

# Estimating a small DSGE model under rational and measured expectations: some comparisons<sup>1</sup>

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## **Abstract**

Using European panel data and GMM system estimation, we explore the empirical performance of the standard three-equation New Keynesian macro model under different informational assumptions. As a benchmark, we consider the performance of the model under rational expectations and revised (final) data. Alternatively, instead of imposing rational expectations hypothesis we use real time information ie Consensus Economics survey data to get empirical proxies for expectations in the model and the current output gap in the Taylor rule. We demonstrate that contrary to the assumption of rational expectations, the errors in measured expectations and real time current output gaps are positively autocorrelated. We get evidence that the use of real time variables (including measured expectations) improves the empirical performance of the New Keynesian model. Relaxation of the rational expectations hypothesis makes a noticeable difference for the parameters of the New Keynesian model, especially in the Taylor rule.

**Key words:** DSGE model, survey expectations, GMM system estimation

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# 1 Introduction

DSGE models of the New Keynesian type are now widely used in macroeconomic research and monetary policy analysis (Clarida, Galí and Gertler 2000, Galí 2002, Walsh 2003, Woodford 2003, Smets and Wouters 2003 to mention just few). In empirical model analysis various techniques have been used, like maximum likelihood (Ireland 2001), Bayesian techniques (Smets and Wouters 2003) and instrumental variable methods (McCallum and Nelson 1998). Typically research is based on revised (ie final) data and the rational expectations assumption. The empirical relevance of the DSGE models has thus far not been firmly established (See eg Rabanal and Rubio-Ramirez 2005, Del Negro, Schorfheide, Smets and Wouters 2006).

In DSGE models economic behavior is assumed to be based largely on expectations. In the standard three-equation model both expected inflation and the expected output gap have a central role in the model and the assumption of expectations formation is of crucial importance for model dynamics. If we assume that expectations are rational, and if that is in fact not the case, we may get biased parameter estimates in estimating the model, and the policy implications from the model may be distorted. As expectations are formed on the basis of information available at the time, measures of real time information are essential, when analyzing expectations. Measures of expectations actually belong to the category of real time variables.

The relevance of rational expectations assumption for the empirical validity of the New Keynesian DSGE model has not received a lot of attention until quite recently. This is partly due to difficulty to measure expectations. Expectations formation has been studied using the learning approach (Evans and Honkapohja 2001, 2003 and Milani 2007), limited information channels (Woodford 2002, Adam 2007) and sticky information models (Mankiw and Reis 2001, 2002). Also the heterogeneity of expectations (Branch 2004) and the so-called epidemiology approach (Carroll 2001), in which information spreads slowly from experts to the general public, have been examined.

This paper approaches the issue of real-time information and measured

expectations by estimating a three-equation New Keynesian DSGE model using European panel data. All real time variables needed in estimation are obtained from the same source: Consensus Economics survey data. More precisely, we use survey-based inflation and output gap expectations for the next period and real time output gap estimates for the current period. As these real time variables do not include possible subsequent revisions, they reflect information available at the time, when economic decisions were made. The contribution of this paper is to use real time information both for expected inflation and expected output gaps and also for the current output gap in the Taylor rule in the model framework. In this approach we can relax the assumption of rationality (while encompassing it, of course) without making any specific assumption of expectations formation.

Measured expectations have not been studied in the European DSGE model context before. Real time proxies for inflation expectations have been used to analyze European inflation dynamics, but only in a single-equation context (see Paloviita and Mayes 2005, and Paloviita 2006, 2007) or in the VAR model framework (Paloviita and Virén 2005, 2007). Real time current output gap estimates have been analyzed in various studies (Orphanides and van Norden 2002, 2005) but the use of real time output gap expectations, especially in a forward-looking model framework, have been virtually absent.

The empirical analysis in this study focuses both on real time expectational errors and the estimation of the three-equation model. Using the data on real time expectations, it is possible to study expectational errors, ie differences between real time and corresponding revised variables. For comparison, the model itself is estimated under both rational and measured expectations. Alternative Taylor rule specifications, incorporating only current variables, or the inflation forecast, are evaluated. In the rational expectations alternative, the analysis is based on revised data and GMM. When using real time expectations, both least squares and GMM methods are used. A robustness analysis considers how sensitive the results are with respect to alternative specifications of the model equations. Under the real time approach, we also examine whether the results are dependent on the way we treat possible measurement errors and simultaneity problems in estimation.

The results suggest, first of all, that expectational errors with respect to inflation

and the output gap are clearly positively autocorrelated. This implies that deviations from rationality are potentially important, when estimating the model. Indeed, we do find in our model estimations that the relaxation of the rational expectations hypothesis makes a noticeable difference for the model parameters.

