Stock Market Volatility and Learning

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Introduction

Research agenda:

Convince the profession: learning-induced small deviations from rationality can significantly improve understanding of economic phenomena.

The strategy:

Simple models of learning can explain what appear to be puzzles from the viewpoint of the (fully) rational expectations literature

Previous Examples:

Marcet and Nicolini (2003): **repeated hyperinflations in South America and their termination**

Adam (2005, 2007): **response of output & inflation to MP shocks**

persistence of output and inflation

Introduction

The aim of this talk:

A very simple (Lucas) asset pricing model can replicate many basic asset pricing moments, once small deviations from RE allowed for

Consumption-based asset pricing models with constant discount factors and RE => 'asset pricing puzzles'

(PD ratio, return volatility, return predictability, equity premium)

Here: one parameter extension of the basic model!

Basic predictions of the Lucas model not robust to small departures from full forecast rationality

&

Non-robustness is empirically encouraging!

Introduction

Standard asset pricing model:

Lucas endowment economy with time-separable preferences

and i.i.d. dividend growth

&

Standard learning scheme:

Agents forecast future price and use OLS to estimate forecast functions

Learning:

Converges to RE, but takes long time & transitional dynamics very different

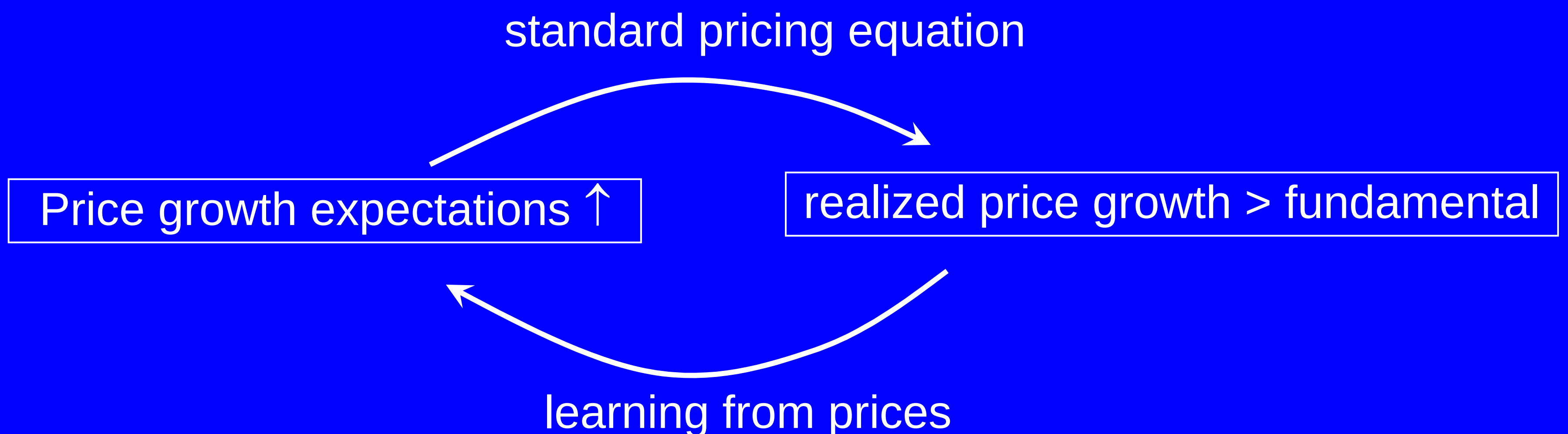
Introduction

Along the convergence process:

Shiller's 'naturally occuring Ponzi schemes'

"Investors, their confidence and expectations buoyed by past price increases, bid up speculative prices further, thereby enticing more investors to do the same, so that the cycle repeats again and again, .. "

Irrational Exuberance, 2005, p.56



Introduction

Statistics	US Data 1925:4-2000:4	Learning Model
$E(r^s)$	2.41	2.41
$E(r^b)$	0.18	0.48
$E(PD)$	113.20	95.93
σ_{r^s}	11.65	13.21
σ_{PD}	52.98	62.19
$\rho_{PD_t,-1}$	0.92	0.94
c_2^5	-0.0048	-0.0067
R_5^2	0.1986	0.3012

