Monetary Policy and the Financing of Firms*

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Abstract

How should monetary policy respond to changes in financial conditions? In this paper we consider a simple model where firms are subject to idyosincratic shocks which may force them to default on their debt. Firms' assets and liabilities are assumed to be denominated in nominal terms and to be predetermined when shocks occur. Monetary policy can therefore affect the real value of funds used to finance production. Furthermore, policy affects the loan and deposit rates. We find that maintaining price stability at all times is not optimal; that the optimal response to adverse financial shocks is to engineer a short period of inflation, if policy rates are at the zero bound and cannot be lowered further; that the Taylor rule may implement allocations that have opposite cyclical properties to the optimal ones.

Keywords: Financial stability; debt deflation; bankruptcy costs; price level volatility; optimal monetary policy; stabilization policy.

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1 Introduction

During financial crises, credit conditions tend to worsen for all agents in the economy. There are frequent calls for a looser monetary policy stance in the popular press, on the grounds that this would help avoid the risks of a credit crunch and a deep recession. The intuitive argument is that lower interest rates will tend to make it easier for firms to obtain external finance, thus countering the effects of the crisis on spreads. Arguments tracing back to Fisher (1933) can also be used to call for some degree of inflation during financial crises, so as to avoid an excessive increase in firms' leverage through a devaluation of their nominal liabilities.

It is less clear, however, whether these arguments would withstand a more formal analysis. In this paper, we present a model that can be used to evaluate them. More specifically, we address the following questions: How should monetary policy respond to financial shocks? How should it respond to real shocks, when financial conditions are relevant? Are there reasons to allow for some inflation during recessions (and vice versa during expansions)? How can policy be effective when interest rates are at the zero bound?

To answer these questions, we use a model where monetary policy has the ability to affect the financing conditions of firms. Our set-up has three distinguishing features. First, firms' internal and external funds are imperfect substitutes. This is due to the presence of information asymmetries between firms and banks on firms' productivity and because monitoring is a costly activity for banks. Second, firms' internal and external funds are nominal assets. Third, those funds, both internal and external, as well as the interest rate on bank loans, are predetermined when aggregate shocks occur.

We find, not surprisingly, that maintaining price stability at all times is not optimal in our model. In response to technology shocks, for example, the price level should move to adjust the real value of total funds. If the shock is negative, the price level increases on impact to lower real funds as well as the real wage. Subsequently, there is inflation in order to increase the real wage at the same pace as productivity converges back to steady state. Along the adjustment path, deposit and loan rates, spreads, financial markups, leverage, and bankruptcy rates remain stable. Therefore, under the optimal policy, if technology shocks were the only shocks, bankruptcies would be acyclical.

The optimal response to an exogenous reduction in internal funds, which amounts to an increase in firms' leverage, also involves an increase in the price level on impact, in order to lower real funds and the real wage. The short period of controlled inflation mitigates the adverse consequences of the shock on bankruptcy rates and allows firms to de-leverage more quickly.

We also find that a policy response according to a simple Taylor-type rule can be costly, in the sense of inducing more persistent deviations in real variables from their optimal values and higher bankruptcy rates. In response to technology shocks, bankruptcies become countercyclical under the simple rule. In response to an exogenous reduction in internal funds, there is deflation initially, which increases the real value of total funds and leads to a much larger increase in leverage. The reduction in output is smaller than under optimal policy and markups decrease. Bankruptcy rates are higher.

In one version of our model the optimal deposit rate is zero, corresponding to the Friedman rule. After setting interest rates at zero, policy can still choose the price level as described above because assets are nominal and predetermined. For given nominal interest rate, there are many possible equilibrium allocations, and therefore ample room for policy.

The Friedman rule is not optimal when we assume that government consumption is an exogenous share of production. In this case, there is a reason for proportionate taxation. The deposit rate acts as a tax on consumption and therefore the optimal steady state deposit rate is positive.

When the optimal average interest rate is away from the lower bound, it may be optimal for it to fluctuate in response to shocks. This is indeed the case for financial shocks, but not for technology shocks. In response to technology shocks, it is optimal to keep rates constant even if they could be lowered. For all financial shocks, the flexibility of moving the nominal interest rate downwards allows policy to speed up the adjustment. Moreover, the effect of these shocks on output can be completely reversed. For instance, a shock that reduces the availability of internal funds is persistently contractionary when the short term nominal rate is kept fixed at zero, while it becomes mildly expansionary and very short-lived when the average interest rate is away from the lower bound and the short term nominal rate is reduced.

In order to understand the mechanisms responsible for these results, we analyze two benchmark models: i) a model where assets are predetermined, but where internal and external funds are perfect substitutes (i.e. monitoring costs are zero); and ii) a model with real exogenous internal funds and where neither assets nor interest rates are predetermined, as in De Fiore and Tristani (2008).

We use the first benchmark model to illustrate that the two assumptions of nominal denomination and predetermination of funds used to finance production are sufficient for the result that changes in the price level affect allocations. We also use this benchmark model to assess the role played by asymmetric information and monitoring costs in explaining business cycle fluctuations. Although these imperfections play a quantitatively minor role in determining the cyclical behavior of non-financial variables, they tend to amplify the reaction of the economy to shocks.

When we consider the second benchmark model, we find very different results from those obtained in our general case. In this economy, there is no additional role for monetary policy, other than setting the deposit rate. When government consumption is an exogenous share of production, it is optimal to raise the policy rate after positive technology shocks and to reduce it after negative shocks. This contrasts with the general model where, in reaction to a technology shock, optimal policy keeps the policy rate constant and changes the price level. This benchmark model also has very different cyclical properties. Bankruptcy rates are procyclical in response to technology shocks, while they are acyclical (under the optimal policy) or countercyclical (under a Taylor rule) in the general model.

Our paper relates to the literature that analyzes the effect of financial factors on the transmission of shocks. In our model, financial factors play a role because of asymmetric information and costly state verification, as in Bernanke et al (1999) and Calstrom and Fuerst (1997, 1998). We also contribute to the recent literature that analyzes the role of financial factors for optimal monetary policy (see e.g. Curdia and Woodford (2008), De Fiore and Tristani (2008), Ravenna and Walsh (2006), and Faia (2008)). The main differences relative to those models are the nominal denomination of debt, as in Christiano et al (2003) and De Fiore and Tristani (2008), and the assumption that assets are decided at the end of each period, before observing the aggregate shocks, as in Svensson (1985). It follows that, in our setup, monetary policy affects allocations by setting the nominal interest rate but also by choosing an appropriate path for prices. This has important implications for the cyclical properties of the economy under the optimal policy.