Our results are consistent with the findings of Paloviita and Mayes (2005), who use the single-equation approach to study alternative Phillips curve specifications. In their study, real time information is used for all Phillips curve variables and also as instruments in GMM estimation. They argue that the use of real time information is especially important in the expectations term: compared with revised data, real time expectations suggest more forward-looking and better determined inflation dynamics. On the other hand, the effects of real time information for the Taylor rule, found in this study, are compatible with the results by Orphanides (2001). He argues that informational problems and the use of real time variables are essential in the analysis of monetary policy rules - using real-time data for the output gap avoids many problems involved by the rational expectations/perfect foresight approach.

This paper proceeds as follows. Section 2 presents the three-equation model. Section 3 describes the data and analyzes errors in expectations. System estimation results and robustness analysis are reported in section 4. Section 5 concludes.

## 2 The model under rational and measured expectations

We examine a standard three-equation New Keynesian model including the IS and Phillips curves, and a monetary policy rule. Similar three-equation structural models have been analyzed in many empirical studies (see, for example Lindé 2005, and Cho and Moreno 2006). We estimate the model with GMM and two alternative approaches based on rational and measured expectations. Under rationality we assume there are no systematic errors in expectations, which implies that expectational errors are white noise. Under measured expectations, by

contrast, we use survey data to get direct proxies for expectations. Measured expectations may be rational, but they do not have to be. Apart from expectations, measured in real time, the real time version of the model also includes the perceived output gap (in the Taylor rule) instead of the revised (final) output gap estimate. Under both approaches all equations can be formulated with or without endogenous persistence.

When assuming rational expectations and endogenous persistence in the IS and Phillips curves, the model has the following form:

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*) \quad (1)$$

$$\pi_t = (1 - \delta)E_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t \quad (2)$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMU}, \quad (3)$$

where the term  $y_t$  refers to the output gap, and  $r_t$  to the nominal interest rate. The equilibrium interest rate is denoted by  $r^*$  and the inflation rate by  $\pi_t$ .  $E_t$  stands for the rational expectations operator conditional on the information set at time  $t$ . In the Taylor rule the dummy  $D_{EMU}$  refers to the years 1999-2004 ie the Stage Three of the European Monetary Union. Correspondingly, inflation and the output gap series in the Taylor rule,  $\pi_t^{EMU}$  and  $y_t^{EMU}$  are pooled individual country variables until the year 1998 and after that they are euro area aggregates, which are based on official ECB weights<sup>2</sup>.

The IS curve, ie equation (1), can be derived from intertemporal utility maximization of a representative agent (household) with external habit persistence (see Fuhrer 2000). The IS curve relates current output gap to expected and lagged output gaps and the ex post real interest rate. The expected output gap coefficient and the level of habit persistence are inversely related. The basic New Keynesian version of the IS curve without habit formation is obtained when the parameter  $\mu = 0$ .

The New Keynesian Phillips curve (equation 2) is based on staggered price setting, as each monopolistically competitive firm maximizes profits subject to

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<sup>2</sup> More precisely, in aggregation we used ECB GDP weights, based on actual exchange rates.

constraints on the frequency of price adjustments (Calvo 1983). We use the hybrid specification of the New Keynesian Phillips curve (Galí and Gertler 1999) which relates current inflation to expected future inflation, lagged inflation and the current output gap. In the hybrid Phillips curve only some firms are assumed to be forward looking and set their prices optimally. The rest are assumed to be backward looking and use rule of thumb or indexation in their pricing decisions. Thus, there is endogenous persistence also in the hybrid Phillips curve. The specification nests the basic New Keynesian Phillips curve without persistence, when the parameter  $\delta = 0$ .

Equation (3) describes a simple Taylor rule, in which the central bank reacts to current economic conditions ie to inflation and the output gap in the current period. In the model we also consider an alternative form of monetary policy rule with expected inflation (forecast) as an argument (see Clarida, Galí and Gertler 2000, Bernanke and Boivin 2003):

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta E_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU}, \quad (4)$$

According to Taylor principle (Taylor 1999) the central bank can stabilize inflation if it raises the nominal interest more than one for one in response to higher inflation. In fact, if the Taylor principle holds, the central bank increases not only nominal but also the real interest rate.

As an alternative approach we analyze the model under measured expectations. Under this approach equations (1) – (3) must be modified slightly:

$$y_t = (1 - \mu) \bar{E}_t y_{t+1} + \mu y_{t-1} - \phi (r_t - \bar{E}_t \pi_{t+1} - r^*) \quad (5)$$

$$\pi_t = (1 - \delta) \bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t \quad (6)$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMUrt}, \quad (7)$$

where the expectations operator  $\bar{E}$  refers to survey-based real time expectations prevailing at time  $t$  and the term  $y_t^{EMUrt}$  indicates real time estimate for the current output gap, which is used in the monetary policy rule due to informational

limitations and data uncertainty. Real time information is even more important in the alternative form of the Taylor rule also used in this paper:

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E} \pi_{t+1}^{EMU} + \gamma_t^{EMUrt} \quad (8)$$

In estimation, we mainly focus on the models with endogenous persistence only in the IS and Phillips curves. We can incorporate endogenous persistence also in the monetary policy rule. In this case, for example, equation (8) would get the form

$$r_t = \rho r_{t-1} + (1 - \rho)(\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E} \pi_{t+1}^{EMU} + \gamma_t^{EMUrt}). \quad (9)$$

As will be shown, under measured expectations, endogenous persistence (ie interest rate smoothing) seems not to be needed in the Taylor rule.