Introduction

Literature: models of learning for theoretical analysis to select between REE

Criticism about models of learning to explain empirical facts:

can choose appropriate learning rule to fit any facts

introduces free parameters

Our response:

- use most standard learning rule: OLS
- introduce a <u>single</u> free parameter
- impose restrictions on learning: small deviations from rationality

OLS is best estimator in the long-run, beliefs converges to RE

transitional departures small: initial belief rational

high (not complete) confidence in initial belief

free parameter set to zero: learning model = RE model

Related Literature

- Timmermann (1993,1996)
- Bullard and Duffy (2001)
- Brock and Hommes (1998)
- Brennan and Xia (2001)
- Cogley and Sargent (2006)
- Carceles and Giannitsarou (2006)

Main differences to literature:

- agents forecast future price: crucial !
- fully non-linear model
- standard representative agent assumption
- correctly specified forecasting models
  & emphasis on small deviations from rationality

Outline of talk

I. Basic RE model ⇔ basic facts

II. Basic model with learning:
analytical results

III. Risk neutral model with learning: illustrate

IV. Calibrate learning model with risk aversion

I. Basic model & facts

Stochastic endowment economy (Lucas 1978):

div./cons. growth i.i.d.

$$\frac{D_t}{D_{t-1}} = a\varepsilon_t$$

$$\log \varepsilon_t \sim N(-\tfrac{s^2}{2}, s^2)$$

Time-separable utility

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{(C_t)^{1-\sigma} - 1}{1-\sigma}$$

$$s.t.: P_t S_t + C_t = (P_t + D_t) S_{t-1}$$

FOC at C=D

$$P_t = \delta E_t\left[\left(\frac{D_t}{D_{t+1}}\right)^{\sigma}(P_{t+1} + D_{t+1})\right]$$