The paper proceeds as follows. In section 2, we outline the environment and describe the equilibria. Then, we derive implementability conditions and we characterize optimal monetary policy. In section 3, we provide numerical results on the response of the economy to various shocks. We describe results both under the optimal monetary policy and a sub-optimal (Taylor) rule. We compare the case when the level of government expenditures is exogenous and the optimal monetary policy is the Friedman rule, to the case when government expenditures are a fixed share of output and the optimal average interest rate is away from zero. In section 4, we introduce the two benchmark models and use them to explain the results obtained in the general model. In section 5, we conclude.

2 Model

We consider a model where firms need internal and external funds to produce and they fail if they are not able to repay their debts. Both internal funds and firm debt are nominal assets. There is a goods market in the beginning of the period and an assets market at the end, where funds are decided for the following period. Funds are predetermined.

Production uses labor only with a linear technology. Aggregate productivity is stochastic. In addition, each firm faces an idiosyncratic shock that is private information.

The households have preferences over consumption, labor and real money. For convenience we assume separability for the utility in real balances.[1]

Banks are financial intermediaries. They are zero profit, zero risk operations. Banks take deposits from households and allocate them to entrepreneurs on the basis of a debt contract where the entrepreneurs repay their debts if production is sufficient and default otherwise, handing in total production to the banks, provided these pay the monitoring costs. Because there is aggregate uncertainty, we assume that the government can make lump sum transfers between the households and the banks that ensure that banks have zero profits in every state. This way the banks are able to pay a risk free rate on deposits.

Entrepreneurs need to borrow in advance to finance production. The payments on outstanding debt are not state dependent. Entrepreneurs accumulate internal funds indefinitely. A proportionate tax on these funds ensures that there is always a need for external funds.

Monetary policy can affect the real value of total funds available for the production of firms, but it can also affect the real value of debt that needs to be repaid. Furthermore, monetary policy also affects the deposit and loan rates.

[1] We also assume a negligible contribution to welfare. This does not mean that the economy is cashless since firms face a cash in advance constraint.

2.1 Households

At the end of period t at the assets market, households decide on holdings of money M_t that they will be able to use at the beginning of period $t+1$ in the goods market. They also decide on a portfolio of nominal state-contingent bonds $\overline{A}_{t+1}$ each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency D_t that will pay $R_t^d D_t$ in the assets market in period $t+1$. Deposits are riskless, in the sense that banks do not fail.

The budget constraint at period t is

$$M_t + E_t Q_{t,t+1}\overline{A}_{t+1} + D_t \leq \overline{A}_t + R_{t-1}^d D_{t-1} + M_{t-1} - P_t c_t + W_t n_t - T_t, \tag{1}$$

where c_t is the amount of the final consumption good purchased, P_t is its price, n_t is hours worked, W_t is the nominal wage, and T_t are lump-sum nominal taxes collected by the government.

The household's problem is to maximize utility, defined as

$$E_0 \left\{ \sum_0^\infty \beta^t \left[u\left(c_t, m_t\right) - \alpha n_t \right] \right\}, \tag{2}$$

subject to (1). Here $u_c > 0$, $u_m \geq 0$, $u_{cc} < 0, u_{mm} < 0$, $\alpha > 0$ and $m_t \equiv M_{t-1}/P_t$ denotes real balances. Throughout we will assume that the utility function is separable in real money, m_t, and that the contribution to welfare is negligible.

Optimality requires that $R_t = R_t^d$ for all t, and the following conditions must hold:

$$\frac{u_c\left(t\right)}{\alpha} = \frac{P_t}{W_t}, \tag{3}$$

$$\frac{u_c\left(t\right)}{u_c\left(t+1\right)} = \beta Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}}, \tag{4}$$

$$\frac{u_c\left(t\right)}{P_t} = R_t^d \beta E_t \frac{u_c\left(t+1\right)}{P_{t+1}}, \tag{5}$$

$$\frac{E_t u_m\left(t+1\right)}{E_t u_c\left(t+1\right)} = R_t^d - 1. \tag{6}$$

2.2 Production

The production sector is composed of a continuum of competitive firms, indexed by $i \in [0,1]$. Each firm is endowed with a stochastic technology that transforms $N_{i,t}$ units of labor into $\omega_{i,t} A_t N_{i,t}$ units of output. The random variable $\omega_{i,t}$ is i.i.d. across time and across firms, with distribution Φ, density ϕ and mean unity. A_t is an aggregate productivity shock. The shock $\omega_{i,t}$ is private information, but its realization can be observed by the financial intermediary at the cost of a share μ of the firm's output.

The firms decide in the assets market at $t-1$ the amount of internal funds to be available in period t, $B_{i,t-1}$. Lending occurs through the financial intermediary, which is able to obtain a safe return. The existence of aggregate shocks occurring during the duration of the contract implies that the intermediary's return from the lending activity is not safe, despite its ability to differentiate across the continuum of firms facing i.i.d. shocks. We assume the existence of a deposit insurance scheme that the government implements by completely taxing away the intermediary's profits whenever the aggregate shock is relatively high, and by providing subsidies up to the point where profits are zero when the aggregate shock is relatively low. Such policy guarantees that the intermediary is always able to repay the safe return to the household, thus insuring households' deposits from aggregate risk.