## 3 Data description and analysis of expectational errors

### 3.1 Data description

Annual pooled revised data until the year 2004 are constructed using OECD National Accounts. When constructing real time variables, Consensus Economics survey data are used<sup>3</sup>. Consumer price changes and 12 month money market rates are used for inflation and the interest rate, respectively, and output gaps are based on HP filtering. When collecting real time expected inflation series for each country, June forecasts for the next year's consumer price inflation are used. Correspondingly, when constructing current and next year's output gap estimates (periods  $t$  and  $t+1$ ), the revised real GDP estimates are used until the period  $t-1$ , and real GDP forecasts for the current and next year are obtained from the June survey. Survey information is available since 1993 for Greece and since 1990 for

the other countries. Thus, in empirical analysis the sample is from 1990 till 2004 and the number of observations is 156.

[Insert Figure 1 here]

Figure 1 gives median values of all model variables. In spite of a small peak in 2000, price developments have been quite stable in euro area countries since the beginning of the 1990s. Inflation expectations have had a quite similar pattern. However, decreasing inflation was overestimated in expectations before the year 2000. Correspondingly, in recent years slightly higher inflation was underestimated. European output gap, which peaked in 2000, was negative in six subsequent years in the 1990s and in the end of the sample. Real time output gaps, which have correct signs in almost all years, have been somewhat less volatile. Two-digit interest rates were experienced in the beginning of the sample, but during the 1990s the level of European interest rates fell sharply.

## 3.2 Analysis of expectational errors

With directly measured expectations we are able to examine how real time and corresponding revised variables are related and whether real time variables are accurate and unbiased as estimates of the revised variables. We can also investigate time series properties of expectational errors. Under rationality, expectational errors should be white noise. If, however, expectational errors are autocorrelated, we get evidence of informational problems and deviation from rationality, which may affect empirical performance of the DSGE model if rational expectations are imposed on it.

[Insert Table 1 here]

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<sup>3</sup> Consensus Economics survey data is available for all euro area countries excluding Luxembourg and Slovenia.



Table 1 shows that the correlation coefficient between revised and expected inflation rates is relatively high: 0.84. This is fairly obvious, since inflation series are typically not subject to substantial revisions<sup>4</sup>. Interestingly, the two real time output gap estimates, current and expected, are also highly correlated (0.85), although they are based on clearly different information sets, which are twelve months apart.

Current output gap estimate is surveyed in June of the current year (ie in the middle of the period, which is surveyed), when some initial information is available from the first months of the year in question. By contrast, corresponding real time expectation for the output gap is formed in June of the previous year, on the basis of information available at the time. Clearly lower correlation between real and revised output gaps reflects revisions, which are common and often non-negligible in real GDP data<sup>5</sup>. The low correlation possibly reflects data uncertainty of economic activity and difficulties to measure potential output in real time. The interest rate is highly positively correlated with inflation rates, but hardly at all correlated with alternative output gaps.

The accuracy of real time variables is investigated by constructing root mean squared errors (RMSE), which are defined as the square roots of the arithmetic averages of the squared differences between real time variables and corresponding revised variables<sup>6</sup>. Both real time current output gap and expected output gap are quite inaccurate (RMSE values are 1.908 and 1.970). Instead, real time and revised inflation rates are basically the same (RMSE = 1.009).

[Insert Table 2 here]

Under rationality, expectations should be unbiased. Unbiasedness of all real time variables is analyzed first estimating, by ordinary least squares, simple

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<sup>4</sup> High correlation of revised and expected inflation series can be also found in Paloviita and Mayes (2005), who use OECD forecasts as a proxy for inflation expectations.

<sup>5</sup> This finding is also consistent with Paloviita and Mayes (2005).

<sup>6</sup>More precisely,  $RMSE = \left( (1/T) \sum_{i=1}^T \{x_i - x_i^*\}^2 \right)^{1/2}$  .

equations of the form  $x = a + bx^*$  where  $x$  refers to a revised variable and  $x^*$  to the corresponding real time variable. The Wald test is used to analyze whether the hypotheses that the constant  $a$  is equal to zero, and the coefficient  $b$  is equal to one, hold. As reported in table 2, in none of the cases the Wald test rejects the unbiasedness at 5 per cent level<sup>7,8</sup>.

On the whole, tables 1 and 2 give at least weak support to the rationality of expectations in a static sense. However, since a more detailed examination of rationality is needed, we continue the analysis by examining also the time series properties of errors in real time variables. If expectational errors are autocorrelated, we find evidence against rationality.

[Insert Table 3 here]

Ljung-Box Q-statistics is used to test time series properties of both expectational errors and residuals of unbiasedness tests. As reported in table 3, for all cases expectational errors to seem be clearly positively autocorrelated, as the test rejects the null hypothesis of no autocorrelation up to order 1-3. Thus, even for the case of inflation rates, we find evidence against rational expectations hypothesis. According to table 3 strong autocorrelation seems to be also present, when we examine residuals of unbiasedness tests.