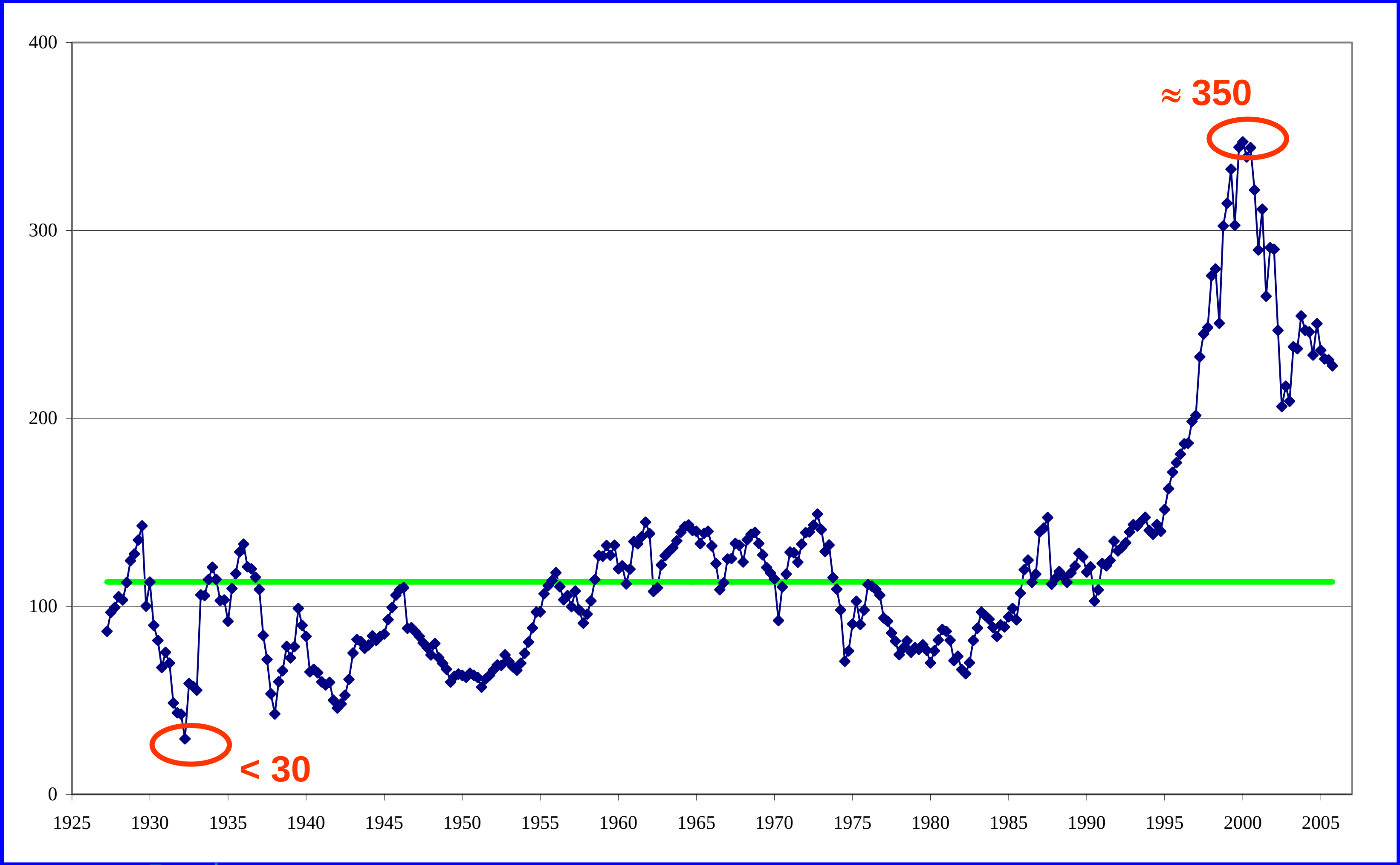
Unique stationary solution

$$P_t = \frac{\delta\beta}{1-\delta\beta} D_t$$

$$\beta = a^{1-\sigma} e^{-\sigma(1-\sigma)\frac{s^2}{2}}$$

I. Basic model & facts

Quarterly U.S. Price Dividend Ratio 1927:1-2005:4



Persistence

I. Basic model & facts

Fact 1: The PD ratio is very volatile

E(PD)	113.2
σ_{PD}	52.9

Response to the volatility puzzle in the literature:

- habit models: volatile MRS
- MRS only (!) degree of freedom for RE theorist (if *iid* D/C growth)

$$E_0 \sum_{t=0}^{\infty} \delta^t \frac{(\mathbb{C}_t)^{1-\sigma} - 1}{1-\sigma}$$

$$\mathbb{C}_t = H(C_t, C_{t-1}, C_{t-2}, \dots)$$

Abel (1990):

$$\mathbb{C}_t = \frac{C_t}{C_{t-1}^{\kappa}}$$

$$\frac{P_t}{D_t} = A(a\varepsilon_t)^{\kappa(\sigma-1)}$$

PD ratio

I. Basic model & facts

Fact 2: The PD ratio is very persistent

ρ_{PD}	0.92

REE models need volatile & <u>persistent</u> MRS

Campbell & Cochrane (1999) successfully engineer habit function

- delivers all the facts that we address in this paper

- complex/many parameters <u>link</u>

- high effective (relative) risk-aversion: 35 in SS & higher out of SS

I. Basic model & facts

Shiller (1981), LeRoy & Porter (1981): prices move 'too much'

Redid Shiller-style variance bound analysis:

25 yrs after publication (Japan, EMU, US, 1984-2005) still true!

$$r_t^s = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} = \left[\frac{\frac{P_t}{D_t} + 1}{\frac{P_{t-1}}{D_{t-1}}} \right] \frac{D_t}{D_{t-1}} - 1$$

Time separable utility & iid dividends: PD ratio constant

$VAR(r^s) \approx VAR(D_t/D_{t-1})$

Fact 3: Stock returns are 'excessively' volatile

$\sigma_{r\wedge s}$	**11.65**
$\sigma_{\Delta D/D}$	**2.98**

I. Basic model & facts

Fact 4: Excess returns predictable over long horizons

$$X(t,t+s) = c_0^s + c_1^s \cdot PD(t)$$

Years	Coefficient on PD, c_1^s	R^2
1	-0.0008	0.0438
3	-0.0023	0.1196
5	-0.0048	0.1986
10	-0.0219	0.3285