2.2.1 The financial contract

The firms pay wages in advance of production. They have internal funds and borrow to be able to pay wages. Each firm i needs $X_{i,t-1}$ total funds, internal plus external, at the assets market in period $t-1$, to be available in period t. It is restricted to hire and pay wages according to

$$W_t N_{i,t} \leq X_{i,t-1}. \tag{7}$$

The firms borrow $X_{i,t-1}-B_{i,t-1}$. The loan contract stipulates a payment of $R^l_{i,t-1}\left(X_{i,t-1}-B_{i,t-1}\right)$, where $R^l_{i,t-1}$ is not contingent on the state at t, when the firm is able to meet those payments, i.e. when $\omega_{i,t} \geq \overline{\omega}_{i,t}$, where $\overline{\omega}_{i,t}$ is the minimum productivity level such that the firm is able to pay the fixed return to the bank, so that

$$P_t A_t \overline{\omega}_{i,t} N_{i,t} = R^l_{i,t-1}\left(X_{i,t-1}-B_{i,t-1}\right). \tag{8}$$

Otherwise the firm goes bankrupt, and hands out all the production $P_t A_t \omega_{i,t} N_{i,t}$, in units of currency. In this case, a constant fraction μ_t of the firm's output is destroyed in monitoring, so that the bank gets $\left(1-\mu_t\right) P_t A_t \omega_{i,t} N_{i,t}$.

Define the average share of production accruing to the bank and to the firms, respectively, as

$$f\left(\overline{\omega}_{i,t}\right) = \int_{\overline{\omega}_{i,t}}^{\infty} \left(\omega_{i,t}-\overline{\omega}_{i,t}\right) \Phi\left(d\omega\right). \tag{9}$$

and

$$g\left(\overline{\omega}_{i,t};\mu_t\right)=\int_0^{\overline{\omega}_{i,t}}\left(1-\mu_t\right)\omega_{i,t}\Phi\left(d\omega\right)+\int_{\overline{\omega}_{i,t}}^{\infty}\overline{\omega}_{i,t}\Phi\left(d\omega\right). \tag{10}$$

Total output is split between the firm, the bank, and monitoring costs

$$f\left(\overline{\omega}_{i,t}\right)+g\left(\overline{\omega}_{i,t};\mu_t\right)=1-\mu_t G\left(\overline{\omega}_{i,t}\right),$$

where $G\left(\overline{\omega}_{i,t}\right)=\int_0^{\overline{\omega}_{i,t}}\omega_{i,t}\Phi\left(d\omega\right)$. On average, $\mu_t G\left(\overline{\omega}_{i,t}\right)$ of output is lost in monitoring.

The optimal contract is a vector $\left(R^l_{i,t-1},X_{i,t-1},\overline{\omega}_{i,t},N_{i,t}\right)$ that solves the following problem: Maximize the expected production accruing to firms

$$\max\ E_{t-1}\left[f\left(\overline{\omega}_{i,t}\right)P_tA_tN_{i,t}\right]$$

subject to

$$W_tN_{i,t} \leq X_{i,t-1} \tag{11}$$

$$E_{t-1}\left[g\left(\overline{\omega}_{i,t};\mu_t\right)P_tA_tN_{i,t}\right] \geq R^d_{t-1}\left(X_{i,t-1}-B_{i,t-1}\right) \tag{12}$$

$$E_{t-1}\left[f\left(\overline{\omega}_{i,t}\right)P_tA_tN_{i,t}\right] \geq R^d_{t-1}B_{i,t-1} \tag{13}$$

where $g\left(\overline{\omega}_{i,t};\mu_t\right)$ and $f\left(\overline{\omega}_{i,t}\right)$ are given by (10) and (9), respectively, and $\overline{\omega}_{i,t}$ is given by (8).

The problem above is written under the assumption that it is optimal to produce, rather than just hold the funds. This is true as long as

$$P_tA_tN_{i,t}\geq X_{i,t-1}.$$

If it is optimal to produce, then the financial constraint (11) holds with equality, so that it is optimal to produce as long as

$$\frac{P_tA_t}{W_t}\geq 1.$$

As long as the economy is sufficiently away from the first best without financial costs, this condition should be satisfied.

The informational structure in the economy corresponds to the standard costly state verification (CSV) problem. The optimal contract maximizes the entrepreneur's expected return subject to the borrowing-in-advance constraint for firms, (11), the financial intermediary receiving an amount not lower than the repayment requested by the household (the safe return on deposits), (12), and the entrepreneur being willing to sign the contract, (13).

Perfect competition among financial intermediaries implies that the zero-profit condition (12) holds with equality in equilibrium.

The decisions on $X_{i,t-1}$ and $B_{i,t-1}$ are made in period $t-1$ at the assets market. Can replace $N_{i,t}=\frac{X_{i,t-1}}{W_t}$ and divide everything by $X_{i,t-1}$ to get

$$\max \ E_{t-1}\left[\frac{P_tA_t}{W_t}X_{i,t-1}f\left(\overline{\omega}_{i,t}\right)\right]$$

subject to

$$E_{t-1}\left[\frac{P_tA_t}{W_t}g\left(\overline{\omega}_{i,t};\mu_t\right)\right] \geq R^d_{t-1}\left(1-\frac{B_{i,t-1}}{X_{i,t-1}}\right) \tag{14}$$

$$E_{t-1}\frac{P_tA_t}{W_t}f\left(\overline{\omega}_{i,t}\right) \geq R^d_{t-1}\frac{B_{i,t-1}}{X_{i,t-1}} \tag{15}$$

where $g\left(\overline{\omega}_{i,t};\mu_t\right)$ and $f\left(\overline{\omega}_{i,t}\right)$ are given by (10) and (9), respectively, and $\overline{\omega}_{i,t}$ is

$$\overline{\omega}_{i,t}=\frac{R^l_{i,t-1}}{\frac{P_tA_t}{W_t}}\left(1-\frac{B_{i,t-1}}{X_{i,t-1}}\right)$$

Given that $B_{i,t-1}$ is exogenous to this problem and is predetermined, we can multiply and divide the objective by $B_{i,t-1}$, so that the problem is written in terms of $\frac{B_{i,t-1}}{X_{i,t-1}}$, $R^l_{i,t-1}$, and $\overline{\omega}_{i,t}$, only. The objective and the constraints of the problem are the same for all firms. The only firm specific variable would be $B_{i,t-1}$ in the objective, but this would be irrelevant for the maximization problem. Hence, the solution for $\frac{B_{i,t-1}}{X_{i,t-1}}$, $R^l_{i,t-1}$, and $\overline{\omega}_{i,t}$ is the same across firms.

