the notion of the natural rate of interest. We find that the response of the natural rate is markedly different in a model with nominal debt contracts relative to a corresponding real business cycle model, pointing to a sensitivity of the natural rate concept to the underlying modelling assumptions. This result is particularly relevant in view of a recent literature showing that financial market frictions are important to explain business cycle volatility. By estimating DSGE models of the US and the euro area with different sources of frictions, Christiano et al. (2006) and Quejo (2004) find that financial frictions are relevant for both areas. Financial markets provide an important source of shocks but also affect the propagation of non-financial shocks. Levin et al. (2004) estimate the structural parameters of a debt-contracting model with informational frictions and reject the null hypothesis of frictionless financial markets.

The paper is organized as follows. In section 2, we outline the model and present the conditions characterizing the equilibrium. In section 3, we argue that the definition of natural rate used in models without nominal debt is not useful for the conduct of monetary policy, as it is not independent from the nominal interest rate. We propose an alternative definition, which is independent from monetary policy, and derive the conditions under which using this latter as an intercept of a Taylor rule delivers price stability. In section 4, we present a numerical analysis. First, we evaluate the importance of credit frictions when they take the form of real debt contracts, by comparing the impulse responses to a technology shock arising in a standard RBC economy to those arising in our model under a Taylor-type rule that includes the natural rate as an intercept (what we call a "natural rate rule"), when the spread is kept constant. Second, we evaluate the role of transactions frictions by computing impulse responses to a technology shock under a natural rate rule, when the spread between the policy rate and

the return on money is either constant or time-varying. Third, we assess the role of nominal frictions by comparing the impulse responses to a technology shock arising under a natural rate rule to those arising under a Taylor rule with a constant intercept. Finally, we compare the reaction of the natural rate of interest to alternative shocks, namely a preference shock, a labor supply shock, a technology shock and a financial shock. In section 5, we conclude.

## 2 The model with nominal debt

The economy is inhabited by a representative infinitely-lived household, a representative firm producing final consumption goods, a continuum of monopolistically competitive intermediate goods firms owned by the household, a competitive investment sector owned by infinitely-lived risk-neutral entrepreneurs, and a zero-profit financial intermediary.

Households have preferences defined over a final consumption good and leisure. At the beginning of the period, they decide how to split their wealth into the available nominal assets. They also decide how much to consume and to devote to investment goods. Households do not have access to a technology to produce capital goods. Hence, they need to purchase capital from firms endowed with such technology, which operate in an investment sector.

The production sector is composed of a representative firm that produces the final consumption good by aggregating a continuum of intermediate goods produced by firms in a monopolistically competitive sector. Because of product differentiation, intermediate goods firms have market power and are therefore price makers. In their price-setting activity, however, they are not free to change their price at will, because prices are subject to Calvo contracts.

The investment sector is made of a continuum of competitive firms endowed with a technology that transforms final consumption goods into capital goods. The internal funds of the firms are not sufficient to finance the desired amount of investment, so entrepreneurs need to raise external finance. Households are not willing to lend directly to firms, because they face idiosyncratic productivity shocks and thus default risk. Lending occurs through the financial intermediary, which is able to diversify the firms' idiosyncratic risk by providing funds to all of them. The loan takes the form of risky debt, which is the optimal contractual arrangement between lenders and borrowers in the presence of asymmetric information and costly state verification. After lending has occurred, the idiosyncratic shock is realized and the firm decides whether or not to repay the debt. In case of bankruptcy, the firm is monitored by the bank

and it loses its entire capital output. Firms that have not defaulted sell the newly produced capital to households and rent their own capital stock to intermediate goods firms.

## 2.1 Households

At the beginning of period  $t$ , the financial market opens. First, the interest on nominal financial assets acquired at time  $t-1$  is paid. The households, holding an amount  $W_t$  of nominal wealth, choose to allocate it among existing nominal assets, namely money  $M_t$ , a portfolio of nominal state-contingent bonds  $A_{t+1}$  each paying a unit of currency in a particular state in period  $t+1$ , and one-period deposits denominated in units of currency  $D_t$ .

As in Woodford (2003), we enable the government to remunerate end-of-period private holdings of money at the rate  $i_t^m$ . We also assume that the government can provide households with a subsidy that compensates for the effect of expected inflation on their wealth. More specifically, under such subsidy, households that decide to deposit an amount  $D_t$  of currency at the financial intermediaries receive back  $\Omega_t (1 + i_t^d) D_t$  at the end of the period, where  $i_t^d$  is the net interest rate paid by the financial intermediary on deposits and  $\Omega_t - 1$  is the subsidy paid by the government for each unit of currency returned to households by the financial intermediary.<sup>1</sup>

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period  $t+1$  is equal to

$$M_t + P_t (w_t h_t + \rho_t k_{h,t}) - P_t (I_{h,t} + c_t) + Z_t - T_t, \quad (1)$$

where  $c_t$  is the amount of final consumption good purchased,  $P_t$  is its price,  $h_t$  is hours worked,  $k_{h,t}$  is the capital stock held by the household,  $w_t$  is the real wage,  $\rho_t$  is the return on capital,  $Z_t$  are nominal profits transferred from intermediate goods producers to households,  $I_{h,t}$  is the amount of consumption good purchased by the household for investment purposes, and  $T_t$  are lump-sum nominal taxes collected by the government.

New capital can be purchased from firms in the investment sector at the end of the period, in exchange of consumption goods. The accumulation of household capital follows the law of

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<sup>1</sup>As we will discuss in section 3.1, we do not need, nor argue that such a subsidy is realistic. Nor our results on the stabilization properties of a Wicksellian monetary policy rely on the existence of such subsidy. We use it as a conceptual device to provide an appropriate definition of the natural rate of interest.

motion

$$I_{h,t} = q_t [k_{h,t+1} - (1 - \delta) k_{h,t}],$$

where  $q_t$  is the price of capital in units of the consumption good.

We can write nominal wealth at the beginning of period  $t + 1$  as

$$W_{t+1} = A_{t+1} + \Omega_t \left(1 + i_t^d\right) D_t + (1 + i_t^m) [M_t + P_t (\rho_t k_{h,t} + w_t h_t) + Z_t - T_t - P_t (I_{h,t} + c_t)]. \quad (2)$$

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^{\infty} \beta^t [u(c_t, m_t; \xi_t) - v(h_t; \psi_t)] \right\}, \quad (3)$$

subject to the budget constraints

$$M_t + D_t + E_t [Q_{t,t+1} A_{t+1}] \leq W_t, \quad (4)$$

for all  $t \geq 0$ . Here  $u_c > 0$ ,  $u_m \geq 0$ ,  $u_{cc} < 0$ ,  $u_{mm} < 0$ ,  $v_h > 0$ ,  $v_{hh} > 0$ ,  $m_t \equiv M_t/P_t$  denotes real balances,  $\xi_t$  is a preference shock, and  $\psi_t$  is a labor supply shock.

Define  $\pi_t \equiv \frac{P_t}{P_{t-1}}$ ,  $\Delta_{m,t} \equiv \frac{i_t - i_t^m}{1 + i_t}$  and  $\Delta_{d,t} \equiv \frac{(1 + i_t) - (1 + i_t^d) \Omega_t}{1 + i_t}$ . Combining (2) and (4), and imposing a no Ponzi game condition, we can write the following intertemporal budget constraint

$$\begin{aligned} & \sum_{s=t}^{\infty} E_t Q_{t,s} \{ \Delta_{m,s} M_s + \Delta_{d,s} D + (1 - \Delta_{m,s}) \{ P_s (I_{h,s} + c_s) + T_s \} \} \\ & \leq W_t + \sum_{s=t}^{\infty} E_t Q_{t,s} \{ (1 - \Delta_{m,s}) \{ P_s w_s h_s + P_s \rho_s k_{h,s} + Z_s \} \}. \end{aligned} \quad (5)$$

The absence of arbitrage opportunities requires the existence of a discount factor  $Q_{t,t+1}$  such that the price of any portfolio of financial assets with random value  $A_{t+1}$  in the following period is given by  $E_t [Q_{t,t+1} A_{t+1}]$ . The riskless nominal interest rate corresponds to the solution to the equation

$$\frac{1}{1 + i_t} = E_t [Q_{t,t+1}]. \quad (6)$$

Optimality requires that, at each  $t$ , (5) holds as equality, (6) is satisfied, either  $\Delta_{d,t} = 0$  or  $D_t = 0$ , and the following equations hold

$$\frac{v_h(h_t; \psi_t)}{u_c(c_t, m_t; \xi_t)} = w_t \quad (7)$$

$$(1 + i_t)^{-1} = \beta E_t \left\{ \frac{u_c(c_{t+1}, m_{t+1}; \xi_{t+1}) + u_m(c_{t+1}, m_{t+1}; \xi_{t+1}) \frac{1}{\pi_{t+1}}}{u_c(c_t, m_t; \xi_t) + u_m(c_t, m_t; \xi_t)} \right\} \quad (8)$$



$$\frac{u_m(c_t, m_t; \xi_t)}{u_c(c_t, m_t; \xi_t)} = \frac{\Delta_{m,t}}{1 - \Delta_{m,t}} \quad (9)$$

$$u_c(c_t, m_t; \xi_t) q_t = \beta E_t \{ u_c(c_{t+1}, m_{t+1}; \xi_{t+1}) [q_{t+1} (1 - \delta) + \rho_{t+1}] \}. \quad (10)$$

Notice that condition (9) can be written as  $m_t = L(c_t, \Delta_{m,t}; \xi_t)$ . We can then define the following functions

$$U_c(c_t, \Delta_{m,t}; \xi_t) \equiv u_c(c_t, L(c_t, \Delta_{m,t}; \xi_t); \xi_t) \quad (11)$$

$$U_m(c_t, \Delta_{m,t}; \xi_t) \equiv u_m(c_t, L(c_t, \Delta_{m,t}; \xi_t); \xi_t). \quad (12)$$

## 2.2 The production sector

Final consumption goods are produced by a representative, competitive firm using a Dixit-Stiglitz aggregator of differentiated intermediate goods

$$y_t = \left[ \int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (13)$$

where  $y_t(j)$  denotes the quantity of the differentiated good  $j$ . Profit maximization of firms producing final goods requires that

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} y_t, \quad (14)$$

where  $P_t(j)$  is the price of good  $j$ . Using condition (14), we can obtain an expression for the consumer price index,  $P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$ .

Each differentiated intermediate good  $j$  is produced by monopolistic competitive firms using the technology

$$y_t(j) = A_t l_t(j)^\alpha k_t(j)^{1-\alpha}, \quad (15)$$

where  $l_t(j)$  and  $k_t(j)$  denote the amount of labor and capital rented on the market by firm  $j$ , while  $A_t$  is an aggregate exogenous productivity shock.