I. Basic model & facts

Although not the focus of the paper, we also look at

Fact 5: Equity premium puzzle

$E(r^b)$	**0.18**
$E(r^s)$	**2.41**

Known since Prescott and Mehra (1985)…

U.S. asset pricing facts, 1927:2-2000:4

(quarterly real values, growth rates & returns in percentage terms)

Fact 1	Volatility of PD ratio	$E(PD)$ σ_{PD}	113.20 52.98
Fact 2	Persistence of PD ratio	$\rho_{PD_t,PD_{t-1}}$	0.92
Fact 3	Excessive return volatility	σ_{r^s} $\sigma_{\frac{\Delta D}{D}}$	11.65 2.98
Fact 4	Excess return predictability	c_2^5 R_5^2	-0.0048 0.1986
Fact 5	Equity premium	$E[r^s]$ $E[r^b]$	2.41 0.18

Standard RE model

PD ratio constant

very persistent (trivially so)

Returns as volatile as dividend growth

No excess return predictability

Tiny equity premium for reasonable risk aversion

II. Simplest learning model

Illustrate economic mechanism that explains quantitative success of learning model

Most basic asset pricing model: risk neutral model

Add general learning scheme

& derive analytical results

II. Simplest learning model

Risk-neutral asset pricing

$$P_t = \delta \tilde{E}_t [P_{t+1} + D_{t+1}]$$

$$D_t / D_{t-1} = a \varepsilon_t$$

Literature: **(Bayesian) learning about dividend process (Timmermann, Sargent & Cogley, Brennan & Xia)**

$$P_t = \tilde{E}_t \sum_{j=1}^{\infty} \delta^j D_{t+j}$$

- **overall limited asset pricing implications**
- **no feed-back from prices into beliefs**

II. Simplest learning model

Learning here: **abstract from dividend learning (baseline)**

$$P_t = \delta \tilde{E}_t(P_{t+1}) + \delta E_t(D_{t+1})$$

Study the implications of forecasting future price
(what real investors' seem to care about)

RE: $E_t\left[\frac{P_{t+1}}{P_t}\right] = a$ Learning: $\tilde{E}_t\left[\frac{P_{t+1}}{P_t}\right] = \beta_t$

Forecasting price:

$\Rightarrow$ **endogenous vars: Bayesian/rational learning not well defined**

$\Rightarrow$ **near-rational learning, rational only asymptotically:**
agents use price growth observed in the past to estimate β_t

II. Simplest learning model

The evolution of β_t determined by learning scheme $f_t(\cdot)$

$$\Delta\beta_t = f_t\left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right)$$

$$f_t(0) = 0$$
$$f_t' > 0$$

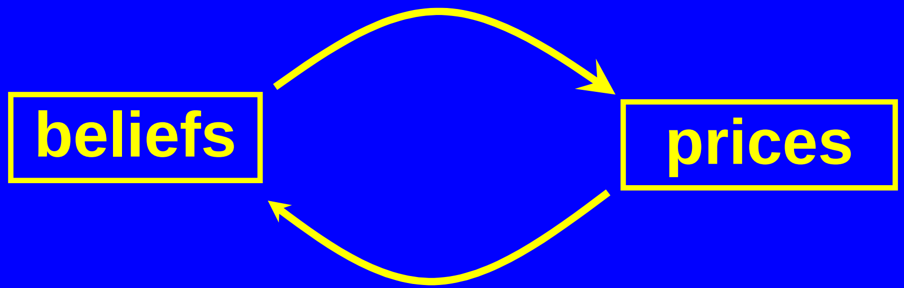
$$f_t \text{ s.t. } 0 < \beta_t < \delta^{-1}$$

Quantitative section:

- **add restrictions to have only small deviations from rationality**
- **will show that asymptotically rationality is obtained**
- **initial beliefs at the REE and**
- **parameterize distance of learning model from RE model**

II. Simplest learning model

Crucial feature of learning : self-referential & dynamic



'Naturally occuring Ponzi schemes' & 'data like' behavior

II. Simplest model (learning)

Dynamics of price growth under learning:

$$P_t = \delta \tilde{E}_t(P_{t+1}) + \delta E_t(D_{t+1})$$ **&** $$\tilde{E}_t\left[\frac{P_{t+1}}{P_t}\right] = \beta_t$$ **=>** $$P_t = \frac{\delta a}{1-\delta\beta_t} D_t$$