Name $b_{t-1}\equiv\frac{B_{i,t-1}}{X_{i,t-1}}$ and $v_t\equiv\frac{P_tA_t}{W_t}$. We can then rewrite $\overline{\omega}_{i,t}$ as

$$\overline{\omega}_t=\frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}. \tag{16}$$

Define this function as $\overline{\omega}\left(R^l_{t-1},b_{t-1};v_t\right)$ and rewrite the problem as

$$\max \ E_{t-1}\left[v_t\frac{1}{b_{t-1}}f\left(\overline{\omega}\left(R^l_{t-1},b_{t-1};v_t\right)\right)\right]$$

subject to

$$E_{t-1}\left[v_tg\left(\overline{\omega}\left(R^l_{t-1},b_{t-1};v_t\right);\mu_t\right)\right] \geq R^d_{t-1}\left(1-b_{t-1}\right) \tag{17}$$

$$E_{t-1}v_tf\left(\overline{\omega}\left(R^l_{t-1},b_{t-1};v_t\right)\right) \geq R^d_{t-1}b_{t-1} \tag{18}$$

where

$$g\left(\overline{\omega}\left(R^l_{t-1},b_{t-1};v_t\right),\mu_t\right)=\int_0^{\frac{R^l_{t-1}(1-b_{t-1})}{v_t}}\left(1-\mu_t\right)\omega_t\Phi\left(d\omega\right)+\frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}\left(1-\Phi\left(\frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}\right)\right),$$

$$f\left(\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right)\right) = \int_{\frac{R^l_{t-1}(1-b_{t-1})}{v_t}}^{\infty} \omega_t \Phi\left(d\omega\right) - \frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}\left(1-\Phi\left(\frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}\right)\right).$$

Define as $\lambda_{1,t-1}$ and $\lambda_{2,t-1}$ the Lagrangean multipliers of (17) and (18) respectively. Conjecturing that $\lambda_{2,t-1} = 0$, the first-order conditions are

$$E_{t-1}\left[-\frac{v_t}{b^2_{t-1}} f\left(\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right)\right) + \frac{v_t}{b_{t-1}} f_2\left(R^l_{t-1}, b_{t-1}; v_t\right)\right] + \lambda_{1t-1} E_{t-1}\left[v_t g_2\left(R^l_{t-1}, b_{t-1}; v_t, \mu_t\right) + R^d_{t-1}\right] = 0$$

$$E_{t-1}\left[\frac{v_t}{b_{t-1}} f_1\left(R^l_{t-1}, b_{t-1}; v_t\right)\right] + \lambda_{1t-1} E_{t-1}\left[g_1\left(R^l_{t-1}, b_{t-1}; v_t, \mu_t\right) v_t\right] = 0$$

$$E_{t-1} g\left(\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right); \mu_t\right) v_t = R^d_{t-1}\left(1-b_{t-1}\right)$$

where f_j and g_j, with $j = 1, 2$, are the derivatives of f and g with respect to the first and second argument of the function $\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right)$.

Assuming $1 \neq b_t$, we can rewrite these conditions as

$$\lambda_{1t-1} R^d_{t-1} b_{t-1} = E_{t-1}\left[\frac{v_t}{b_{t-1}} f\left(\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right)\right)\right],$$

$$R^l_{t-1}\left(1-b_{t-1}\right)\lambda_{1t-1} E_{t-1}\left[\frac{\mu_t}{v_t}\phi\left(\frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}\right)\right] + \left(\frac{1}{b_{t-1}} - \lambda_{1t-1}\right) E_{t-1}\left[1-\Phi\left(\frac{R^l_{t-1}\left(1-b_{t-1}\right)}{v_t}\right)\right] = 0,$$

$$E_{t-1}\left[g\left(\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right); \mu_t\right) v_t\right] = R^d_{t-1}\left(1-b_{t-1}\right).$$

From the second condition, since $b_{t-1} < 1$ and $\lambda_{1t-1} > 0$, $R^l_{t-1}\left(1-b_{t-1}\right)\lambda_{1t-1} E_{t-1}\left[\frac{\mu_t}{v_t}\phi\left(\frac{R^l_{t-1}(1-b_{t-1})}{v_t}\right)\right] > 0$. Moreover, $1 > \Phi\left(\frac{R^l_{t-1}(1-b_{t-1})}{v_t}\right)$ so that $\lambda_{1t-1} - \frac{1}{b_{t-1}} > 0$ and $\lambda_{1t-1} b_{t-1} > 1$. It follows that $R^d_{t-1} b_{t-1} < E_{t-1}\left[v_t f\left(\overline{\omega}\left(R^l_{t-1}, b_{t-1}; v_t\right)\right)\right]$, which verifies the conjecture that $\lambda_{2t-1} = 0$.

Using the definition of the threshold, (16), the first-order conditions can be written as

$$E_{t-1}\left[v_t f\left(\overline{\omega}_t\right)\right] = \frac{R^d_{t-1}}{1 - \frac{E_{t-1}\left[\mu_t \overline{\omega}_t \phi\left(\overline{\omega}_t\right)\right]}{E_{t-1}\left[1-\Phi\left(\overline{\omega}_t\right)\right]}} b_{t-1} \tag{19}$$

$$E_{t-1}\left[v_t g\left(\overline{\omega}_t; \mu_t\right)\right] = R^d_{t-1}\left(1-b_{t-1}\right). \tag{20}$$

2.3 Entrepreneurial decisions

Entrepreneurs are infinitely lived and have linear preferences over consumption with rate of time preference β^e. We assume β^e sufficiently low so that the return on internal funds is always higher than the preference discount $\frac{1}{\beta^e}$. Entrepreneurs then accumulate their entire share of production as internal funds and never consume. They pay taxes which prevents their wealth to grow indefinitely.