Because of product differentiation, each intermediate good firm has some market power. Profits are distributed to the households, who are the owners of the firms. We assume that each retailer can change its price with probability  $1 - \theta$ , following Calvo (1983). Let  $P_t^*$  denote the price for good  $j$  set by a firm that can change the price at time  $t$ , and  $y_t^*(j)$  the demand faced given this price. Then each intermediate good firm chooses its price to maximize expected discounted profits, given by

$$E_t \sum_{k=0}^{\infty} \theta^k \bar{Q}_{t,t+k} [P_t^* y_{t+k}(j) - P_{t+k} \bar{\chi}_{t+k}(j)],$$

where  $\bar{Q}_{t,t+k} = \beta \frac{U_c(c_{t+k}, \Delta_{m,t+1}; \xi_{t+k})}{U_c(c_t, \Delta_{m,t}; \xi_t)}$  and  $\bar{\chi}_{t+k}(j)$  denote total real costs of production. It follows that  $\bar{\chi}_t(j)$  is given by

$$\bar{\chi}_t(j) = \min_{k,l} [w_t l_t(j) + \rho_t k_t(j)]$$

subject to (15). Denote  $\chi_t$  as the lagrangean multiplier associated to constraint (15), or the real cost of producing one unit of output. This latter is not firm-specific, as all firms face the same real wage and price of capital. Optimality implies that

$$\begin{aligned} w_t &= \chi_t \alpha \frac{y_t(j)}{l_t(j)} \\ \rho_t &= \chi_t (1 - \alpha) \frac{y_t(j)}{k_t(j)}. \end{aligned}$$

Using (14), the firm's profit maximization problem can be written as

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \bar{Q}_{t,t+k} \left( \frac{P_t^*}{P_{t+k}} - \chi_{t+k} \right) \left( \frac{P_t^{*-\varepsilon}}{P_{t+k}^{-\varepsilon}} \right) y_{t+k}.$$

The first-order conditions lead to

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \bar{Q}_{t,t+k} \frac{P_t^{-\varepsilon}}{P_{t+k}^{-\varepsilon}} \chi_{t+k} y_{t+k} \right\}}{E_t \left\{ \sum_{k=0}^{\infty} \theta^k \bar{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} y_{t+k} \right\}}.$$

Now define

$$\begin{aligned} \Theta_{1,t} &\equiv E_t \left\{ \sum_{k=0}^{\infty} \theta^k \bar{Q}_{t,t+k} \frac{\bar{P}_{t+k}}{P_{t+k}^{1-\varepsilon}} P_t^{-\varepsilon} y_{t+k} \right\}, \\ \Theta_{2,t} &\equiv E_t \left\{ \sum_{k=0}^{\infty} \theta^k \bar{Q}_{t,t+k} \frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}} y_{t+k} \right\}. \end{aligned}$$

Using the expression for the aggregate price index,  $P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$ , and substituting out  $\frac{P_t^*}{P_t}$ , we obtain the following conditions

$$1 = \theta \pi_t^{\varepsilon-1} + (1 - \theta) \left( \frac{\varepsilon}{\varepsilon - 1} \frac{\Theta_{1,t}}{\Theta_{2,t}} \right)^{1-\varepsilon} \quad (16)$$

$$\Theta_{1,t} = \chi_t y_t + \theta E_t \left[ \bar{Q}_{t,t+1} \pi_{t+1}^{\varepsilon} \Theta_{1,t+1} \right] \quad (17)$$

$$\Theta_{2,t} = y_t + \theta E_t \left[ \bar{Q}_{t,t+1} \pi_{t+1}^{\varepsilon-1} \Theta_{2,t+1} \right]. \quad (18)$$

### 2.3 The investment sector

The investment sector is composed of an infinite number of competitive firms, each endowed with a stochastic technology that transforms  $I$  units of the final consumption good into  $\omega I$  units of capital. The random variable  $\omega$  is i.i.d. across time and across entrepreneurs, with distribution  $\Phi$ , density  $\phi$  and mean unity. The shock  $\omega$  is private information, but its realization can be observed by the financial intermediary at the cost of  $\mu I$  units of capital.

The amount of internal funds available to firm  $i$  is given by its net worth,

$$n_{i,t} = [q_t (1 - \delta) + \rho_t] z_{i,t},$$

where  $z_{i,t}$  is the stock of capital owned by firm  $i$  at the beginning of period  $t$ . The firm's net worth (i.e. the value of the accumulated capital stock plus the return from renting the capital stock to intermediate goods firms) is not sufficient to produce the desired amount of investment goods. Hence, firms need to raise external finance. However, since firms face an idiosyncratic productivity shock and thus default risk, households are not willing to lend directly to them. Lending occurs through the financial intermediary, which is able to ensure a safe return by providing funds to the continuum of firms facing idiosyncratic shocks. The loan takes the form of risky debt, which is the optimal contractual arrangement between lenders and borrowers in the presence of asymmetric information and costly state verification.

#### 2.3.1 The financial contract

Loans are stipulated in units of currency after all aggregate shocks have occurred, and repaid at the end of the same period. The amount of funds lent to firm  $i$  is  $P_t (I_{i,t} - n_{i,t})$ . The capital output is then sold directly to households at the price  $P_t q_t$  and used to repay the agreed upon amount (unless default occurs). Define

$$f(\bar{\omega}) \equiv \int_{\bar{\omega}}^{\infty} \omega \Phi(d\omega) - \bar{\omega} [1 - \Phi(\bar{\omega})]$$

$$g(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega \Phi(d\omega) - \mu \Phi(\bar{\omega}) + \bar{\omega} [1 - \Phi(\bar{\omega})]$$

as the fraction of the expected net capital output accruing respectively to an entrepreneur and to a lender, after stipulating a contract that sets the fixed repayment at  $P_t q_t \bar{\omega} I_{i,t}$  units of currency. In case of default, a constant fraction  $\mu$  of the production input is destroyed in

monitoring. At the individual firm level, total output is split between the entrepreneur, the lender, and monitoring costs,

$$f(\bar{\omega}) + g(\bar{\omega}) = 1 - \mu\Phi(\bar{\omega}).$$

Hence, on average,  $\mu\Phi(\bar{\omega})$  of the produced capital is lost in monitoring.

Since the contract is intra-period, we can drop the price level from the formulation of the problem. The optimal contract is the pair  $(I_{i,t}, \bar{\omega}_{i,t})$  that solves the following CSV problem:

$$\max q_t f(\bar{\omega}_{i,t}) I_{i,t}$$

subject to

$$q_t g(\bar{\omega}_{i,t}) I_{i,t} \geq \frac{(1+i_t)}{\Omega_t} (I_{i,t} - n_{i,t}) \quad (19)$$

$$f(\bar{\omega}_{i,t}) + g(\bar{\omega}_{i,t}) + \mu\Phi(\bar{\omega}_{i,t}) \leq 1 \quad (20)$$

$$q_t f(\bar{\omega}_{i,t}) I_{i,t} \geq n_{i,t}, \quad n_{i,t} \geq 0. \quad (21)$$

The optimal contract maximizes the entrepreneur's expected return subject to the financial intermediary's expected return exceeding the repayment requested by the household, (19), the feasibility condition, (20), and the entrepreneur being willing to sign the contract, (21). Notice that the intermediary needs to pay back to the household a gross return equal to the safe interest on deposits,  $1 + i_t^d$ . Since in equilibrium  $\frac{(1+i_t)}{\Omega_t} = 1 + i_t^d$ , the financial intermediary's expected return on each unit of loans cannot be lower than  $\frac{(1+i_t)}{\Omega_t}$ , as stated in condition (19).

The informational structure corresponds to a standard costly state verification (CSV) problem. The solution is a debt contract (see e.g. Gale and Hellwig (1985)) that is characterized by three properties. First, the repayment to the financial intermediary is constant in states when monitoring does not occur. Second, the firm is declared bankrupt if and only if the fixed repayment cannot be honored. Third, in case of bankruptcy, the financial intermediary commits to monitor and completely seizes the output in the hands of the firm. The first and third properties ensure that the contract is incentive compatible. The second property ensures that monitoring is done in as few circumstances as possible to avoid the deadweight loss.

The first-order conditions of the problem can be written as

$$q_t = \frac{(1+i_t)/\Omega_t}{1 - \mu\Phi(\bar{\omega}_{i,t}) + \frac{\mu\bar{\omega}_{i,t}f(\bar{\omega}_{i,t})\phi(\bar{\omega}_{i,t})}{f'(\bar{\omega}_{i,t})}}, \quad (22)$$

$$I_{i,t} = \left\{ \frac{(1+i_t)/\Omega_t}{(1+i_t)/\Omega_t - q_t g(\bar{\omega}_{i,t})} \right\} n_{i,t}. \quad (23)$$

Notice from equation (22) that the terms of the contract depend on the state of the economy only through the price of capital  $q_t$  and the return  $(1 + i_t) / \Omega_t$ . Hence, they are the same for all firms, i.e.  $\bar{\omega}_{i,t} = \bar{\omega}_t$ , for all  $i$ . From equation (23), it is clear that the contract only differs among firms because of their initial wealth. The larger is  $n_{i,t}$ , the larger is the investment  $I_{i,t}$  that can be financed. Condition (22) can thus be rewritten as

$$q_t = \frac{(1 + i_t) / \Omega_t}{1 - \mu \Phi(\bar{\omega}_t) + \frac{\mu \bar{\omega}_t f(\bar{\omega}_t) \phi(\bar{\omega}_t)}{f'(\bar{\omega}_t)}}. \quad (24)$$

The net nominal interest rate implicit in the contract with firm  $i$ ,  $i_{i,t}^l$ , is given by

$$1 + i_{i,t}^l = \frac{q_t \bar{\omega}_t I_{i,t}}{I_{i,t} - n_{i,t}}. \quad (25)$$

Given linearity in net worth, firm-specific variables can be easily aggregated. Equation (23) leads to the aggregate condition

$$I_t = \left\{ \frac{(1 + i_t) / \Omega_t}{(1 + i_t) / \Omega_t - q_t g(\bar{\omega}_t)} \right\} n_t. \quad (26)$$

### 2.3.2 Entrepreneurial consumption and investment decisions

Entrepreneurs are infinitely lived, risk-neutral and more impatient than households. They discount the future at a rate  $\beta \gamma_t$  and their utility is linear in entrepreneurial consumption. Here  $\beta$  is the discount factor of households and  $\gamma_t$  is an exogenous shock to entrepreneurs impatience with mean  $\gamma$  and support  $[0, 1]$ . The higher degree of impatience relative to households induces entrepreneurs to consume a sufficiently high share of their net worth, so that they do not accumulate capital up to the point where there is no need for external finance and agency costs become irrelevant.

The problem of the entrepreneur is to maximize the expected value of the discounted stream of future utilities,

$$E_0 \sum_{t=0}^{\infty} (\beta \gamma_t)^t e_{i,t}, \quad 0 < \gamma < 1, \quad (27)$$

subject to the budget constraint

$$e_{i,t} + q_t z_{i,t+1} = q_t f(\bar{\omega}_t) I_{i,t}, \quad (28)$$

Here  $e_{i,t}$  denotes entrepreneurial consumption of the final good and  $z_{i,t+1}$  is investment in physical capital to be used in period  $t + 1$ .