Realized price growth:

$$\frac{P_t}{P_{t-1}} = \underbrace{\left(a + \frac{a\delta\,\Delta\beta_t}{1-\delta\beta_t}\right)}_{=:T(\beta_t,\Delta\beta_t)} \varepsilon_t$$

Belief dynamics:

$$\Delta\beta_{t+1} = f_{t+1}\left(T(\beta_t,\Delta\beta_t)\varepsilon_t - \beta_t\right)$$

- **2nd order non-linear diff eqn: no closed form solution**
- **highly non-linear: T-map has asymptote at $\delta\beta_t = 1$**
- **beliefs dynamics <=> dynamics of PD ratio**

Analytic results about belief/price dynamics:

Qualitative: illustrate potential to generate interesting data-like behavior

To show that results come from learning:

deterministic dynamics ($\varepsilon_t = 1$)

II. Simplest model (learning)

(1) Around the REE: stock price changes display momentum

For all $0 < \beta_t < \delta^{-1}$:

$$\text{if } \Delta\beta_t > 0, \text{ then } \frac{P_t}{P_{t-1}} > a$$

$$\text{if } \Delta\beta_t < 0, \text{ then } \frac{P_t}{P_{t-1}} < a$$

Momentum at the REE ('naturally occuring Ponzi scheme'):

$$\beta_t = a \text{ and } \Delta\beta_t > 0 \Rightarrow \Delta\beta_{t+1} > 0$$

$$\beta_t = a \text{ and } \Delta\beta_t < 0 \Rightarrow \Delta\beta_{t+1} < 0$$

beliefs ↑ -> price growth ↑ -> future beliefs ↑

II. Simplest model (learning)

(2) Prices and beliefs display mean reversion in the long-run

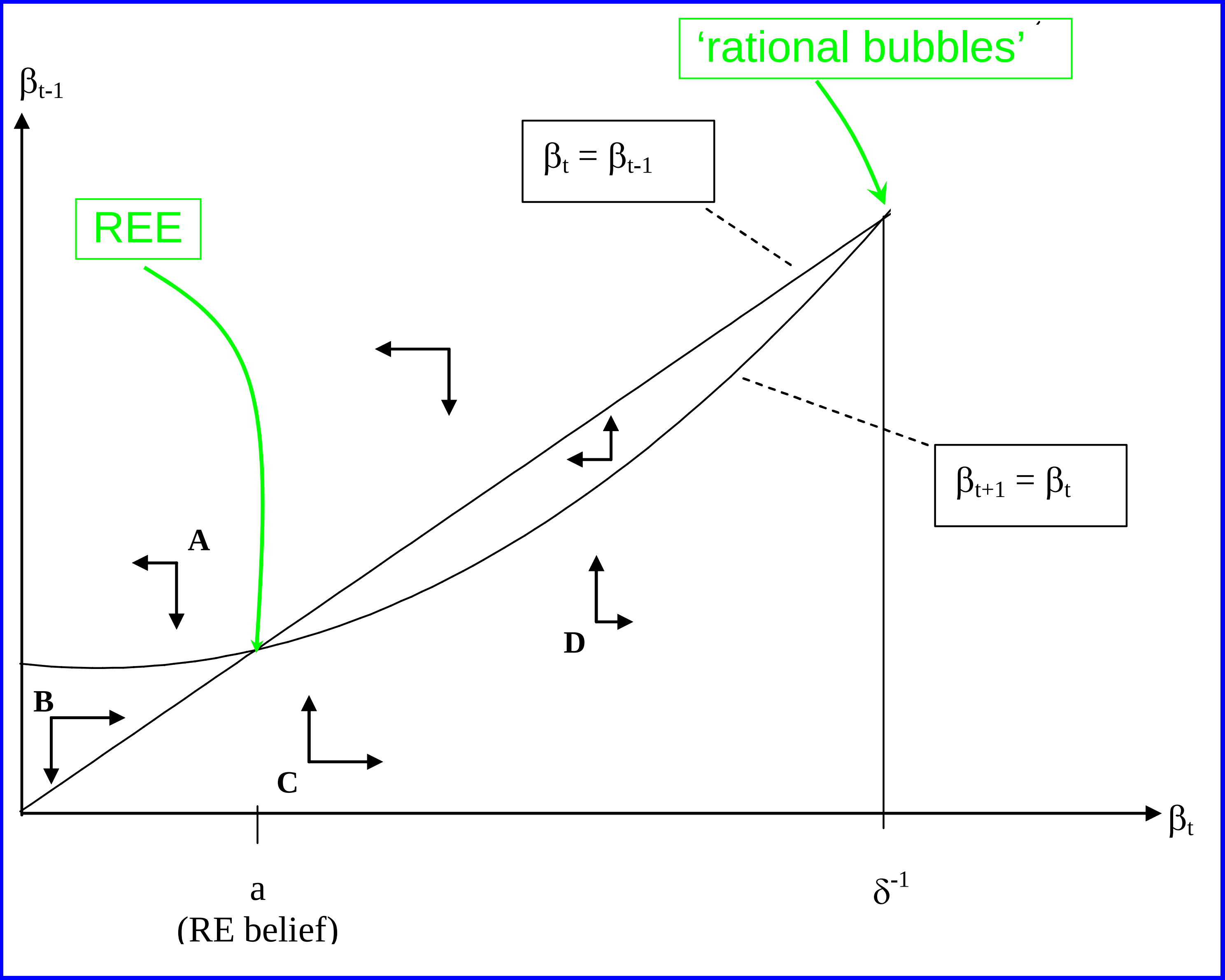
For any $\eta > 0$ and t such that $\beta_t > a + \eta$,
there is a finite t' such that $\beta_{t'} < a + \eta$