The accumulation of internal funds is given by

$$B_t = f(\overline{\omega}_t) P_t A_t N_t - T_t, \tag{21}$$

which can be written as

$$B_t = f(\overline{\omega}_t) \frac{v_t}{b_{t-1}} B_{t-1} - T_t. \tag{22}$$

The tax revenues are

$$T_t = \gamma_t f(\overline{\omega}_t) \frac{v_t}{b_{t-1}} B_{t-1}. \tag{23}$$

They are transferred to the households or used for government consumption. The entrepreneurs do not internalize that they are being taxed at the rate γ_t. The accumulation of funds is, then, as follows

$$B_t = (1-\gamma_t) f(\overline{\omega}_t) \frac{v_t}{b_{t-1}} B_{t-1}. \tag{24}$$

The resource constraint is

$$A_t n_t [1 - \mu_t G(\overline{\omega}_t)] = c_t + G_t,$$

where G_t denotes government expenditures. We assume that government expenditures is a share g_t of total production,

$$G_t = g_t A_t n_t [1 - \mu_t G(\overline{\omega}_t)].$$

The resource constraint is then given by

$$c_t = (1-g_t) A_t n_t [1 - \mu_t G(\overline{\omega}_t)]. \tag{25}$$

2.4 Equilibria

The equilibrium conditions are given by equations (3)-(6), (7), (16), (19), (20), together with

$$B_{i,t} = b_t X_{i,t}, \tag{26}$$

equation (25), and

$$M_t + B_t = M_t^s$$

$$D_t = X_t - B_t,$$

where $\int B_{i,t}di = B_t$, $\int N_{i,t}di = n_t$, $\int X_{i,t}di = X_t$, and where $g(\overline{\omega}_t;\mu_t)$ and $f(\overline{\omega}_t)$ are given by (10) and (9), respectively, with $\overline{\omega}_t$ replacing $\overline{\omega}_{it}$.

Aggregating across firms and imposing market clearing, we can write conditions (7) and (26) as

$$\bar{b}_t = b_{t-1}\frac{A_t}{v_t}n_t$$

and

$$b_t = \frac{B_t}{X_t}, \tag{27}$$

where we have defined $\bar{b}_t = \frac{B_{t-1}}{P_t}$.

2.5 Implementability

The equilibrium conditions can be summarized by

$$\frac{u_c(t)}{\alpha} = \frac{v_t}{A_t},$$

$$\frac{u_c(t)}{P_t} = R_t^d \beta E_t \frac{u_c(t+1)}{P_{t+1}},$$

$$E_{t-1}\left[v_t f(\overline{\omega}_t)\right] = \frac{R_{t-1}^d}{1 - \frac{E_{t-1}[\mu_t \overline{\omega}_t \phi(\overline{\omega}_t)]}{E_{t-1}[1-\Phi(\overline{\omega}_t)]}} b_{t-1}$$

$$E_{t-1}\left[v_t g(\overline{\omega}_t;\mu_t)\right] = R_{t-1}^d(1-b_{t-1})$$

$$\overline{\omega}_t = \frac{R_{t-1}^l(1-b_{t-1})}{v_t}$$

$$N_t = \frac{v_t X_{t-1}}{A_t P_t}$$

$$B_{t-1} = b_{t-1}X_{t-1}$$

$$B_{t-1} = \left(1-\gamma_{t-1}\right) f(\overline{\omega}_{t-1}) \frac{v_{t-1}}{b_{t-2}} B_{t-2}$$

$$(1-g_t) A_t N_t \left[1 - \mu_t G(\overline{\omega}_t)\right] = c_t$$

with $g(\overline{\omega}_t;\mu_t)$ and $f(\overline{\omega}_t)$ being given by

$$f(\overline{\omega}_t) = \int_{\overline{\omega}_t}^{\infty} (\omega_t - \overline{\omega}_t)\,\Phi(d\omega).$$

and

$$g\left(\overline{\omega}_t;\mu_t\right)=\int_0^{\overline{\omega}_t}\left(1-\mu_t\right)\omega_t\Phi\left(d\omega\right)+\int_{\overline{\omega}_t}^{\infty}\overline{\omega}_t\Phi\left(d\omega\right).$$

The other conditions determine the remaining variables:

$$\frac{u_c\left(t\right)}{\beta u_c\left(t+1\right)}=Q_{t,t+1}^{-1}\frac{P_t}{P_{t+1}},$$

determines $Q_{t,t+1}^{-1}$,

$$\frac{E_t u_m\left(t+1\right)}{E_t u_c\left(t+1\right)}=R_t^d-1.$$

restricts m_{t+1},

$$v_t=\frac{A_t P_t}{W_t}$$

determines W_t.

At $t=0$, given the values b_{-1}, X_{-1} and R_{-1}^l, the optimal allocation c_0, N_0, v_0, $\overline{\omega}_0$, must satisfy

$$\frac{u_c\left(0\right)}{\alpha}=\frac{v_0}{A_0}$$

$$\overline{\omega}_0=\frac{R_{-1}^l\left(1-b_{-1}\right)}{v_0}$$

$$\left(1-g_0\right)A_0N_0\left[1-\mu_0G\left(\overline{\omega}_0\right)\right]=c_0.$$

These are 4 contemporaneous variables and 3 contemporaneous conditions. If P_0 is set exogenously, then using

$$N_0=\frac{v_0X_{-1}}{A_0P_0},$$

all variables have a single solution.

The restrictions can be written as a single constraint,

$$\left(1-g_0\right)A_0N_0\left[1-\mu_0G\left(\frac{R_{-1}^l\left(1-b_{-1}\right)}{\frac{u_c(0)}{\alpha}A_0}\right)\right]\geq c_0.$$

The first-order conditions imply

$$-\frac{u_c(0)}{v_n(0)}A_0=\frac{1+\left(1-g_0\right)A_0N_0\mu_0G'\left(0\right)\frac{\overline{\omega}_0}{c_0}}{\left(1-g_0\right)\left[1-\mu_0G\left(\overline{\omega}_0\right)\right]}. \tag{28}$$

Notice that $\frac{U_c(t)}{\alpha}A_t=v_t$. Hence, at $t=0$ the optimal markup is higher than it would be with an exogenous $\overline{\omega}$. This helps to lower bankruptcies.