Assuming an interior solution, optimality requires that

$$q_t = \beta \gamma_t E_t \left\{ q_{t+1} f(\bar{\omega}_{t+1}) \left[ \frac{[q_{t+1}(1-\delta) + \rho_{t+1}](1+i_{t+1})/\Omega_{t+1}}{(1+i_{t+1})/\Omega_{t+1} - q_{t+1}g(\bar{\omega}_{t+1})} \right] \right\}. \quad (29)$$

## 2.4 A Wicksellian monetary policy regime

The central bank follows a monetary policy rule described by

$$i_t = v_t \phi \left( \frac{\pi_t}{\pi_t^*} \right), \quad (30)$$

where  $\phi(\cdot)$  is a non-negative-valued, non-decreasing function,  $\pi_t^*$  is the target path for the inflation rate, and  $v_t$  captures either a random disturbance to the policy rule or a systematic response of monetary policy to exogenous shocks.

Monetary policy also needs to specify an additional rule for either  $i_t^m$  or  $M_t^s$ . It is convenient to express this rule in terms of  $\Delta_{m,t}$ . We consider a general rule of the form

$$\Delta_{m,t} = \Gamma(1+i_t), \quad (31)$$

where  $\Gamma(\cdot)$  is a non-negative-valued function.

## 2.5 Fiscal policy

The fiscal authorities decide on the subsidy  $\Omega_t$ . They also use lump-sum taxes  $T_t$  to balance the government's budget at each period. This latter is given by

$$M_{t+1}^s - M_t^s + (\Omega_t - 1) \left( 1 + i_t^d \right) D_t + i_t^m m_t = T_t. \quad (32)$$

## 2.6 Market clearing

Market clearing for money, bonds, capital, investment, labor, loans and goods requires that

$$M_t = M_t^s \quad (33)$$

$$B_t = 0 \quad (34)$$

$$k_t = k_{h,t} + z_t \quad (35)$$

$$k_{t+1} = (1-\delta)k_t + I_t [1 - \mu\Phi(\bar{\omega}_{i,t})] \quad (36)$$

$$h_t = l_t \quad (37)$$

$$D_t = P_t(I_t - n_t) \quad (38)$$

$$\left[ \int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = c_t + e_t + I_t. \quad (39)$$

## 2.7 Equilibrium

An equilibrium is a solution to the system of equations (7)-(10), (13)-(18), (24)-(26), (29)-(39), together with definitions (11) and (12). A log-linearized reduced-form system of equilibrium conditions is reported in Appendix A.

## 3 The natural rate of interest

In the sticky price model considered by Woodford (2003), the natural rate of interest, defined as the equilibrium real rate of return when prices are fully flexible, acts as a summary statistics of the underlying economic conditions and provides a useful tool in the conduct of monetary policy. A monetary policy rule such that the policy instrument reacts proportionally to movements in the natural rate is able to ensure the achievement of price stability.

In the simplest version of the sticky price model, where labor is the only input in production and financial frictions are absent, the natural rate of interest can be expressed solely as a function of the exogenous stochastic processes. In a version of the model that also includes capital, the natural rate depends on the exogenous stochastic processes and on the current level of the capital stock. In none of these cases, monetary policy exerts a direct influence on the natural rate. In fact, it is always possible to rewrite the flexible price version of these models in two separate blocks of equilibrium conditions. The first can be used to solve for the real variables (and for the equilibrium real rate of interest). The second consists of a Fisher relation, linking the real interest rate to the nominal rate and to expected inflation, and a monetary policy rule. This latter block provides a solution for the path of inflation and the nominal interest rate.

In our model, the dichotomy between the natural rate of interest and monetary policy is lost. One way to realize this is by observing that the flexible price version of this model differs from the one previously described only through the retail price setting equations (16)-(18). These need to be replaced by the condition  $\chi_t = \chi = \frac{\varepsilon-1}{\varepsilon}$ , for all  $t$ . Since the nominal interest rate affects the real side of the economy through the terms of the financial contract and the policy spread  $\Delta_{m,t}$ , the equilibrium real rate of interest under flexible prices can only be computed after specifying the policy rule adopted by the monetary authorities.

The lack of dichotomy in the flexible price version of our model is due to the presence of two frictions. The first arises because of the transactions role of money, which is captured by

the timing of households' decisions as adopted in Lucas and Stokey (1987). Money demand is an integral part of the system of equilibrium conditions, implying that the natural rate depends on real balances and thus on monetary policy. There are circumstances, however, under which money continues to play no role on the real allocation of resources even under Lucas and Stokey's timing. Woodford (2003) shows that the dependence of the equilibrium upon real balances is eliminated if actual money balances are remunerated at a constant spread below the policy instrument.<sup>2</sup> In our model, we can obtain a similar result by remunerating money holdings - more precisely by setting a constant spread  $\Delta_{m,t} = \Delta_m$ , for all  $t$ .

The second friction arises because of nominal debt contracts between firms and financial intermediaries. Nominal debt introduces an interdependence between the real and nominal sides of the economy, which is not due to price rigidities. To the extent that nominal debt contracts provide a realistic characterization of the financing of firms, this interdependence cannot be broken. Nevertheless, we can define a notion of the natural rate of interest which would prevail under ideal conditions, in the same way in which the natural rate is typically defined for an ideal flexible-price economy when prices are sticky in the actual economy.

We demonstrate next that there exists a modified concept of natural rate of interest which can be defined independently of monetary policy. We show that this concept can be used in the conduct of monetary policy and that a natural rate rule (i.e. a standard Taylor-type rule that uses this concept of natural rate as an intercept) ensures price stability at all times. Nonetheless, a necessary condition for the achievement of price stability under this rule is a constant spread between the policy instrument and the return on money. If the spread varies over time, e.g. if money is not remunerated, a natural rate rule is not able to stabilize inflation.

### 3.1 A modified definition of natural rate

Consider an economy where the central bank is able to remunerate money as to maintain a constant spread  $\Delta_{m,t} = \Delta_m$ , for all  $t$ . In such an economy, the presence of agency costs breaks the dichotomy between monetary policy and the real interest rate arising under flexible prices only if firms' debt is denominated in nominal terms. If debt was real, firms' financing costs would be given by the real interest rate and would therefore be unaffected by the underlying level of inflation.

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<sup>2</sup>See Woodford (2003, Chapter 4) for a discussion of the real-balance effects and Woodford (2003, Appendix A.16) for the extension to the alternative timing assumed by Lucas and Stokey (1987).



In the standard sticky-price model, the natural rate is defined as the real interest rate which would prevail in a notional flexible-price equilibrium. In the presence of financial frictions, we propose a modified definition of the natural rate of interest as the real rate of return arising in a model where the central bank is able to remunerate money at a rate that is proportional to the policy rate and all nominal frictions are absent, i.e.: i) prices have always been fully flexible and are expected to remain so in the indefinite future; ii) external finance takes the form of real debt.

We denote with a subscript  $n$  the variables characterizing this natural equilibrium. To solve for the natural rate, we consider an economy where the spread  $\Delta_{m,t}$  is constant, prices are flexible, and the government is able to compensate households for the presence of expected inflation by providing a subsidy equal to the ratio between the real rate of interest and the nominal return on a one-period risk-free asset, i.e.

$$\Omega_{n,t} = \frac{E_t \left[ \frac{U_c(c_{n,t+1}; \xi_{t+1}) + U_m(c_{n,t+1}; \xi_{t+1})}{U_c(c_{n,t}; \xi_t) + U_m(c_{n,t}; \xi_t)} \right]}{E_t \left[ \frac{U_c(c_{n,t+1}; \xi_{t+1}) + U_m(c_{n,t+1}; \xi_{t+1})}{U_c(c_{n,t}; \xi_t) + U_m(c_{n,t}; \xi_t)} \frac{1}{\pi_{n,t+1}} \right]}. \quad (40)$$

Notice that under this subsidy, the effective return paid by the financial intermediary to the household corresponds to the real interest rate, i.e.

$$\left( \frac{1 + i_{n,t}}{\Omega_{n,t}} \right)^{-1} = \beta E_t \left[ \frac{U_c(c_{n,t+1}; \xi_{t+1}) + U_m(c_{n,t+1}; \xi_{t+1})}{U_c(c_{n,t}; \xi_t) + U_m(c_{n,t}; \xi_t)} \right] = (1 + r_{n,t})^{-1}. \quad (41)$$

The non-linear system of equilibrium conditions describing this natural equilibrium is reported in Appendix B. Here, we find it convenient to use the system log-linearized around a steady state with zero inflation, which can be written in matrix form as

$$\begin{bmatrix} E_t \widehat{Z}_{n,t+1} \\ \widehat{X}_{n,t+1} \end{bmatrix} = \underset{(10 \times 10)}{\Upsilon} \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} + \underset{(10 \times 4)}{\Sigma} s_t \quad (42)$$

$$E_t s_{t+1} = \underset{(4 \times 4)}{\Phi_s} s_t + \varepsilon_t, \quad (43)$$

where

$$\begin{aligned} \widehat{Z}_{n,t} &\equiv \left[ \widehat{c}_{n,t} \quad \widehat{c}_{n,t} \quad \widehat{h}_{n,t} \quad \widehat{I}_{n,t} \quad \widehat{q}_{n,t} \quad \widehat{\rho}_{n,t} \quad \widehat{\omega}_{n,t} \quad \widehat{r}_{n,t} \right]', \\ \widehat{X}_{n,t} &\equiv \left[ \widehat{k}_t \quad \widehat{z}_t \right]', \\ s_t &\equiv \left[ A_t \quad \xi_t \quad \psi_t \quad \widehat{\gamma}_t \right]', \end{aligned}$$

$\varepsilon_t$  is a vector of iid random processes,  $\Upsilon$  and  $\Sigma$  are coefficient matrices, and variables with a hat denote percentage deviations of a variable from its steady state level.

Using standard methods (see Appendix C), the system can be solved to yield

$$\widehat{X}_{n,t+1} = \widehat{X}_{n,t+1}^s + \Psi_{xx}\widehat{X}_{n,t}, \quad (44)$$

$$E_t\widehat{Z}_{n,t+1} = E_t\Psi_{zs'}(L^{-1})s_t - (V_1')^{-1}V_2'\Psi_{xx}\widehat{X}_{n,t}, \quad (45)$$

where  $\widehat{X}_{n,t+1}^s$ ,  $\Psi_{xx}$  and  $\Psi_{zs'}$  are defined in appendix C. Here  $\widehat{X}_{n,t+1}^s$  denote the component of the evolution of the endogenous state variables that depend only on exogenous real disturbances, while  $\Psi_{xx}$  and  $\Psi_{zs'}$  are coefficient matrices.

Notice that the equilibrium condition for the natural rate of interest  $\widehat{r}_t^n$ , i.e. the log-linearized version of equation (41), can be written as

$$\widehat{r}_{n,t} = g_0'\widehat{Z}_{n,t} + g_1'E_t\widehat{Z}_{n,t+1} + g_2's_t.$$

Substituting for  $\widehat{Z}_{n,t}$  and  $E_t\widehat{Z}_{n,t+1}$  (using equations (54) and (57) in Appendix C), we obtain

$$\widehat{r}_{n,t} = \widehat{r}_{n,t}^s + \Psi_{rx}\widehat{X}_{n,t}, \quad (46)$$

where  $\widehat{r}_{n,t}^s$  denotes the component of the natural rate that exclusively depends on exogenous real disturbances and  $\Psi_{rx}$  is a coefficient matrix (both defined in Appendix C). Equation (46) solves for the natural rate of interest as a function of the exogenous stochastic processes, given the actual values of the endogenous state variables,  $\widehat{X}_{n,t} \equiv \widehat{X}_t$ . The natural rate is independent of any nominal variable, and thus of the monetary policy rule. Our definition of the natural rate is therefore apt to be used as a policy indicator.