Note: β_t can be arbitrarily high & η arbitrarily small!

Similarly for low beliefs:

For any $\eta > 0$ and t such that $\beta_t < a - \eta$,
there is a finite t'' such that $\beta_{t''} > a - \eta$

Phase diagram



II. Simplest model (learning)

Momentum & mean reversion =>

- **Large & persistent movements in PD ratio (Facts 1+2)**
- **Excess return predictability (Fact 4)**
- **Excess volatility (Fact 3)**

$$Var\left(\ln \frac{P_t}{P_{t-1}}\right) = Var\left(\ln \frac{1-\delta\beta_{t-1}}{1-\delta\beta_t}\right) + Var\left(\ln \frac{D_t}{D_{t-1}}\right)$$

- **Simulation results show: also equity premium – a surprise to us.**

III. Simulating the Risk Neutral Model

Combine:

(1) Most standard pricing model: risk neutral model

&

(2) Most standard learning scheme:

Ordinary least squares (OLS)

Simulate: ***learning dramatically improves asset price behavior!***

III. Simulating the Risk Neutral Model

Expectations function

$$\tilde{E}_t\left[\frac{P_{t+1}}{P_t}\right] = \beta_t$$

Learning rule $f_t(\cdot)$:

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t}\left[\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right]$$

PJF: no updating if implied PD>500

Similar constraints in models of Bayesian learning

$$\beta_0 = a$$
$$1/\alpha_1 \in [0,1] \quad \text{given.}$$
$$1/\alpha_t = 1/(\alpha_{t-1}+1) \quad t \geq 2$$

Initial belief: centered at RE value

Confidence in initial belief:

$1/\alpha_1 = 0$: full , learning -> RE

$1/\alpha 1 = 1$: none, pure OLS

β_t: average of observed sample growth rate and α_1 'observations' of fundamental growth a

Single free parameter introduced by learning: *$1/\alpha_1$*

III. Simulating the Risk Neutral Model

Attractive features of OLS learning setup

1. Standard & parsimonious: single free parameter ($1/\alpha_1$)

2. 'Can define small deviations from rationality'

Theorem: 'Asymptotic Rationality'

globally $\beta_t \to a$ almost sure.