The equilibrium conditions for $t \geq 1$ can be summarized by

$$\frac{u_c(t)}{\alpha} = \frac{v_t}{A_t}, \tag{29}$$

$$\frac{u_c(t)}{P_t} = R_t^d \beta E_t \frac{u_c(t+1)}{P_{t+1}},$$

$$E_{t-1}[v_t f(\overline{\omega}_t)] = \frac{R_{t-1}^d}{1 - \frac{E_{t-1}[\mu_t \overline{\omega}_t \phi(\overline{\omega}_t)]}{E_{t-1}[1-\Phi(\overline{\omega}_t)]}} b_{t-1} \tag{30}$$

$$E_{t-1}[v_t g(\overline{\omega}_t; \mu_t)] = R_{t-1}^d (1 - b_{t-1}) \tag{31}$$

$$\overline{\omega}_t = \frac{R_{t-1}^l (1 - b_{t-1})}{v_t}$$

$$N_t = \frac{v_t X_{t-1}}{A_t P_t}$$

$$B_{t-1} = b_{t-1} X_{t-1}$$

$$B_{t-1} = \left(1 - \gamma_{t-1}\right) f(\overline{\omega}_{t-1}) \frac{v_{t-1}}{b_{t-2}} B_{t-2}$$

$$(1 - g_t) A_t N_t [1 - \mu_t G(\overline{\omega}_t)] = c_t$$

in the following variables: c_t, N_t, v_t, $\overline{\omega}_t$, P_t, B_{t-1}, R_{t-1}^d, b_{t-1}, R_{t-1}^l, X_{t-1}. These are 5 contemporaneous variables and 5 predetermined variables, restricted by 4 contemporaneous conditions and 5 predetermined conditions.

If P_t are set exogenously, all the other variables have a single solution. Alternatively, set R_{t-1}^d, plus P_t in as many states as $\#S^t - \#S^{t-1}$.

We can use (**??**), and combine (30) and (31), to obtain a smaller set of implementability conditions:

$$E_{t-1}\left[\frac{u_c(t) A_t}{\alpha}\left[1 - \mu_t G(\overline{\omega}_t) - f(\overline{\omega}_t) \frac{E_{t-1}[\mu_t \overline{\omega}_t \phi(\overline{\omega}_t)]}{E_{t-1}[1 - \Phi(\overline{\omega}_t)]}\right]\right] = R_{t-1}^d$$

$$\frac{E_{t-1}\left[\frac{u_c(t) A_t}{\alpha} g(\overline{\omega}_t; \mu_t)\right]}{E_{t-1}\left[\frac{u_c(t) A_t}{\alpha} f(\overline{\omega}_t)\right]} = \frac{\frac{1-b_{t-1}}{b_{t-1}}}{1 - \frac{E_{t-1}[\mu_t \overline{\omega}_t \phi(\overline{\omega}_t)]}{E_{t-1}[1-\Phi(\overline{\omega}_t)]}}$$

$$\overline{\omega}_t = \frac{R_{t-1}^l (1 - b_{t-1})}{\frac{u_c(t) A_t}{\alpha}}$$

$$\frac{u_c(t-1)}{P_{t-1}} = R_{t-1}^d \beta E_{t-1} \frac{u_c(t)}{P_t}, \tag{32}$$

$$N_t = \frac{u_c(t)}{\alpha} \frac{B_{t-1}}{b_{t-1} P_t} \tag{33}$$

$$B_{t-1} = \left(1-\gamma_{t-1}\right) f\left(\overline{\omega}_{t-1}\right) \frac{v_{t-1}}{b_{t-2}} B_{t-2} \tag{34}$$

and

$$\left(1-g_t\right) A_t N_t \left[1-\mu_t G\left(\overline{\omega}_t\right)\right] = c_t$$

The other conditions determine other variables:

$$\frac{u_c\left(t\right)}{\alpha} = \frac{v_t}{A_t},$$

determines v_t;

$$B_{t-1} = b_{t-1} X_{t-1}$$

determines X_{t-1}.

We can use conditions (32), (33) and (34) to get the smallest set of implementability conditions in c_t, N_t, $\overline{\omega}_t$, R^d_{t-1}, b_{t-1}, R^l_{t-1}, $t \geq 1$,

$$E_{t-1}\left[\frac{u_c\left(t\right)A_t}{\alpha}\left[1-\mu_t G\left(\overline{\omega}_t\right) - f\left(\overline{\omega}_t\right)\frac{E_{t-1}\left[\mu_t \overline{\omega}_t \phi\left(\overline{\omega}_t\right)\right]}{E_{t-1}\left[1-\Phi\left(\overline{\omega}_t\right)\right]}\right]\right] = R^d_{t-1}, \tag{35}$$

$$\frac{E_{t-1}\left[\frac{u_c(t)A_t}{\alpha} g\left(\overline{\omega}_t;\mu_t\right)\right]}{E_{t-1}\left[\frac{u_c(t)A_t}{\alpha} f\left(\overline{\omega}_t\right)\right]} = \frac{\frac{1-b_{t-1}}{b_{t-1}}}{1-\frac{E_{t-1}\left[\mu_t\overline{\omega}_t\phi(\overline{\omega}_t)\right]}{E_{t-1}\left[1-\Phi(\overline{\omega}_t)\right]}}$$

$$\overline{\omega}_t = \frac{R^l_{t-1}\left(1-b_{t-1}\right)}{\frac{u_c(t)A_t}{\alpha}}$$

$$N_{t-1}\left(1-\gamma_{t-1}\right) f\left(\overline{\omega}_{t-1}\right) v_{t-1} = R^d_{t-1} b_{t-1} \beta E_{t-1} N_t,$$

$$\left(1-g_t\right) A_t N_t \left[1-\mu_t G\left(\overline{\omega}_t\right)\right] = c_t$$

There are 3 predetermined conditions and 2 contemporaneous conditions for 3 predetermined variables and 3 contemporaneous variables.

Condition

$$P_t = \frac{u_c\left(t\right)}{\alpha}\frac{B_{t-1}}{b_{t-1}N_t}$$

is satisfied by the choice of P_t.

2.6 Optimal policy

Suppose exogenous government consumption in each state is at some level G_t not proportional to output. Then, the Friedman rule, $R^d_{t-1} = 1$, is optimal, as in the calibrated version we analyze below. Setting the nominal interest rate at its lower bound does not exhaust monetary policy. Because the funds are nominal and predetermined there is still a role for policy.

In particular, in response to a technology shock, the optimal price level policy is aimed at keeping the nominal wage constant. The price level adjusts so that the real wage moves with productivity. As a result, labor does not move, wages do not move and therefore, nominal predetermined funds are ex-post optimal.