Finally, we examine orthogonality of all real time variables. Rationality implies that expectational errors are orthogonal to all past information known at the time, when the expectations are formed. We test orthogonality simply by regressing expectational errors on lagged values of all model variables. Using the Wald test we analyze, whether all coefficients included in the regression are jointly equal to

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<sup>7</sup> If Belgium and France, which have the lowest correlations between expected and revised inflation rates, are excluded from the sample, unbiasedness gets clearly more support (in this case F-statistic is 1.607 and probability 0.205).

<sup>8</sup> Qualitatively similar results with pooled euro area data can be found in Paloviita (2006). When OECD inflation forecasts are used to proxy inflation expectations, she gets evidence that in 1977–1990, when inflation was high and volatile in many European countries, inflation expectations were biased. By contrast, the hypothesis of unbiasedness cannot be rejected in the euro area for the period 1991–2003.

zero, as suggested by the rational expectations hypothesis. The null hypothesis is strongly rejected for all three cases<sup>9</sup>.

All in all, the analysis of this section suggests that the rationality of expectations may not be a reasonable assumption to use in the DSGE model context. Deviations from rationality are not necessarily large, but it is worth comparing, whether noisy information and uncertainty, especially in the case of output, makes a noticeable difference for estimated parameters of the model.

## 4 System estimation results

### 4.1 Basic analysis

First, the model is estimated under rational expectations with revised (final) data. In GMM estimation the two alternative specifications of the Taylor rule are used, and the same instrument sets and same modification of standard errors<sup>10</sup> are used in both cases in order to enable a reasonable comparison.

[Insert Table 4 here]

Overall, as reported in table 4, GMM estimation results under rational expectations are broadly reasonable with a few problematic features (in model C the monetary policy is based on current and in model E on expected inflation). As regards the Taylor rule, the Taylor principle holds for both variants of the Taylor rule. However, the estimated parameter for the output gap is always low and imprecise or incorrectly signed. Regarding the IS curve, the results with rational expectations give a weight of slightly over 50% for the forward-looking term (lead of the output gap); a relatively low and not very significant coefficient for

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<sup>9</sup> For the joint hypothesis that all coefficients are equal to zero F-statistics are 83.293 (real time output gap), 40.644 (expected output gap) and 16.724 (expected inflation). In every case the corresponding p-value is equal to zero.

<sup>10</sup> The standard errors are modified using a Bartlett kernel with variable bandwidth (without prewhitening).

the ex post real interest rate, -0.04, is obtained, and the estimated equilibrium interest rate is rather high, always clearly above 3.

The results concerning the Phillips curve with rational expectations give a weight of slightly above 50% for the forward-looking inflation term, matching the degree of persistence in the IS curve. The coefficient of the current output gap is very small, although significant. According to J-statistics for any of the models overidentifying restrictions are not rejected at 5 per cent level.

Next, in view of the evident non-rationality of expectations, as they are measured, we check how the estimated results change if repeated with measured (real-time) expectations instead of imposing the rational expectations assumption. A further motivation for this is the weak performance of the output gap in the Taylor rule when operationalized with the revised (final) data. As a point of reference, our three-equation model is first estimated with least squares (LS) using measured expectations and unrevised ie real time current output gap in the Taylor rule.

[Insert Table 5 here]

As reported in table 5, for both specifications, estimated parameters are correctly signed. Compared with results under rational expectations (table 4), for both cases we get higher coefficients for the ex post real interest rate in the IS curve. Moreover, higher output gap coefficients are always obtained both for the Phillips curve and the Taylor rule. Under measured expectations the equilibrium interest rates are below 3, which is a more reasonable (lower) level than in table 4. In the case of the IS curves, some estimated parameters are slightly imprecise. Under measured expectations endogenous persistence is very dominating in the aggregate demand. By contrast, forward looking expectations now seem to strongly dominate the inflation process, as the relative weight of the lagged inflation term in the Phillips curve is clearly below 0.5. For both specifications, the results suggest that the Taylor principle holds and monetary policy decisions put clearly more weight on inflation compared to that of the output gap. When monetary policy rule is based only on current economic conditions (equation 7),

in the Taylor rule we get lower estimated parameters for both inflation and output gap, which is quite imprecise. Also, higher determinant residual covariance is obtained in this case.

All in all, least squares estimation results under measured expectations are quite plausible. When real time information is used in the model instead of imposing rational expectations hypothesis, we obtain reasonable coefficients even without assuming endogenous persistence in the Taylor rule. On the other hand, we get evidence that monetary policy rule is more precisely estimated using forward looking rules.

Of course, the LS method is not necessarily an appropriate estimation in the present context, however. Examination of estimated LS residuals indicates that residuals are strongly autocorrelated (not reported here). This implies that the model may be mis-specified. On the other hand, results may be also be biased due to measurement errors and/or simultaneity between right hand side variables of some or all of the equations. In order to overcome these estimation problems, we estimate the model using system GMM in two alternative ways.