$1/\alpha_1 \to 0$: reduces to RE

3. Asymptotically learning optimal: returns are iid and OLS estimate is posterior mode of Bayesian estimate

III. Simulating the Risk Neutral Model

Calibration	*RE model*	*Learning*
Mean div growth rate (a)	*0.35%*	*idem*
Std div growth rate (s)	*2.98%*	*idem*
Discount factor (δ)	*0.9877*	*idem*
Initial gain ($1/\alpha_1$)	*0.00*	*0.02*

Statistic	US Data	RE model
$E(r^s)$	2.41	1.24
$E(r^b)$	0.18	1.24
$E(PD)$	113.20	113.20
σ_{r^s}	11.65	3.01
σ_{PD}	52.98	0.00
$\rho_{PD,-1}$	0.92	-
c_2^5	-0.0048	-
R_5^2	0.1986	0.00

III. Simulating the Risk Neutral Model

The equity premium…

$$\prod_{t=1}^{T} \frac{P_t + D_t}{P_{t-1}} = \underbrace{\prod_{t=1}^{T} \frac{D_t}{D_{t-1}}}_{=R_1} \cdot \underbrace{\left(\frac{PD_T + 1}{PD_0} \right)}_{=R_2} \cdot \underbrace{\prod_{t=1}^{T-1} \frac{PD_t + 1}{PD_t}}_{=R_3}$$

R1: independent of expectations formation

R2: positive premium of $PD_T > PD_0 = PD^{RE}$, but….

R3: positive premium if on average $PD < PD^{RE}$ under learning

‘convergence from below’ (as in Cogley and Sargent)

convex in PD: volatility of PD helps generate equity premium

IV. Model with Risk Aversion

Evaluate quantitative performance of learning model

Introduce risk-aversion: match volatility in the data

Basic insight from risk-neutral version extend:

- momentum & mean reversion of beliefs/prices
- asymptotic rationality ($\beta_t \to \beta^{RE}$)
- $1/\alpha_1 \to 0$ learning model reduces to RE model

IV. Model with Risk Aversion

Asset pricing equation:

$$P_t = \delta \tilde{E}_t\left(\left(\frac{D_t}{D_{t+1}}\right)^\sigma P_{t+1}\right) + \delta E_t\left(\frac{D_t^\sigma}{D_{t+1}^{\sigma-1}}\right)$$

Learning on 'risk-adjusted' price growth:

$$\tilde{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^\sigma \frac{P_{t+1}}{P_t}\right] = \beta_t$$

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t}\left[\left(\frac{D_{t-2}}{D_{t-1}}\right)^\sigma \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right]$$

Volatility of risk-adj price growth
under RE increases with σ for $\sigma > 1$

IV. Model with Risk Aversion

5 Parameters: $(a, s, \sigma, 1/\alpha_1, \delta)$

- (a, s) → Mean & Std of div growth in the data
- σ → $\sigma = 5$ 'low' risk aversion
- $(1/\alpha_1, \delta)$ → 8 stock price 'moments':

$$\hat{S}' \equiv \left(\hat{E}(r^s), \hat{E}(PD), \hat{\sigma}_{r^s}, \hat{\sigma}_{PD}, \hat{\rho}_{PD_{t,-1}}, \hat{c}_2^5, \hat{R}_5^2, \hat{E}(r^b)\right)$$

$$\min_{\delta \le 1, 1/\alpha_1} \left(S^M - \hat{S}\right) W \left(S^M - \hat{S}\right)'$$

$$W^{-1} = \begin{pmatrix} \hat{\sigma}_{S_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{\sigma}_{S_7} \end{pmatrix}$$

Invariant to rescaling of variables

$\hat{\sigma}_{S_i}$ Estimated from data, calibration literature uses model-implied std…

IV. Model with Risk Aversion

2 parameters

$$\left(\frac{1}{\alpha_1}, \delta\right)$$

8 stock price 'moments':

$$\widehat{\mathcal{S}}' \equiv \left(\widehat{E}(r^s), \widehat{E}(PD), \widehat{\sigma}_{r^s}, \widehat{\sigma}_{PD}, \widehat{\rho}_{PD_{t,-1}}, \widehat{c}_2^5, \widehat{R}_5^2, \widehat{E}(r^b)\right)$$

Criterion of fit is t-ratio:

$$\frac{\widehat{\mathcal{S}}_i - \mathcal{S}_i}{\widehat{\sigma}_{\mathcal{S}_i}}$$

Fit OK if t-ratio below 2 or 3….