The optimality of the Friedman rule derives from the fact that steady state bankruptcies are independent of monetary policy. The steady state $\overline{\omega}$ can be found as a solution of the following equation

$$\frac{1-\gamma}{\beta}=1-\frac{\mu\overline{\omega}\phi\left(\overline{\omega}\right)}{1-\Phi\left(\overline{\omega}\right)}$$

which is obviously independent of the nominal interest or inflation.

Suppose we maximized utility

$$E_{t-1}\left[u\left(c_t\right)-\alpha n_t\right] \tag{36}$$

subject to the resource constraint only, for given $\overline{\omega}_t$. Then, optimality would require that

$$\frac{u_c\left(t\right)A_t}{\alpha}=\frac{1}{1-\mu_t G\left(\overline{\omega}_t\right)}.$$

From

$$\frac{u_c\left(t\right)A_t}{\alpha}=\frac{R_{t-1}^d}{1-\mu_t G\left(\overline{\omega}_t\right)-f\left(\overline{\omega}_t\right)\frac{E_{t-1}\left[\mu_t\overline{\omega}_t\phi\left(\overline{\omega}_t\right)\right]}{E_{t-1}\left[1-\Phi\left(\overline{\omega}_t\right)\right]}}, \tag{37}$$

$$E_{t-1}\left[\frac{u_c\left(t\right)A_t}{\alpha}\left[1-\mu_t G\left(\overline{\omega}_t\right)-f\left(\overline{\omega}_t\right)\frac{E_{t-1}\left[\mu_t\overline{\omega}_t\phi\left(\overline{\omega}_t\right)\right]}{E_{t-1}\left[1-\Phi\left(\overline{\omega}_t\right)\right]}\right]\right]=R_{t-1}^d, \tag{38}$$

this could only be satisfied if either $\mu_t=0$ or $\overline{\omega}_t=0$, and $R_{t-1}^d=1$, for each state. When credit frictions are present, and $f\left(\overline{\omega}_t\right)\frac{E_{t-1}\left[\mu_t\overline{\omega}_t\phi\left(\overline{\omega}_t\right)\right]}{E_{t-1}\left[1-\Phi\left(\overline{\omega}_t\right)\right]}\neq 0$, it is still optimal to set $R_{t-1}^d=1$. It is a corner solution. The Friedman rule is optimal.

With $g_t>0$ it is optimal to tax on average. The same argument as above cannot go through. The optimal condition just using the resource constraint would require that

$$\frac{u_c\left(t\right)A_t}{\alpha}=\frac{1}{\left(1-g_t\right)\left[1-\mu_t G\left(\overline{\omega}_t\right)\right]}. \tag{39}$$

In spite of the reason to subsidize, due to $f\left(\overline{\omega}_t\right)\frac{E_{t-1}\left[\mu_t\overline{\omega}_t\phi\left(\overline{\omega}_t\right)\right]}{E_{t-1}\left[1-\Phi\left(\overline{\omega}_t\right)\right]}$, if g_t is high enough, it is optimal to tax, which in this model can only be done using the nominal interest rate. Then it will be optimal to tax at different rates, in response to shocks.

From condition (35) at the lower bound,

$$E_{t-1}\left[\frac{u_c(t)\,A_t}{\alpha}\left[1-\mu_t G(\overline{\omega}_t)-f(\overline{\omega}_t)\frac{E_{t-1}[\mu_t\overline{\omega}_t\phi(\overline{\omega}_t)]}{E_{t-1}[1-\Phi(\overline{\omega}_t)]}\right]\right]=1,$$

we can understand what is at stake in optimal policy. $\frac{u_c(t)A_t}{\alpha}$ is the wedge between the marginal rate of substitution and the marginal rate of transformation if the financial technology is not taken into account. The term $\frac{1}{1-\mu_t G(\overline{\omega}_t)-f(\overline{\omega}_t)\frac{E_{t-1}[\mu_t\overline{\omega}_t\phi(\overline{\omega}_t)]}{E_{t-1}[1-\Phi(\overline{\omega}_t)]}}$ is the financial markup present in models with asymmetric information and bankruptcy costs. The wedge has to be equal to the financial markup, on average, but not always in response to shocks. As will be clear from the numerical results, the optimal policy in response to technology shocks will be to stabilize the financial markup, therefore stabilizing bankruptcy rates, and setting the wedge equal to the stabilized financial markup.

3 Numerical results

The model calibration is very standard. We assume utility to be logarithmic in consumption and linear in leisure. Following Carlstrom and Fuerst (1997), we calibrate the volatility of idiosyncratic productivity shocks and the rate of accumulation of internal funds, $1-\gamma_t$, so as to generate an annual steady state credit spread of approximately 2% and a quarterly bankruptcy rate of approximately 1%.[2] The monitoring cost parameter μ is set at 0.15 following Levin *et al.* (2004).

In the rest of this section, we always focus on adverse shocks, i.e. shocks which tend to generate a fall in output. Impulse responses under optimal policy refer to an equilibrium in which policy is described by the first order conditions of a Ramsey planner deciding allocations for all times $t \geq 1$, but ignoring the special nature of the initial period $t = 0$. Responses under a Taylor rule refer to an equilibrium in which policy is set according to the following simple interest rate rule:

$$\widehat{i}_t = 1.5 \cdot \widehat{\pi}_t \tag{40}$$

where hats denote logarithmic deviations from the non-stochastic steady state.

In all cases, we only study the log-linear dynamics of the model.

[2] The exact values are 1.8% for the annual spread and 1.1% for the bankruptcy rate.