In the next section, we explore its properties when used as the intercept of a Taylor-type rule.

### 3.2 Price stability under a Wicksellian monetary policy regime

Here we show that our definition of the natural rate of interest can be used in the conduct of monetary policy. More specifically, if the spread between the policy instrument and the return on money is kept constant, a policy such that the short-term real interest rate tracks our definition of the natural rate is able to ensure price stability in an economy with price rigidities and nominal debt contracts. If money cannot be remunerated or if the spread varies over time, however, price stability cannot be achieved. To show that this result is independent

of the subsidy  $\Omega_t$ , we assume that no such subsidy is available in the actual economy, i.e.  $\Omega_t = 1$ .

As a first step, it is useful to write the log-linearized system of equilibrium conditions in terms of gaps between the observed variables and those arising in an equilibrium with flexible prices and subsidized nominal debt.

Define the vector of policy instruments as  $\widehat{\delta}_t \equiv \left[ \widehat{i}_t \quad \widehat{\Omega}_t \quad \widehat{\Delta}_{m,t} \right]'$  and denote with a tilde the gap of each variable relative to its natural level. Notice that  $\widetilde{\chi}_t = \widehat{\chi}_t$ ,  $\widetilde{\pi}_t = \widehat{\pi}_t$ ,  $\widetilde{\Omega}_t = -\widehat{\Omega}_{n,t}$  and  $\widetilde{\Delta}_{m,t} = \widehat{\Delta}_{m,t}$ . The system can thus be written as

$$\begin{bmatrix} E_t \widetilde{Z}_{1,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} = \begin{matrix} \Upsilon' \\ (11x11) \end{matrix} \begin{bmatrix} \widetilde{Z}_{1,t} \\ \widetilde{X}_t \end{bmatrix} + \begin{matrix} \Xi' \\ (11x3) \end{matrix} \widetilde{\delta}_t + \begin{matrix} \Psi' \\ (11x1) \end{matrix} \widetilde{r}_t \quad (47)$$

$$\widetilde{r}_t = \widehat{i}_t - \kappa'_0 E_t \widetilde{Z}_{1,t+1} - \widehat{r}_{n,t} \quad (48)$$

$$\widehat{i}_t = v_t + \phi_\pi \kappa'_1 \widetilde{Z}_{1,t} \quad (49)$$

where for convenience we have listed separately the condition for the gap between the real rate of interest and its natural level. Here

$$\begin{aligned} \widetilde{Z}_{1,t} &\equiv \left[ \widehat{c}_t - \widehat{c}_{n,t} \quad \widehat{e}_t - \widehat{e}_{n,t} \quad \widehat{h}_t - \widehat{h}_{n,t} \quad \widehat{I}_t - \widehat{I}_{n,t} \quad \widehat{q}_t - \widehat{q}_{n,t} \quad \widehat{\rho}_t - \widehat{\rho}_{n,t} \quad \widehat{\omega}_t - \widehat{\omega}_{n,t} \quad \widehat{\pi}_t \quad \widehat{\chi}_t \right]' \\ \widetilde{X}_t &\equiv \left[ \widehat{k}_t - \widehat{k}_{n,t} \quad \widehat{z}_t - \widehat{z}_{n,t} \right]', \quad \widetilde{\delta}_t = \left[ 0 \quad -E_t \widehat{\pi}_{t+1} \quad \widehat{\Delta}_{m,t} \right]'. \end{aligned}$$

Notice that  $\kappa'_0$  in equation (48) and  $\kappa'_1$  in equation (49) denote vectors with all zeros except for the coefficients on current and expected inflation, respectively. Also, since our definition of a natural equilibrium is based on the current value of the predetermined variables,  $\widetilde{X}_t = 0$ .

We now investigate whether an interest rate rule that uses the natural rate as an intercept, i.e. a rule such that  $v_t = \widehat{r}_{n,t}$  is consistent with price stability.

**Case I:**  $\widehat{\Delta}_{m,t} = 0$ .

Complete stabilization of inflation implies that  $\widehat{\pi}_t = 0$ , for all  $t$ , so that  $\widetilde{\delta}_t = 0$  and

$$\begin{aligned} \begin{bmatrix} E_t \widetilde{Z}_{1,t+1} \\ \widetilde{X}_{t+1} \end{bmatrix} &= \overline{\Upsilon}' \begin{bmatrix} \widetilde{Z}_{1,t} \\ 0 \end{bmatrix} + \overline{\Psi}' \widetilde{r}_t \\ \widetilde{r}_t &= \widehat{i}_t - \widehat{r}_{n,t} \\ \widehat{i}_t &= \widehat{v}_t \end{aligned}$$

A necessary condition for the gaps to be closed at all times, i.e. for  $E_t \widetilde{Z}_{1,t+1} = \widetilde{Z}_{1,t} = 0$  for all  $t$ , is

$$\widehat{v}_t = \widehat{r}_{n,t}. \quad (50)$$

Notice that a monetary policy rule satisfying condition (50) is consistent with an equilibrium with zero inflation at all times, but it might be consistent also with many other equilibria. To ensure that the zero-inflation equilibrium is the only one decentralized by a natural rate rule, we need to impose restrictions on the policy parameters that achieve determinacy of rational-expectations equilibrium. To do so, substitute condition (50) into equations (47)-(49). Using the definition of the vector of policy instruments  $\tilde{\delta}_t$ , we can rewrite the system as

$$\begin{bmatrix} E_t \tilde{Z}_{1,t+1} \\ \tilde{X}_{t+1} \end{bmatrix} = \begin{matrix} \Upsilon' \\ (11 \times 11) \end{matrix} \begin{bmatrix} \tilde{Z}_{1,t} \\ 0 \end{bmatrix} + \begin{matrix} \Xi' \\ (11 \times 3) \end{matrix} \begin{bmatrix} 0 \\ E_t \tilde{Z}_{1,t+1} \\ 0 \end{bmatrix} + \begin{matrix} \Psi' \\ (11 \times 1) \end{matrix} \tilde{r}_t,$$

$$\tilde{r}_t = \phi_\pi \kappa_1 \tilde{Z}_{1,t} - \kappa_0 E_t \tilde{Z}_{1,t+1},$$

or as

$$\begin{bmatrix} E_t \tilde{Z}_{2,t+1} \\ \tilde{X}_{t+1} \end{bmatrix} = \begin{matrix} F' \\ (12 \times 12) \end{matrix} \begin{bmatrix} \tilde{Z}_{2,t} \\ 0 \end{bmatrix},$$

where now  $\tilde{Z}_{2,t} \equiv [\tilde{Z}_{1,t}, \tilde{r}_t]$ . Since the system includes two pre-determined variables, equilibrium determinacy under a natural rate rule is ensured if and only if the matrix  $F'$  has exactly 10 eigenvalues outside the unit circle (i.e. with modulus greater than one in absolute value).

**Case II:**  $\hat{\Delta}_{m,t} = \frac{\Gamma'}{\Gamma} \hat{i}_t$ .

Complete stabilization of inflation implies that  $\hat{\pi}_t = 0$ , for all  $t$ , and

$$\begin{bmatrix} E_t \tilde{Z}_{1,t+1} \\ \tilde{X}_{t+1} \end{bmatrix} = \bar{\Upsilon}' \begin{bmatrix} \tilde{Z}_{1,t} \\ 0 \end{bmatrix} + \bar{\Xi}' \begin{bmatrix} 0 \\ 0 \\ \frac{\Gamma'}{\Gamma} \hat{v}_t \end{bmatrix} + \bar{\Psi}' \tilde{r}_t$$

$$\tilde{r}_t = \hat{i}_t - \hat{r}_{n,t}$$

$$\hat{i}_t = \hat{v}_t$$

It follows that a policy where condition (50) holds leads to the following path of the endogenous variables

$$E_t \tilde{Z}_{1,t+1} = \begin{matrix} \Upsilon'_{11} \\ (9 \times 9) \end{matrix} \tilde{Z}_{1,t} + \begin{matrix} \Xi'_{13} \\ (9 \times 1) \end{matrix} \frac{\Gamma'}{\Gamma} \hat{r}_{n,t}.$$

Hence, a natural rate rule is not consistent with complete stabilization of prices, since it does not implement an equilibrium where  $E_t \tilde{Z}_{t+1} = \tilde{Z}_t = 0$ , for all  $t$ .

Our results show that remuneration of the monetary base such that the spread  $\Delta_{m,t}$  remains constant is necessary for price stability. On the contrary, the amount of government subsidy

$\Omega_t$  is irrelevant. A Wicksellian monetary policy rule such as the one described above is able to ensure inflation stabilization also if the government is unwilling or unable to compensate households for the effect of expected inflation. The subsidy is only a necessary conceptual device for defining a natural rate of interest that is independent of monetary policy.

## 4 Numerical analysis

Here, we illustrate our results through a numerical analysis. We consider the polar cases where money balances are remunerated ( $\Delta_m = \text{const}$  case) and where their own return is zero ( $i_m = 0$  case). We solve the model either under a Taylor rule with constant intercept and a 1.5 coefficient on inflation (for the sake of clarity, there is no response to deviations of output from its natural level) or under a natural rate rule (i.e. the same rule where the intercept is equal to the natural rate of interest).

Structural parameter values are set following Carlstrom and Fuerst (1997). More specifically, monitoring costs are set at 25% of the firm's output and we calibrate the standard deviations of idiosyncratic shocks and the average entrepreneurs' time preference so that approximately 1% of firms go bankrupt each quarter and the annualized spread between loan rates and the policy interest rate is approximately 2%.<sup>3</sup> As to monopolistic competition and retail pricing, we assume  $\varepsilon = 7$ , implying a steady-state mark-up of 17%, and a probability of not being able to re-optimize prices  $\theta = 0.66$ , implying that prices are changed on average every 3 quarters. Finally, the money demand function  $L(c_t, \Delta_{m,t}; \xi_t)$  is calibrated so that steady state real balances are approximately 1% of steady state output.

Concerning shocks, the technology process is specified as in typical RBC calibrations (again following Carlstrom and Fuerst, 1997), namely as an autoregressive process with autocorrelation coefficient  $\rho_a = 0.95$  and standard deviation  $\sigma_a = 0.01$ . All the other shock processes are assumed to have an autocorrelation coefficient equal to 0.9; their standard deviations are normalized so as to generate an impact response of the natural rate similar to its response to a technology shock.

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<sup>3</sup>This implies  $\mu = 0.25$ ,  $\gamma = 0.947$  and  $\sigma_{\omega_i} = 0.207$ .

## 4.1 Impulse responses

Figures 1 and 2 show the impulse responses to a 1-standard deviation technology shock under a Taylor rule with an intercept equal to the natural rate for our benchmark model with credit frictions and constant  $\Delta_m$ , and for the corresponding standard RBC model without credit frictions.