In the first instance, we assume that all real time variables, ie current output gap in the Taylor rule and expected inflation and expected output gap are exogenous, as if the information set on which these expectations are based was dated in the beginning of the current period. So, in this case the variables treated as endogenous (and instrumented) comprise the current revised and lagged output gaps, current and lagged inflation rates, the nominal interest rate and the equilibrium interest rate. However, exogeneity of real time variables may not be a reasonable assumption in reality, since in annual data at least, the simultaneous interaction of measured expectations with current outcomes cannot be ruled out. Therefore, as a second alternative, also the real time variables ie real time current output gap, expected output gap and expected inflation rate are treated in estimation as endogenous variables due to possible simultaneity (and possible measurement errors in these variables)<sup>11</sup>.

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<sup>11</sup> Compared with table 4 the same instrument set and same modification of error terms were used in tables 6-9 in order to enable reasonable comparisons across alternative cases.

[Insert Table 6 here]

Table 6 summarizes system GMM results when real time variables are treated as exogenous. For the most part, the pattern of results is similar to that obtained with LS. The weight of the lagged output gap in the IS curve remains above 0.5, and is very significant, meaning that there is evidence of habit formation behavior in the aggregate demand. One notes that, in the case of an expectations-based Taylor rule specification (model E), we get a large standard error for the coefficient of the real interest rate in the IS curve and also the estimate for the equilibrium real interest rate is low and imprecise. In the Phillips curve, we find evidence of slightly more backward looking inflation dynamics than in the LS specification, but still forward looking expectations seem to clearly dominate the inflation process. The coefficient of the output gap in the Phillips curve remains much higher than in the rational expectations specification, a result which was already found with LS.

Compared with LS results in table 5, we now get more differences between the two Taylor rule specifications. With GMM, the coefficients for inflation and output gap are always higher than with LS. This suggests that the least squares bias is the most severe in the case of the Taylor rule. The forward-looking (ie expectations-based) specification of the Taylor rule yields clearly higher coefficients for both inflation and the output gap than the specification based on current inflation.

[Insert Table 7 here]

Estimation results from the case where real time variables are treated as endogenous, are reported in table 7. The results are statistically somewhat better than in the previous alternative. In particular, all coefficients are now significant, also the coefficient of the real interest rate in the IS curve which was small and insignificant in the previous case. Also the estimates of the equilibrium real interest rate are now very reasonable (2.8 and 2.2 per cent) and its standard error is small. Turning to the Phillips curve, the results are almost unaffected by

treating the expectations variables as endogenous. The result obtained above, that using measured expectations in the Phillips curve, instead of assuming rational expectations, gives a clearly bigger and more significant coefficient for the output gap holds here too. In the Taylor rule, the coefficient of the output gap is again significant only in the forward-looking version of the rule. Actually, in the standard version of the rule, where only current variables enter, the output gap coefficient is even less significant now than in the case when the real-time variables are treated as exogenous. So, under real time expectations, the use of forward looking variables seems to improve the empirical success of the Taylor rule. Also the diagnostic is better now than in the previous alternative. According to J-statistics, in none of the cases overidentifying restrictions are rejected at 1 per cent level.

All in all, the estimation results in tables 5 - 7 indicate that the empirical performance of the three-equation DSGE model is quite reasonable and robust when applied to real time data, including measured expectations. Contrary to the estimation results under rational expectations, we always obtain correctly signed coefficients for all equations of the model. Even for the Taylor rule, which seems to be the equation most affected by noisy information and data uncertainty, especially in the output gap, we obtain reasonable coefficients, when expectations are based on real time information. Quantitatively, the most important difference between the results with rational expectations and those with real-time variables is that the coefficients of the fundamentals (real interest rate in the IS curve, the output gap in the Phillips curve and the output gap in the Taylor rule) are much greater when the rational expectations assumption is relaxed. We get qualitatively similar results with all estimation methods we applied: least squares and the two varieties of GMM.

## 4.2 Robustness analysis

Next, we want to consider, whether the above results are sensitive with respect to the model specification ie endogenous persistence in the IS and Phillips curves.

We also analyze how the results are changed, when endogenous persistence is included in the Taylor rule.

Table 8 in Appendix 1 summarizes system GMM results for the model without endogenous persistence in the Phillips curve. Compared with the results under rational expectations, under measured expectations we always get more endogenous persistence, higher real interest rate and lower equilibrium interest rate in the IS curve. Moreover, more reasonable parameters for the Phillips curve are obtained (only slightly too high coefficient for expected inflation and correctly signed, statistically significant output gap coefficients). When the monetary policy rule does not include forward looking variables, the Taylor rule coefficients are almost the same in both cases. However, with expected inflation plausible results are obtained only under measured expectations, since in the other case the output gap coefficient is incorrectly signed. When estimating both approaches without external habit formation, we always get unreasonable results (not reported here): unreasonably high equilibrium interest rate and extremely low  $R^2$  in the IS curve.

