

Discussion of 'Learning in an Estimated Medium-Scale DSGE Model'

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Conference

Joe Pearlman

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Contents of Paper

- ▶ NK Model of US, compatible with long-run growth
- ▶ Estimation and Model Evaluation using DSGE-VAR
- ▶ Learning when agents have full information (MSV Learning)
- ▶ Learning when agents use the same information as econometricians (VAR Learning)
- ▶ Large deviations, escapes and indeterminacy
- ▶ Estimation under learning, including optimised initial beliefs

Structure of Discussion

- ▶ Discussion of individual issues
 1. MSV (or Saddlepath) Learning and Indeterminacy
 2. VAR learning
 3. Optimised initial beliefs
- ▶ Gaps that can be filled

Alternative General Setup

- ▶ A potentially clearer setup is that due to Blanchard and Kahn (1980):

$$\begin{bmatrix} y_t^s \\ E_t y_{t+1}^f \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} y_{t-1}^s \\ y_t^f \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \varepsilon_t$$

- ▶ Note that in the paper there are 5 forward-looking equations - in c, i, Q^k, π, w - but an additional 3 forward-looking variables - L, r^k, u ; after pushing the other equations one step forward and then taking expectations, one can substitute for the latter.
- ▶ The learning assumption on forward expectations is

$$E_t y_{t+1}^f = -N_{t-1} y_t^s$$

Learning Theorems

The following relate to learnability of N :

- ▶ (McCallum, 2006) Determinacy implies E-Stability and Learnability
- ▶ (McCallum, 2006) Indeterminacy implies E-Stability does not always lead to a unique learnable saddlepath
- ▶ (Ellison and Pearlman, 2007) Indeterminacy implies iterative E-stability and a unique learnable saddlepath (MSV/MOD). But sunspot solutions are also learnable.
- ▶ The latter justifies S&W's exclusion of indeterminate systems.

Slobodyan & Wouters: MSV Learning and Indeterminacy

- ▶ The MSV learning concerns learning about the saddle path i.e. y_{t+1}^f is regressed on y_t^s , but not on y_t^f . This is E-stable, as described earlier.
- ▶ Note that sunspots are ruled out because Dynare penalises indeterminacy.
- ▶ As I understand it, the plots in Figures 7 and 8 depend on the the dynamics generated by $G_{11} - G_{12}N_t$. Presumably these figures relate to median values of the parameters obtained at the end of the MCMC algorithm.

Slobodyan & Wouters: VAR Learning

- ▶ Agents regress their optimal estimates $y_{j+1,j+1}^f$ on y_j^{obs} , enabling a forecast of $y_{t+1,t}^f$ based on y_j^{obs} . This yields expressions for $y_{t,t}^f$ and $y_{t,t}^s$, which presumably should be filtered together with y_t^{obs} to improve these estimates.
- ▶ The filtering means that current estimates of shocks can be recovered.
- ▶ However there is nothing to which these estimates will obviously converge, so it is no surprise that IRFs in this case are quite different from RE.

Slobodyan & Wouters: VAR Learning Continued

- ▶ An alternative is to assume that the structure of the economy and all parameters are known by all agents, and they use their latest estimate of N_t , and their best estimate of $y_{t,t}^s$ to form the projection $y_{t+1,t}^f = -N_t y_{t,t}^s$
- ▶ This leads to the behavioural relationship
$$-N_t y_{t,t}^s = G_{21} y_t^s + G_{22} y_t^f + H_2 \varepsilon_t$$
- ▶ This can be combined with the expression for y_{t+1}^s into Kalman filter form to yield best estimates of $y_{t,t}^s$ and $y_{t,t}^f$
- ▶ Then run a regression of $y_{j+1,j+1}^f$ on $y_{j,j}^s$ to estimate $-N_{t+1}$.
- ▶ Preliminary results indicate that this has identical E-stability properties to the full information case.

Slobodyan & Wouters: Optimised Initial Beliefs

- ▶ A puzzle. Consider estimation of the parameters of an AR1 process. The log-likelihood function, if one is maximizing over initial beliefs, is given by $-2\ln L = \sum_{t=1}^T \frac{(y_t - \alpha y_{t-1})^2}{\sigma^2} + T \ln \sigma^2$. If we maximize over the initial belief y_0 , then we set $y_0 = y_1/\alpha$. In effect, we drop the first term in the summation.
- ▶ However the usual assumption is that y_1 has the steady state distribution, so that the first term in the summation is $\frac{(1-\alpha^2)y_1^2}{\sigma^2}$. Thus the effect of optimizing over the initial value is to lose one degree of freedom.
- ▶ Thus when one optimizes over initial beliefs, I would expect to see a reduction in the accuracy of the parameter estimates.
- ▶ However this is not the case here from Tables 7 and 8! What is the intuition?

Slobodyan & Wouters: A Gap to be Filled

- ▶ The paper examines two types of information setup.
 1. The usual Dynare setup: agents have full information, while econometricians have information on only a subset of variables (in this case 7).
 2. The VAR learning setup: agents have full information when making decisions, but the same partial information set as econometricians when learning.
- ▶ Now ignore the learning issue. It is perfectly feasible that agents do not have full information on shocks, notably the technology shock, or even capital stock. What is the effect of this on parameter estimates?

- ▶ Pearlman et al (2007) have estimated the S-W model using Euro-area data under the (possibly extreme) assumption that agents and econometricians have the same 7-variable information set. The algorithm incorporates the general results of Pearlman et al (1986) for solution of RE models under partial information.
- ▶ The paper assumes conventional Dixit-Stiglitz preferences.
- ▶ The parameter estimates are broadly the same as for the usual information assumption, with one important exception. The price stickiness parameter $\xi_p = 0.74$ at both the mode and median of MCMC draws, implying a mean time between price changes of less than 4 quarters.

A General Caution for DSGE MCMC

- ▶ Levine et al (2007) have shown that for the simplest NK model with consumption habit, the 2nd-order conditions for a maximum may not be satisfied for zero inflation for some combinations of parameters. (See also Benigno and Woodford, 2007)
- ▶ We have found that as many as 1.5% of MCMC draws for S-W model estimation should be rejected using this criterion.
- ▶ This is a greater percentage than one usually finds for indeterminacy.

Conclusions

- ▶ A fascinating tour, both theoretical and methodological, through all the areas that are now of relevance to business cycle economists.
- ▶ Misgivings about the VAR learning approach.
- ▶ One further link in the chain is required.