In spite of the different formulation of the financial contract, our results concerning real variables are very close to those in Figure 2 of Carlstrom and Fuerst (1997). The notable difference in the impulse responses of the model with agency costs, compared to the RBC model, is the hump-shaped response of investment and output (as well as hours, not shown in the figure). The difference is mainly due to the dynamic behavior of net worth. This is initially constrained by the fact that entrepreneurial capital is fixed. The increased demand for capital, however, leads to an increase in the price of capital and in the return on internal funds, thus, over time, an increase in entrepreneurial capital and net worth. Net worth keeps increasing as long as the price of capital is above the baseline, i.e. for three periods. Thereafter, the impulse responses of output and investment mimic those of the RBC model.

To reap the benefits of the higher return on internal funds and invest more, entrepreneurs also reduce their consumption sharply (50% on impact). The corresponding share of output is partly consumed by households, whose consumption can therefore increase much more, on impact, than in the RBC case. This effect only lasts two periods, because in the third period after the shock entrepreneurs also start consuming more than in steady state. Thereafter, households' consumption follows the same pattern as in the RBC model (albeit at a slightly lower level because of the output share consumed by entrepreneurs).

The behavior of the natural rate (which coincides with the nominal interest rate, since  $\Delta_m$  is constant) is determined by the dynamic response of consumption. In the RBC case, expected consumption growth is positive but decreasing over time, which leads to a protracted increase in the natural rate. On the contrary, the natural rate falls on impact in the model with agency costs, because of the expected fall in household consumption from the peak of the impact response. The dynamics of the natural rate are consistent with those of the RBC model as of the fourth period after the shock.

As demonstrated above, the policy rule which tracks the natural interest rate is indeed consistent with the maintenance of price stability at all times. In the model with agency costs,

the technology shock would tend to create some initial deflationary pressures. The impact fall in the policy interest rate provides the stimulus to aggregate demand which is necessary to ensure that no pressures ensue on marginal costs and final prices.

Figures 3 and 4 illustrate, for the model with agency costs, how the performance of a natural rate rule differs in the two cases of constant  $\Delta_m$  and  $i_m = 0$ . The source of the differences between these two cases is in the behavior of real balances. In the constant  $\Delta_m$  case, real balances mimic the dynamic response of consumption because of a transactions demand motive (this is clearer from the graph at the bottom right corner of figure 2). When  $i_m = 0$ , however, the demand for real balances is also affected by the decrease in their opportunity costs, the nominal interest rate. The graph at the bottom right corner of figure 4 shows that this component is important in our calibration: the increase in real balances is 30 times larger than in the constant  $\Delta_m$  case. The higher demand for real balances is satisfied through an increase in money supply and a decrease in the price of final goods, which induces a mild deflation in the economy. The case where  $i_m = 0$  also generates a larger decline in the nominal interest rate and in the loan rate. The lower cost of external finance leads to a higher demand for investment despite the slow motion of the firm's net worth, generating an increase in the price of capital. The expansionary effect on investment prevails over the lower response of consumption. Thus, aggregate output increases above the level observed when  $\Delta_m$  is constant.

Figures 5 and 6 compare the dynamic behavior of the economy for the case where  $\Delta_m$  is constant under different policy rules, namely a natural rate rule and a Taylor rule with constant intercept and an inflation response coefficient equal to 1.5. Under a simple Taylor rule, the impact reduction of the policy interest rate (and of the loan rate) is smaller than that under the natural rate rule and insufficient to achieve price stability. The ensuing initial deflation, nonetheless, has little effect on the real variables. Compared to the natural rate rule case, the demand for capital and investment rises slightly less on impact. The expansion in aggregate output is also less pronounced (although to a limited extent) than under the natural rate rule.

Finally, figure 7 compares the dynamic responses of the natural rate to various economic shocks in the model with credit frictions and the RBC model. The general message is that the responses in the initial periods are significantly different for any shock considered. This is obvious for shocks originating in the financial block of the model, which is absent in the RBC world. It is also the case, however, for the other shocks. More precisely, in the cases of technology and labour supply shocks, the impact response have the opposite sign compared

to those of the RBC model. For shocks to consumption preferences, the signs of the impact responses are identical, but their size is significantly larger in the model with credit frictions.

## 5 Conclusions

We have argued that the definition of the natural rate of interest as the equilibrium real rate of return in a model with flexible prices is not a useful tool for the conduct of monetary policy if money offers liquidity services and if firms' external finance takes the form of nominal debt contracts. In our sticky price model with transactions frictions and nominal debt contracts, we have shown that, if the spread between the own return on money and the nominal interest rate is constant over time, there is a definition of natural interest rate which: (i) is independent of monetary policy, and (ii) delivers price stability if used as the intercept of an interest rate rule. This result, however, does not hold if the spread cannot be held constant, e.g. if base money were not remunerated. In this case, inflation cannot be stabilized using the natural rate as an intercept of the Taylor rule.

We have also assessed quantitatively the relevance of credit frictions for the notion of the natural rate of interest. Our numerical analysis shows that on impact the response of the natural rate is markedly different in a model with credit frictions relative to a corresponding real business cycle model. However, over the medium-term the dynamic behavior of the natural rate converges to the one observed in the absence of credit frictions.

## Appendix

### A. The reduced-form system of equilibrium conditions

Define  $p_t(j) \equiv \frac{P_t(j)}{P_t}$  and variables with a tilde as the corresponding variables in a log-linear approximation around a steady state with zero inflation. The log-linearized system of equilibrium



conditions can be written as a block of intra-temporal conditions

$$\begin{aligned}
(1 + \bar{v}) \widehat{h}_t &= -\sigma^{-1} \widehat{c}_t - \sigma_{\Delta}^{-1} \widehat{\Delta}_{m,t} + \widehat{y}_t + \widehat{\chi}_t + \bar{h}_c \xi_t - \bar{h}_h \psi_t \\
\widehat{\rho}_t &= \widehat{y}_t + \widehat{\chi}_t - \widehat{k}_t \\
\widehat{y}_t &= \frac{c}{y} \widehat{c}_t + \frac{e}{y} \widehat{e}_t + \frac{I}{y} \widehat{I}_t \\
\widehat{y}_t &= \widehat{A}_t + \alpha \widehat{h}_t + (1 - \alpha) \widehat{k}_t \\
\epsilon_{\omega} \widehat{\omega}_t &= \widehat{i}_t - \widehat{q}_t - \widehat{\Omega}_t \\
\widehat{I}_t &= \frac{qg' \bar{\omega} n}{I} \frac{(1+i)}{\Omega} \widehat{\omega}_t - \left( \frac{I}{n} - 1 \right) (\widehat{i}_t - \widehat{\Omega}_t) + \widehat{z}_t + \frac{q(1-\delta)}{[q(1-\delta) + \rho]} \widehat{q}_t + \frac{\rho}{[q(1-\delta) + \rho]} \widehat{\rho}_t.
\end{aligned}$$

a block of law of motions for the state variables

$$\begin{aligned}
\widehat{k}_{t+1} &= (1 - \delta) \widehat{k}_t + \delta \widehat{I}_t - \mu \phi \bar{\omega} \frac{I}{k} \widehat{\omega}_t \\
\widehat{z}_{t+1} &= \frac{(e + qz)}{qz} \left( \widehat{I}_t + \frac{f' \bar{\omega}}{f} \widehat{\omega}_t \right) - \frac{e}{qz} \widehat{e}_t - \frac{q(z - fI)}{qz} \widehat{q}_t
\end{aligned}$$

a block of intertemporal conditions

$$\begin{aligned}
\widehat{i}_t &= \eta_c^{-1} (E_t \widehat{c}_{t+1} - \widehat{c}_t) + \eta_{\Delta}^{-1} (E_t \widehat{\Delta}_{m,t+1} - \widehat{\Delta}_{m,t}) - \bar{h}_{\xi} (E_t \xi_{t+1} - \xi_t) + E_t \widehat{\pi}_{t+1}, \\
&\sigma_c^{-1} (E_t \widehat{c}_{t+1} - \widehat{c}_t) + \sigma_{\Delta}^{-1} (E_t \widehat{\Delta}_{m,t+1} - \widehat{\Delta}_{m,t}) - \bar{h}_c (E_t \xi_{t+1} - \xi_t) \\
&= \frac{q(1-\delta)}{[q(1-\delta) + \rho]} E_t \widehat{q}_{t+1} - \widehat{q}_t + \frac{\rho}{[q(1-\delta) + \rho]} E_t \widehat{\rho}_{t+1}, \\
\widehat{q}_t &= \widehat{\gamma}_t + \left( \frac{g' \bar{\omega} I \Omega}{n(1+i)} + \frac{f'}{f} \right) E_t \widehat{\omega}_{t+1} + \left( 1 - \frac{I}{n} \right) E_t (\widehat{i}_{t+1} - \widehat{\Omega}_{t+1}) \\
&+ \left[ 1 + \frac{q(1-\delta)}{[q(1-\delta) + \rho]} + \frac{qgI\Omega}{n(1+i)} \right] E_t \widehat{q}_{t+1} + \frac{\rho}{[q(1-\delta) + \rho]} E_t \widehat{\rho}_{t+1}, \\
\widehat{\pi}_t &= \lambda \widehat{\chi}_t + \beta E_t \widehat{\pi}_{t+1},
\end{aligned}$$

and a block of policy functions

$$\begin{aligned}
\widehat{i}_t &= v_t + \phi_{\pi} \pi_t \\
\widehat{\Delta}_{m,t} &= \frac{\Gamma'}{\Gamma} \widehat{i}_t \\
\widehat{\Omega}_t &= E_t \widehat{\pi}_{t+1},
\end{aligned}$$

where the coefficients need to satisfy

$$\begin{aligned}
\sigma_c^{-1} &\equiv -\frac{U_{cc}c}{U_c}, \quad \sigma_\Delta^{-1} \equiv -\frac{U_{c\Delta}\Delta_m}{U_c}, \quad \bar{v} \equiv h \frac{v_{hh}}{v_h} \\
\bar{h}_c &\equiv \frac{U_{c\xi}}{U_c}, \quad \bar{h}_h \equiv \frac{v_{h\psi}}{v_h}, \quad \bar{h}_\xi \equiv \frac{U_{c\xi} + U_{m\xi}}{U_c + U_m} \\
\eta_c^{-1} &\equiv -\left(\frac{U_{cc} + U_{mc}}{U_c + U_m}\right) c > 0, \quad \eta_\Delta^{-1} \equiv -\left(\frac{U_{c\Delta} + U_{m\Delta}}{U_c + U_m}\right) \Delta_m > 0 \\
\epsilon_\omega &\equiv \frac{\mu\bar{\omega}\phi}{\left(1 - \mu\Phi + \frac{f\phi\mu\bar{\omega}}{f'}\right)} \left[\bar{\omega} - 1 + \frac{f}{f'} \left(1 + \frac{\phi'\bar{\omega}}{\phi} - \frac{\phi\bar{\omega}}{f'}\right)\right] \\
\lambda &\equiv \frac{(1 - \theta)(1 - \theta\beta)}{\theta}.
\end{aligned}$$

The system consists of 15 equations and provides a solution for the path of the endogenous variables  $\{\hat{c}_t, \hat{e}_t, \hat{h}_t, \hat{I}_t, \hat{y}_t, \hat{q}_t, \hat{\rho}_t, \hat{\chi}_t, \hat{\omega}_t, \hat{\pi}_t\}$ , the policy variables  $\{\hat{i}_t, \hat{\Omega}_t, \hat{\Delta}_{m,t}\}$  and the state variables  $\{\hat{k}_{t+1}, \hat{z}_{t+1}\}$ , given the law of motion of the exogenous stochastic processes  $\{A_t, \xi_t, \psi_t, \hat{\gamma}_t\}$ .