Next, we add endogenous persistence also in the Taylor rule (see table 9 in Appendix 1). Qualitatively same differences as before across alternative approaches can be found in the IS and Phillips curve coefficients. However, for the monetary policy rule interesting differences can be found. Under rational expectations interest rate smoothing seems to be always needed: the smoothing parameter is relatively high (above 0.3) and significant, and the Taylor principle holds. By contrast, with real time variables, the evidence of interest rate smoothing is weaker, especially in the case of the forward looking rule (the estimated smoothing parameter is very low and the Taylor principle does not hold). In fact, on the basis of the t-test it is not clear that the interest smoothing parameter is needed at all. Thus, according to table 9 endogenous persistence seems to be needed only in the rational expectations model, possible due to informational problems.



## 5 Conclusions

In New Keynesian DSGE models, which are nowadays intensively studied in macroeconomics and monetary policy research, economic behaviour is largely based on dynamic optimisation under rational expectations. However, as the empirical relevance of DSGE models has had conflicting assessments, the rational expectations assumption has been increasingly questioned in recent macroeconomic debate.

In economic analysis we need to explain people's behaviour in the context of what they knew and believed at the time. This is particularly important in the case of policy decisions, as argued by Orphanides (2001). When estimating DSGE models using the most recently revised data, which takes into account all the subsequent revisions and improvements, we may get biased parameter estimates and distorted policy implications. Instead, if we use real time data available at the time, economic relationships are potentially better described.

Addressing real time issues and informational problems is especially central, both conceptually and empirically, when examining expectations. In this study we have analyzed directly measured expectations. Using European panel data and system estimation, we examined the empirical relevance of the three-equation DSGE model under rational expectations and revised data and alternatively, under measured expectations based on survey information. Real time information was also used in the current output gap in the Taylor rule in order to take into account possible informational problems in the monetary policy rule.

Analysis of this study reveals that expectational errors are clearly positive autocorrelated. It casts doubts to the empirical validity of rational expectations hypothesis in the DSGE model framework. When relaxing the rational expectations hypothesis in estimation, the estimated parameters of the model change substantially. Moreover, the results suggest that real time information, especially in the output gap, is essential in the monetary policy rule. The system estimation results of this study are compatible with the single-equation studies by Orphanides (2001) examining monetary policy rules and by Paloviita and Mayes (2005) analysing Phillips curve specifications.

On the basis of our results the New Keynesian DSGE model succeeds somewhat better with directly measured expectations and real time data than with rational expectations assumption and revised (final) data. This is natural considering that the rational expectations assumption fails in rationality tests with real time variables. Quantitatively the biggest difference between the rational expectations and real time variants is the size of the coefficients of the fundamentals in the model (output gap and the real interest rate): these coefficients about double when we move from rational expectations model to the model based on measured expectations. Our results seem to be reasonably robust across different estimation methods and the most relevant alternative specifications.

It is sometimes conjectured that giving up the rational expectations assumption would make it unnecessary to have backward-looking elements in the New Keynesian model (such as habit formation in the aggregate demand, or endogenous inflation persistence in the Phillips curve). Our results show that in fact, this is not the case, since the habit formation term and the lagged inflation term are actually needed in the IS and Phillips curve also when the rationality assumption is replaced by the use of real-time measured expectations. The exception to this is the Taylor rule. Under the rational expectations assumption endogenous persistence is needed in the Taylor rule - it potentially reflects information limitations. This need to use endogenous persistence disappears in the DSGE model framework, when real time information is used.

All in all, our results underline the importance of research into expectations formation because the evidence is against the basic rational expectations version of the New Keynesian DSGE model.

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## Appendix 1 Robustness analysis

Table 8. No endogenous persistence in the Phillips curve

### GMM estimation results under real time expectations:

#### Model C

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = \delta \bar{E}_t \pi_{t+1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMUrt}$$

#### Model E

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = \delta \bar{E}_t \pi_{t+1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMUrt}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.704 (0.055)	-0.069 (0.030)	1.995 (1.636)	1.040 (0.022)	0.147 (0.021)	-0.395 (0.233)	2.166 (0.418)	1.736 (0.068)	0.085 (0.124)	0.018
E	0.672 (0.054)	-0.054 (0.035)	2.302 (1.865)	1.015 (0.020)	0.176 (0.025)	-1.087 (0.211)	-0.041 (0.419)	2.384 (0.104)	0.249 (0.153)	0.001

### GMM estimation results under rational expectations:

#### Model C

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*)$$

$$\pi_t = \delta E_t \pi_{t+1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMU}$$

#### Model E

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*)$$

$$\pi_t = \delta E_t \pi_{t+1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta E_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.480 (0.037)	-0.038 (0.018)	3.717 (1.237)	1.073 (0.022)	0.018 (0.031)	-0.205 (0.231)	2.095 (0.401)	1.747 (0.062)	0.060 (0.081)	0.115
E	0.463 (0.032)	-0.039 (0.015)	3.658 (1.022)	1.082 (0.020)	-0.008 (0.028)	-1.240 (0.238)	1.633 (0.355)	2.158 (0.068)	-0.106 (0.090)	0.216

Note: See table 4.