IV. Model with Risk Aversion

Statistics	US Data	std
$E(r^s)$	2.41	0.45
$E(r^b)$	0.18	0.23
$E(PD)$	113.20	15.15
σ_{r^s}	11.65	2.88
σ_{PD}	52.98	16.53
$\rho_{PD_{t,-1}}$	0.92	0.02
c_2^5	-0.0048	0.002
R_5^2	0.1986	0.083

V. Robustness I

Statistic	US Data	Learning on Div.	t-ratio
$E(r^s)$	2.41	2.41	0.00
$E(r^b)$	0.18	0.48	-1.29
$E(PD)$	113.20	96.17	1.12
σ_{r^s}	11.65	13.23	-0.55
σ_{PD}	52.98	62.40	-0.57
$\rho_{PD_t,-1}$	0.92	0.94	-1.22
c_2^5	-0.0048	-0.0067	0.96
R_5^2	0.1986	0.2982	-1.20
Parameters:			
δ		0.999	
$1/\alpha_1$		0.015	

V. Robustness II

C = D and identified with dividends in the data

In the data C smoother than D…

…helped model matching volatility & equity premium

Now allow for C $\neq$ D as in Campbell&Cochrane (1999)

$$\frac{C_{t+1}}{C_t} = a\varepsilon^c_{t+1} \quad \text{for} \quad \ln\varepsilon^c_t \sim iiN(-\frac{s_c^2}{2}; s_c^2)$$

$$s_c = \frac{s}{7} \text{ and } \rho(\varepsilon^c, \varepsilon) = .2$$

V. Robustness II

Statistics	US Data	$C \neq D$	t-ratio
$E(r^s)$	2.41	2.36	0.12
$E(r^b)$	0.18	1.76	-6.91
$E(PD)$	113.20	63.56	3.28
σ_{r^s}	11.65	8.42	1.12
σ_{PD}	52.98	30.14	1.38
$\rho_{PD_t,PD_{t-1}}$	0.92	0.91	0.49
c_2^5	-0.0048	-0.0073	1.2410
R_5^2	0.1986	0.2641	-0.7911
Parameters:			
δ		1	
$1/\alpha_1$		0.0178	

V. Robustness III

Relaxing the constraint $\delta \leq 1$

Statistics	US Data	$C \neq D$	t-ratio
$E(r^s)$	2.41	2.01	0.89
$E(r^b)$	0.18	0.84	-2.89
$E(PD)$	113.20	112.85	0.02
σ_{r^s}	11.65	10.43	0.42
σ_{PD}	52.98	61.16	-0.49
$\rho_{PD_t,PD_{t-1}}$	0.92	0.95	-1.43
c_2^5	-0.0048	-0.0089	2.0440
R_5^2	0.1986	0.2397	-0.4966
Parameters:			
δ		1.00906	
$1/\alpha_1$		0.0244	

V. Conclusions

- Introducing learning into a simple asset pricing model generates a rich set of qualitatively new dynamics

- Learning - induced transitional dynamics to REE allow to match evidence on

  - Mean, volatility and persistence of PD ratio
  - Stock return volatility
  - Excess return predictability
  - Equity premium

- Empirically, learning model seems more plausible than a standard RE model with similar number of parameters.

VI. Outlook

Learning model: rich interactions between asset price dynamics and aspects of the environment

- trend growth changes
- real interest rates (monetary policy)
- risk aversion

Applications of learning to other settings

- exchange rate models

Campbell & Cochrane preferences

Flow consumption utility

$$\frac{(S_t C_t)^{1-\gamma} - 1}{1-\gamma}$$

Law of motion for surplus

$$S_t = \bar{S}^{1-\phi} S_{t-1}^{\phi} \left(\frac{C_t / C_{t-1}}{g} \right)^{\lambda(S_t)}$$

$$\lambda(S_t) = \begin{cases} \frac{1}{\bar{S}} \sqrt{1 - 2\log\left(S_t / \bar{S}\right)} - 1 & for \quad S_t \le S_{\max} \\ 0 & otherwise \end{cases}$$

back