## B. The natural equilibrium

We define with a subscript  $n$  the variables obtained in a natural equilibrium, where prices are flexible,  $\Delta_{n,m,t} = \Delta_{n,m}$ , and  $\Omega_{n,t}$  is described by (40), given the currently observed level of all exogenous and predetermined variables.

Recall that in a natural equilibrium,  $\chi_t = \chi = \frac{\varepsilon-1}{\varepsilon}$ . We can thus derive the path of the natural rate of interest by solving the following system of equations,

$$\begin{aligned}
\frac{v_h(h_{n,t}; \psi_t)}{U_c(c_{n,t}; \xi_t)} &= \chi \alpha A_t h_{n,t}^{\alpha-1} k_t^{1-\alpha} \\
(1 + r_{n,t})^{-1} &= \beta E_t \left\{ \frac{U_c(c_{n,t+1}; \xi_{t+1}) + U_m(c_{n,t+1}; \xi_{t+1})}{U_c(c_{n,t}; \xi_t) + U_m(c_{n,t}; \xi_t)} \right\} \quad (51) \\
U_c(c_{n,t}; \xi_t) q_{n,t} &= \beta E_t \left\{ U_c(c_{n,t+1}; \xi_{t+1}) [q_{n,t+1} (1 - \delta) + \rho_{n,t+1}] \right\} \\
\rho_{n,t} &= \chi (1 - \alpha) A_t h_{n,t}^\alpha k_t^{-\alpha} \\
A_t h_{n,t}^\alpha k_t^{1-\alpha} &= c_{n,t} + e_{n,t} + I_{n,t} \\
e_{n,t} + q_{n,t} z_{n,t+1} &= q_{n,t} f(\bar{\omega}_{n,t}) I_{n,t} \\
k_{n,t+1} &= (1 - \delta) k_{n,t} + I_{n,t} [1 - \mu\Phi(\bar{\omega}_{n,t})] \\
q_{n,t} \left[ 1 - \mu\Phi(\bar{\omega}_{n,t}) + \frac{\mu\bar{\omega}_{n,t} f'(\bar{\omega}_{n,t}) \phi(\bar{\omega}_{n,t})}{f'(\bar{\omega}_{n,t})} \right] &= 1 + r_{n,t}
\end{aligned}$$

$$I_{n,t} [(1 + r_{n,t}) - q_{n,t}g(\bar{\omega}_{n,t})] = [q_{n,t}(1 - \delta) + \rho_{n,t}] z_t (1 + r_{n,t})$$

$$q_{n,t} = \beta\gamma_t E_t \left\{ q_{n,t+1} f(\bar{\omega}_{n,t+1}) \left[ \frac{[q_{n,t+1}(1 - \delta) + \rho_{n,t+1}](1 + r_{n,t})}{(1 + r_{n,t}) - q_{n,t+1}g(\bar{\omega}_{n,t+1})} \right] \right\}$$

The system has 10 equations and can be solved for the variables  $\{c_{n,t}, e_{n,t}, h_{n,t}, I_{n,t}, q_{n,t}, \rho_{n,t}, \bar{\omega}_{n,t}, r_{n,t}\}$  and  $\{k_{n,t+1}, z_{n,t+1}\}$ , for given law of motion of the stochastic processes  $\{A_t, \xi_t, \psi_t, \gamma_t\}$  and initial value of the predetermined variables  $\{k_t, z_t\}$ .

### C. Solution of the natural equilibrium: the log-linearized system

The log-linearized system characterizing the natural equilibrium can be written as

$$\begin{bmatrix} E_t \widehat{Z}_{n,t+1} \\ \widehat{X}_{n,t+1} \end{bmatrix} = \underset{(10 \times 10)}{\Upsilon} \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} + \underset{(10 \times 4)}{\Sigma} s_t \quad (52)$$

$$E_t s_{t+1} = \underset{(4 \times 4)}{\Phi_s} s_t + \varepsilon_t, \quad (53)$$

where  $\widehat{Z}_{n,t} \equiv [\widehat{c}_{n,t} \ \widehat{e}_{n,t} \ \widehat{h}_{n,t} \ \widehat{I}_{n,t} \ \widehat{q}_{n,t} \ \widehat{\rho}_{n,t} \ \widehat{\omega}_{n,t} \ \widehat{r}_{n,t}]'$ ,  $\widehat{X}_{n,t} \equiv [\widehat{k}_t \ \widehat{z}_t]'$  and  $s_t \equiv [A_t \ \xi_t \ \psi_t \ \widehat{\gamma}_t]'$ ,  $\varepsilon_t$  is a vector of iid random processes, and

$$\underset{(10 \times 10)}{\Upsilon} \equiv \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ (8 \times 8) & (8 \times 2) \\ \Upsilon_{21} & \Upsilon_{22} \\ (2 \times 8) & (2 \times 2) \end{bmatrix}, \quad \underset{(10 \times 4)}{\Sigma} \equiv \begin{bmatrix} \Sigma_1 \\ (8 \times 4) \\ \Sigma_2 \\ (2 \times 4) \end{bmatrix},$$

are coefficient matrices. Let

$$V' \Upsilon = \Lambda V',$$

where all the eigenvalues of  $\Lambda$  are outside the unit circle. The system (52) can be rewritten as

$$V' \begin{bmatrix} E_t \widehat{Z}_{n,t+1} \\ \widehat{X}_{n,t+1} \end{bmatrix} = \Lambda V' \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} + V' \Sigma s_t.$$

Define now

$$\widehat{\varphi}_{n,t} \equiv V' \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix} = \begin{bmatrix} V'_1 & V'_2 \end{bmatrix} \begin{bmatrix} \widehat{Z}_{n,t} \\ \widehat{X}_{n,t} \end{bmatrix}.$$

It follows that

$$\begin{aligned} \widehat{\varphi}_{n,t} &= \Lambda^{-1} \{E_t \widehat{\varphi}_{n,t+1} - V' \Sigma s_t\} \\ &= E_t [\Psi_{\varphi_s} (L^{-1}) s_t] \end{aligned}$$

where  $\Psi_{\varphi s}(L^{-1}) \equiv -\Lambda^{-1} \left( \frac{1}{1-\Lambda^{-1}L^{-1}} \right) V' \Sigma$ , and

$$\begin{aligned}\widehat{Z}_{n,t} &= (V'_1)^{-1} \left( \widehat{\varphi}_{n,t} - V'_2 \widehat{X}_{n,t} \right) \\ &= (V'_1)^{-1} \left[ E_t \left[ \Psi_{\varphi s}(L^{-1}) s_t \right] - V'_2 \widehat{X}_{n,t} \right].\end{aligned}\quad (54)$$

Moreover,

$$\widehat{X}_{n,t+1} = \Upsilon_{21} \widehat{Z}_{n,t} + \Upsilon_{22} \widehat{X}_{n,t} + \Sigma_2 s_t \quad (55)$$

$$= \widehat{X}_{n,t+1}^s + \Psi_{xx} \widehat{X}_{n,t}, \quad (56)$$

where  $\widehat{X}_{n,t+1}^s \equiv \left\{ \Upsilon_{21} (V'_1)^{-1} E_t \Psi_{\varphi s}(L^{-1}) + \Sigma_2 \right\} s_t$  is the component of the evolution of the endogenous state variables that depend only on exogenous real disturbances, and  $\Psi_{xx} \equiv \Upsilon_{22} - \Upsilon_{21} (V'_1)^{-1} V'_2$ . Also,  $E_t \widehat{\varphi}_{n,t+1} = E_t \left[ \Psi_{\varphi s}(L^{-1}) \Phi_s s_t \right]$ . It follows that

$$\begin{aligned}E_t \widehat{Z}_{n,t+1} &= (V'_1)^{-1} E_t \widehat{\varphi}_{n,t+1} - (V'_1)^{-1} V'_2 \widehat{X}_{n,t+1} \\ &= E_t \Psi_{zs'}(L^{-1}) s_t - (V'_1)^{-1} V'_2 \Psi_{xx} \widehat{X}_{n,t},\end{aligned}\quad (57)$$

where  $E_t \Psi_{zs'}(L^{-1}) = (V'_1)^{-1} \left[ E_t \Psi_{\varphi s}(L^{-1}) \Phi_s - V'_2 \left\{ \Upsilon_{21} (V'_1)^{-1} E_t \left[ \Psi_{\varphi s}(L^{-1}) \right] + \Sigma_2 \right\} \right]$ . The complete solution to the system of equilibrium conditions is thus given by (44)-(45) in the main text.

Notice that the equilibrium condition for the equilibrium real interest rate  $\widehat{r}_t^n$ , equation (51), can be written as

$$\widehat{r}_{n,t} = g'_0 \widehat{Z}_{n,t} + g'_1 E_t \widehat{Z}_{n,t+1} + g'_2 s_t.$$

Substituting the solution for  $\widehat{Z}_{n,t}$  and  $E_t \widehat{Z}_{n,t+1}$ , we obtain

$$\widehat{r}_{n,t} = \widehat{r}_{n,t}^s + \Psi_{rx} \widehat{X}_{n,t}.$$

where  $\widehat{r}_{n,t}^s \equiv \left[ g'_0 E_t \Psi_{\varphi s}(L^{-1}) + g'_1 E_t \Psi_{zs'}(L^{-1}) + g'_2 \right] s_t$  denotes the component of the natural rate that exclusively depends on exogenous real disturbances and  $\Psi_{rx} \equiv -g'_0 (V'_1)^{-1} V'_2 - g'_1 (V'_1)^{-1} V'_2 \Psi_{xx}$ .