## Appendix 1 Robustness analysis, cont.

Table 9. Interest rate smoothing in the Taylor rule

### GMM estimation results under real time expectations:

#### Model C

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \rho r_{t-1} + (1 - \rho)(\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMUrt})$$

#### Model E

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \rho r_{t-1} + (1 - \rho)(\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E}_t \pi_{t+1}^{EMU} + \gamma y_t^{EMUrt})$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\rho$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.691	-0.100	2.166	0.385	0.134	0.422	-0.288	1.627	1.479	0.434	0.009
	(0.053)	(0.035)	(1.153)	(0.067)	(0.022)	(0.219)	(0.356)	(0.907)	(0.248)	(0.499)	
E	0.686	-0.116	2.558	0.389	0.145	0.142	-0.143	-0.578	0.954	1.373	0.006
	(0.059)	(0.036)	(0.976)	(0.062)	(0.023)	(0.284)	(2.189)	(4.160)	(3.422)	(3.348)	

### GMM estimation results under rational expectations:

#### Model C

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)E_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \rho r_{t-1} + (1 - \rho)(\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma y_t^{EMU})$$

#### Model E

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*)$$

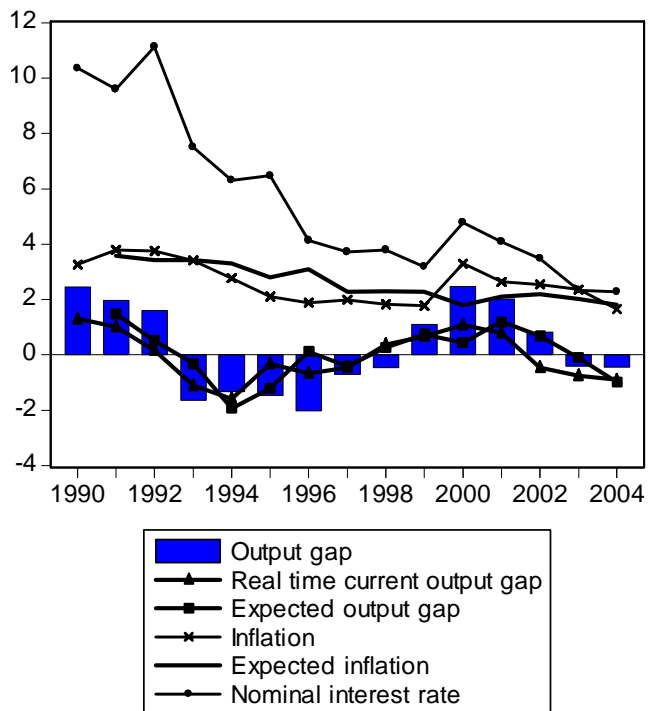
$$\pi_t = (1 - \delta)E_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \rho r_{t-1} + (1 - \rho)(\alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta E_t \pi_{t+1}^{EMU} + \gamma y_t^{EMU})$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\rho$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.471	-0.044	3.579	0.451	0.064	0.340	-0.412	2.010	1.333	0.580	0.325
	(0.034)	(0.016)	(0.822)	(0.029)	(0.015)	(0.158)	(0.397)	(0.679)	(0.298)	(0.405)	
E	0.471	-0.049	3.520	0.460	0.066	0.400	-1.372	1.375	1.864	0.311	0.236
	(0.034)	(0.017)	(0.790)	(0.032)	(0.015)	(0.120)	(0.305)	(0.669)	(0.226)	(0.222)	

Note: See table 4.

Figure 1. Median values of pooled euro area data<sup>1</sup>



<sup>1</sup> Real time variable in year t was surveyed in June of the year t-1.



Table 1. **Correlations of real time variables**

	Infl	Expinfl	Gap	Expgap	Rtgap	Ir
Infl	1.000	0.842	0.107	0.035	0.026	0.785
Expinfl	0.842	1.000	-0.192	-0.115	-0.163	0.854
Gap	0.107	-0.192	1.000	0.526	0.590	-0.111
Expgap	0.035	-0.115	0.526	1.000	0.848	-0.039
Rtgap	0.026	-0.163	0.590	0.848	1.000	-0.101
Ir	0.785	0.854	-0.111	-0.039	-0.101	1.000

Note: Infl= revised inflation, Expinfl = expected inflation, Gap = revised output gap, Expgap = expected output gap, Rtgap = real time current output gap and Ir = nominal interest rate.