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Figure 1: IRF to a technology shock – Nat.rate rule,  $\Delta_m = \text{const}$

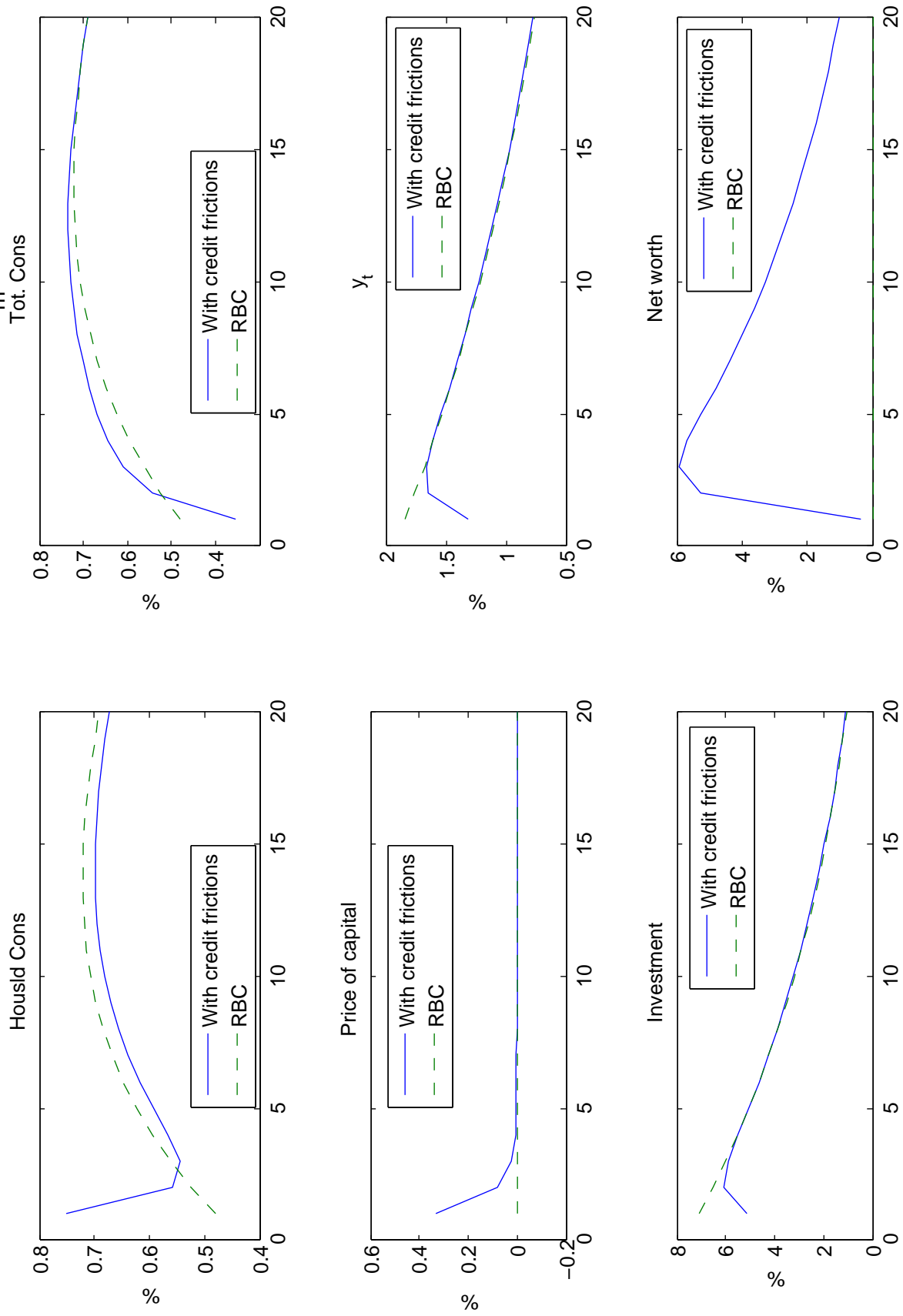


Figure 2: IRF to a technology shock  $-\Delta_m = \text{const}$

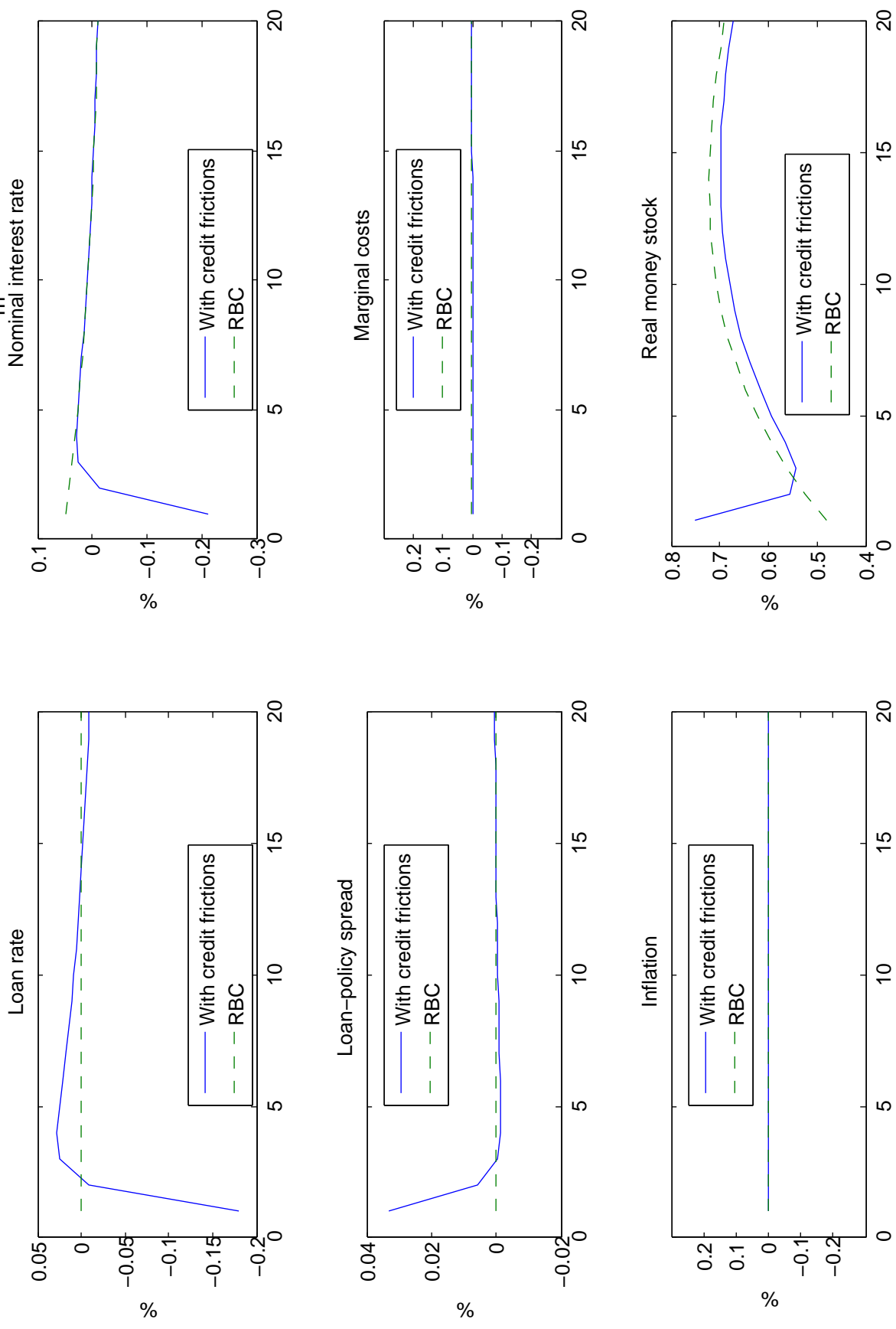




Figure 3: CREDIT MODEL: IRF to a technology shock

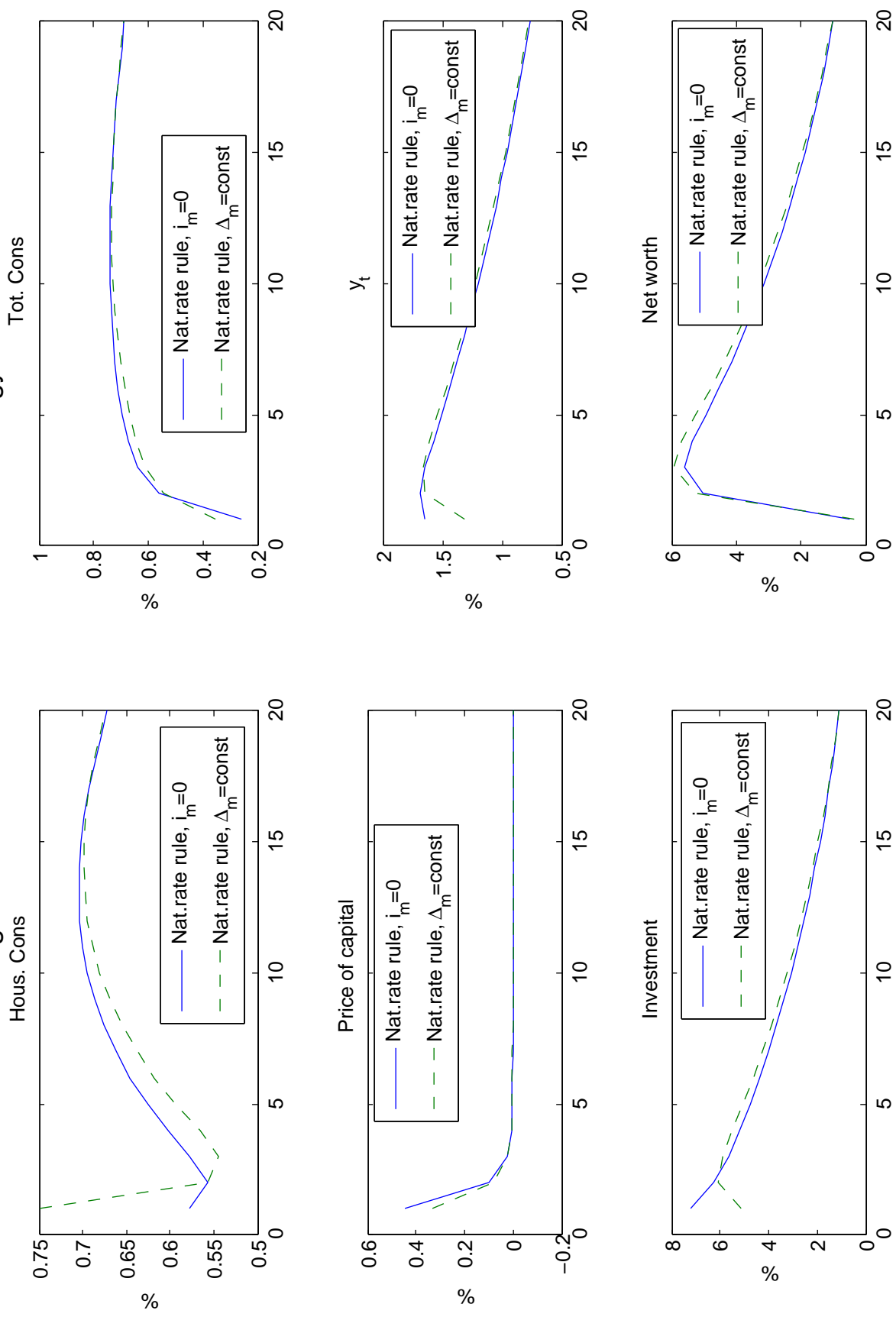


Figure 4: CREDIT MODEL: IRF to a technology shock

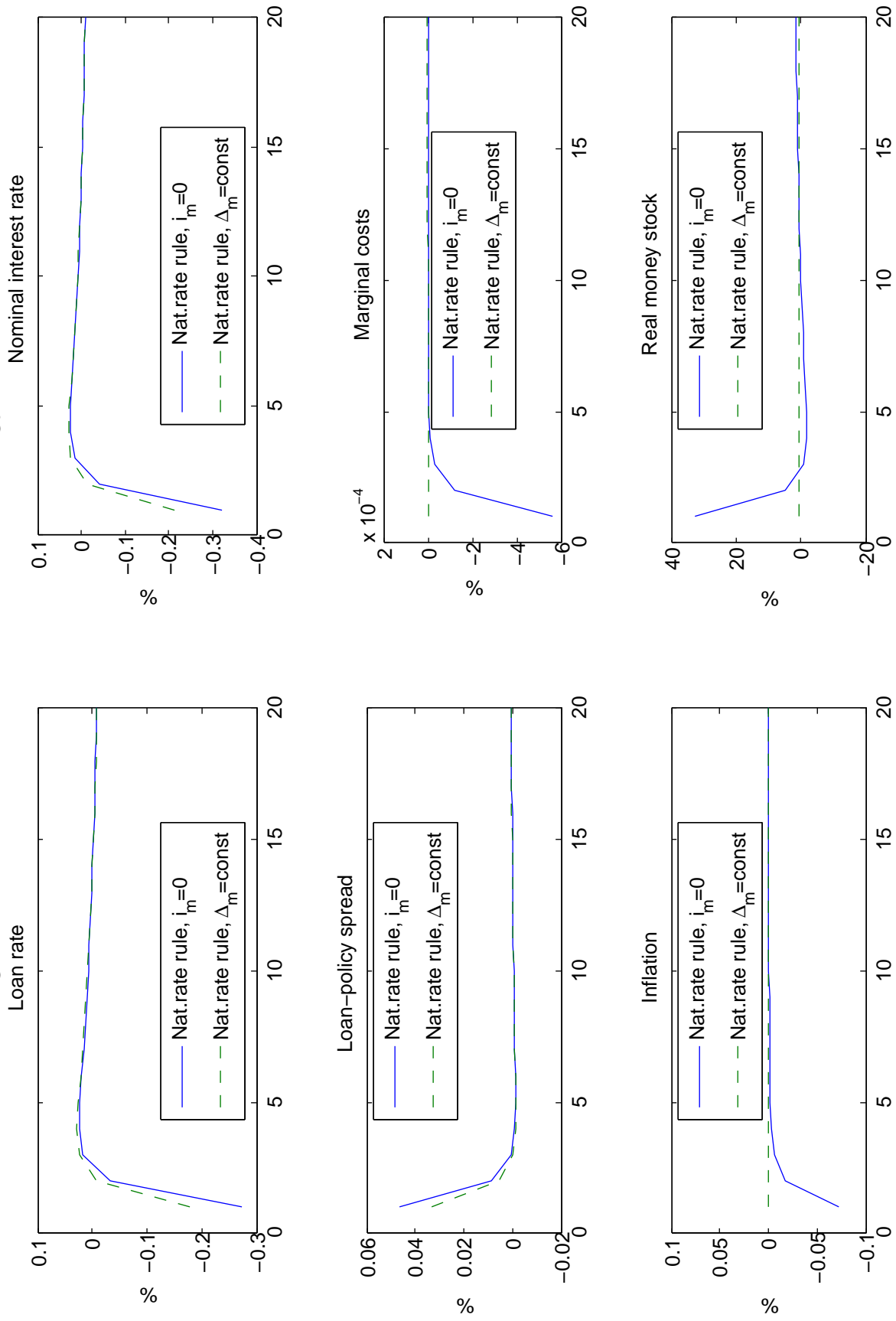


Figure 5: CREDIT MODEL – IRF to a technology shock

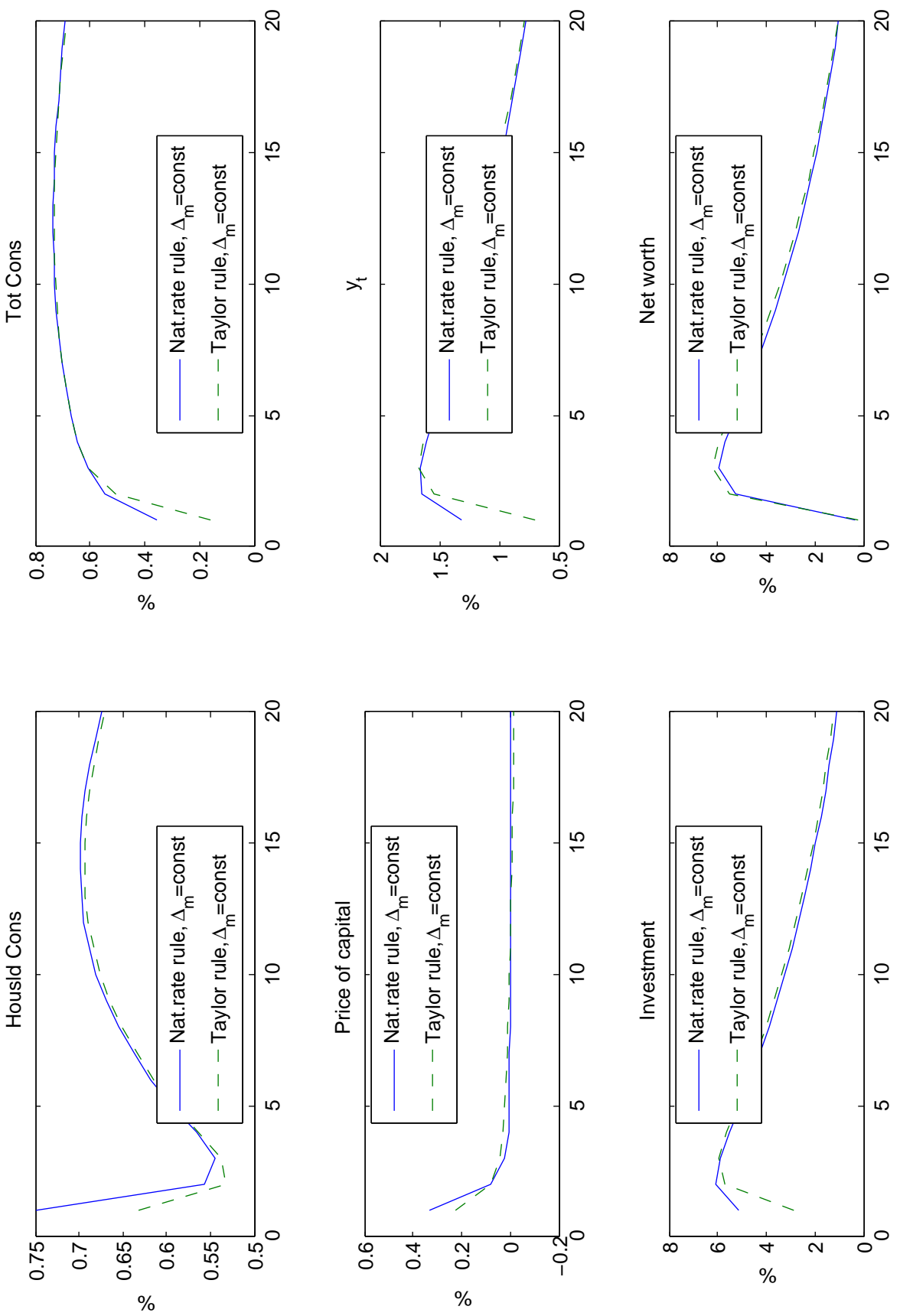


Figure 6: CREDIT MODEL – IRF to a technology shock

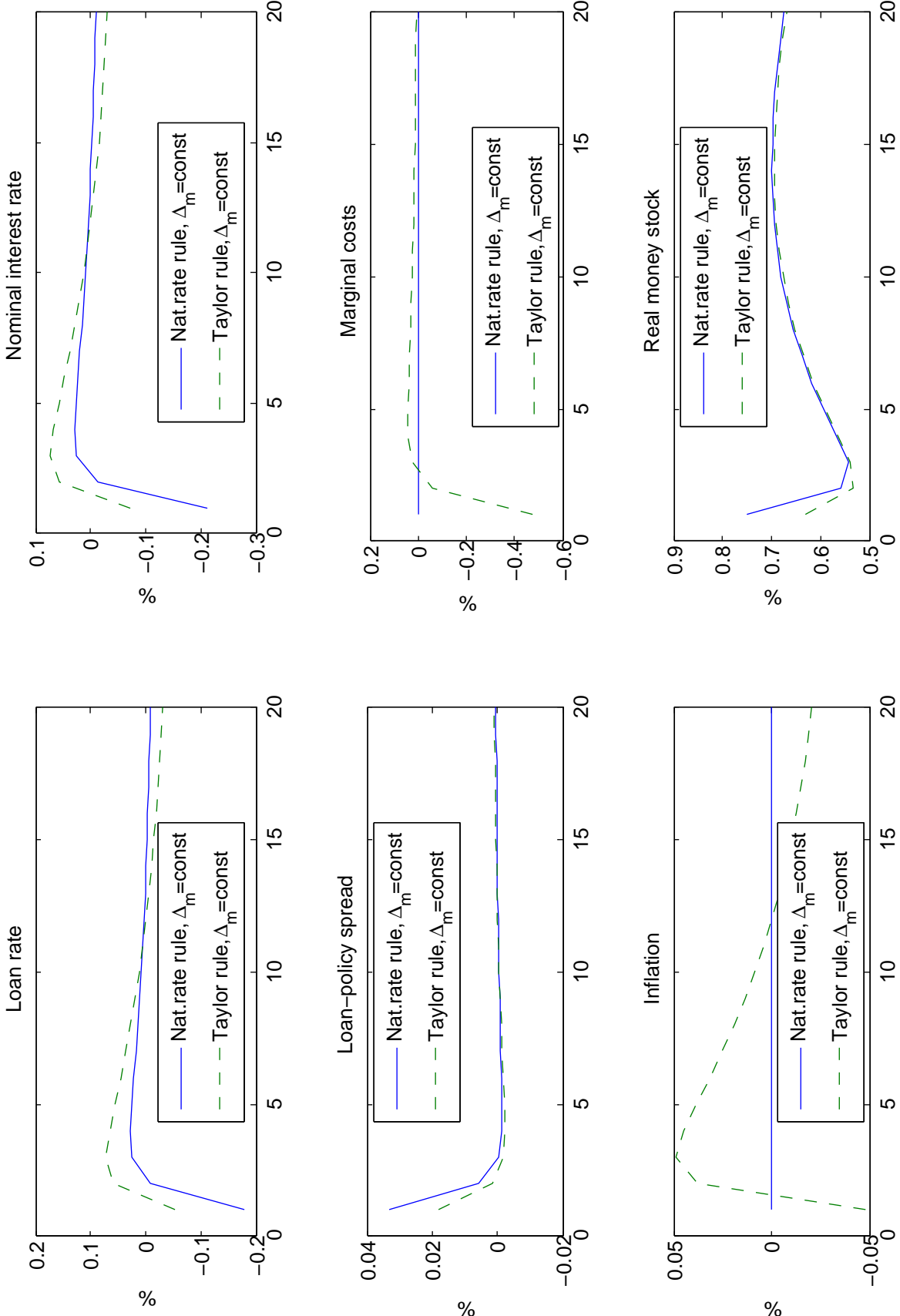


Figure 7: IRF of the natural rate