Table 2. **Wald test**

	F-statistic	Probability
Expected inflation	2.837	(0.062)
Current output gap	1.515	(0.223)
Expected output gap	1.307	(0.274)

Table 3. Ljung-box autocorrelation tests

Expectational errors:

Real time current output gap

Q(1) 86.336\*  
Q(2) 118.59\*  
Q(3) 121.93\*

Expected output gap

Q(1) 66.512\*  
Q(2) 92.553\*  
Q(3) 104.73\*

Expected inflation

Q(1) 13.874\*  
Q(2) 17.571\*  
Q(3) 17.611\*

Residuals of unbiasedness test:

Real time current output gap

Q(1) 86.818\*  
Q(2) 120.37\*  
Q(3) 124.31\*

Expected output gap

Q(1) 68.873\*  
Q(2) 92.655\*  
Q(3) 100.78\*

Expected inflation

Q(1) 13.803\*  
Q(2) 17.197\*  
Q(3) 17.205\*

Note: Q(n) denotes the Ljung-Box autocorrelation test statistics for up to  $n$ th-order autocorrelation.  
\* Significance at 5 per cent level.

Table 4. GMM estimation results under rational expectations

**Model C**

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)E_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma_t^{EMU}$$

**Model E**

$$y_t = (1 - \mu)E_t y_{t+1} + \mu y_{t-1} - \phi(r_t - E_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)E_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta E_t \pi_{t+1}^{EMU} + \gamma_t^{EMU}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.485 (0.036)	-0.040 (0.021)	3.795 (1.361)	0.463 (0.045)	0.077 (0.023)	-0.247 (0.230)	2.168 (0.439)	1.724 (0.073)	0.049 (0.091)	0.053
E	0.465 (0.035)	-0.043 (0.021)	3.444 (1.065)	0.458 (0.040)	0.074 (0.019)	-1.387 (0.273)	1.710 (0.411)	2.155 (0.083)	-0.145 (0.115)	0.117

Note: Standard errors are in parenthesis. The instrument set for the IS curve includes two lags of of PCP, real public consumption and real imports of goods and services (%-changes). For the Phillips curve instruments are two lags of real oil price and import prices of goods and services (%-changes), and two lags of unemployment rate and the output gap (%). The Taylor rule instruments are two lags of general govt financial balances (% of nominal GDP), and two lags of general govt total outlays, labour productivity, private investment in housing and PCP (%-changes). p-value refers to J-test of the overidentifying restrictions.

Table 5. Least squares estimation results under real time expectations

**Model C**

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma_t^{EMUrt}$$

**Model E**

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E}_t \pi_{t+1}^{EMU} + \gamma_t^{EMUrt}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	DRC
C	0.655 (0.034)	-0.063 (0.038)	2.755 (1.692)	0.316 (0.049)	0.136 (0.023)	0.243 (0.242)	2.550 (0.287)	1.617 (0.070)	0.101 (0.109)	1.680
E	0.655 (0.034)	-0.063 (0.038)	2.755 (1.692)	0.316 (0.049)	0.136 (0.023)	-0.654 (0.244)	0.333 (0.335)	2.265 (0.089)	0.238 (0.100)	1.529

Note: DRC refers to determinant residual covariance

Table 6. GMM estimation results under exogenous real time expectations

**Model C**

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma_t^{EMUrt}$$

**Model E**

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E}_t \pi_{t+1}^{EMU} + \gamma_t^{EMUrt}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.623	-0.060	2.022	0.401	0.129	-0.307	1.431	1.839	0.172	0.013
	(0.042)	(0.040)	(1.504)	(0.054)	(0.020)	(0.222)	(0.386)	(0.079)	(0.127)	
E	0.606	-0.028	0.689	0.368	0.147	-0.946	-0.791	2.498	0.246	0.001
	(0.043)	(0.042)	(4.251)	(0.067)	(0.023)	(0.171)	(0.349)	(0.098)	(0.109)	

Note: See table 4.

Table 7. GMM estimation results under endogenous real time expectations

**Model C**

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \pi_t^{EMU} + \gamma_t^{EMUrt}$$

**Model E**

$$y_t = (1 - \mu)\bar{E}_t y_{t+1} + \mu y_{t-1} - \phi(r_t - \bar{E}_t \pi_{t+1} - r^*)$$

$$\pi_t = (1 - \delta)\bar{E}_t \pi_{t+1} + \delta \pi_{t-1} + \lambda y_t$$

$$r_t = \alpha_1 D_{EMU} + \alpha_2 (1 - D_{EMU}) + \beta \bar{E}_t \pi_{t+1}^{EMU} + \gamma_t^{EMUrt}$$

	$\mu$	$\phi$	$r^*$	$\delta$	$\lambda$	$\alpha_1$	$\alpha_2$	$\beta$	$\gamma$	p-value
C	0.671	-0.110	2.838	0.408	0.131	-0.214	2.188	1.731	0.078	0.120
	(0.045)	(0.031)	(0.749)	(0.054)	(0.018)	(0.224)	(0.336)	(0.058)	(0.117)	
E	0.696	-0.086	2.181	0.399	0.137	-1.012	0.167	2.309	0.297	0.019
	(0.051)	(0.034)	(1.279)	(0.069)	(0.021)	(0.218)	(0.441)	(0.102)	(0.128)	

Note: See table 4.