3.1 Impulse responses under optimal policy

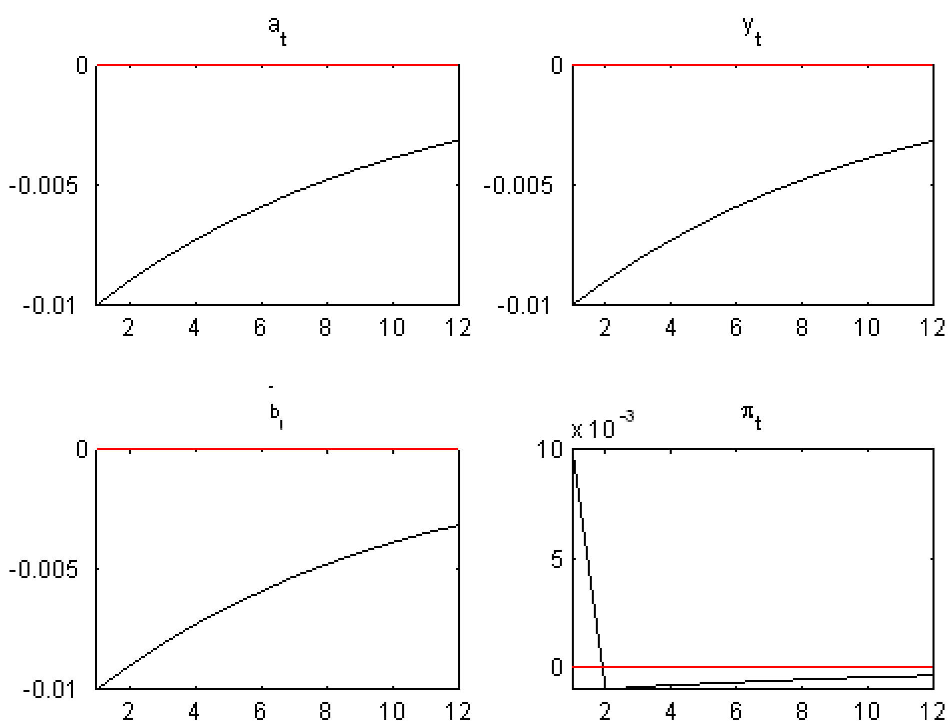
Optimal policy in the calibrated version of the model entails setting the nominal interest rate permanently to zero, as long as government consumption is exogenous. This restriction is imposed when computing impulse responses.

3.1.1 Technology shocks: price stability is not optimal

Figure 1 shows the impulse response of selected macroeconomic variables to a negative, 1% technology shock under optimal policy.

It is important to recall that the model includes many features which could potentially lead to equilibrium allocations that are far from the first best: asymmetric information and monitoring costs; the predetermination of financial decisions; and the nominal denomination of debt contracts. At the same time, the presence of nominal predetermined contracts implies that monetary policy is capable of affecting allocations by choosing appropriate sequences of prices.

Figure 1: Impulse responses to a negative technology shock under optimal policy



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9.

Figure 1 illustrates that optimal policy is able to replicate the first best allocation in which consumption behaves exactly as in the neoclassical benchmark without any frictions. In response to the negative technology shock, since nominal internal and external funds are predetermined, and optimal policy generates inflation for 1 period. As a result, the real value of total funds needed to finance production falls exactly by the amount necessary to generate the correct reduction in output.

In subsequent periods, the real value of total funds is slowly increased through a mild reduction in the price level. Along the adjustment path, leverage remains constant and firms make no losses. Consumption moves one-to-one with technology, while hours worked remain constant. With constant labor and an equilibrium nominal wage that stays constant, the restriction that funds are predetermined is not relevant. The price level adjusts so that the real wage is always equal to productivity. Since total funds are always at the desired level, the accumulation equation for nominal funds never kicks in.

The impulse responses in Figure 1 would obviously be symmetric after a positive technology shock. Hence, perfect price stability – i.e. an equilibrium in which the price level is kept perfectly constant at all points in time – is not optimal in our model (we show below that this is the case for all shocks, not just technology shocks). Short inflationary episodes are useful to help firms adjust their funds, both internal and external, to their production needs. In the case of technology shocks, this policy also prevents any undesirable fluctuations in the economy's bankruptcy rate, financial markup, or the markup resulting from the predetermination of assets.

This result is robust to a number of perturbations of the model. It also holds if there are reasons not to keep the nominal interest rate at zero. And it obviously holds in a model where internal and external funds are perfect substitutes.

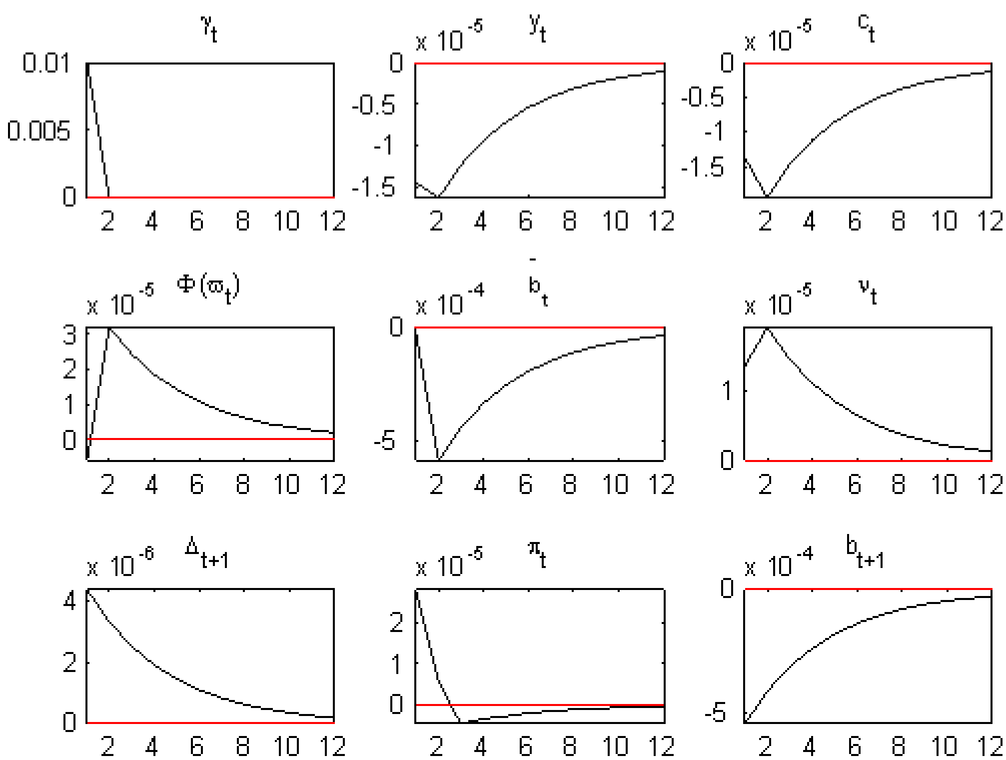
3.1.2 Financial shocks

We analyze the impulse responses to three types of financial shocks which can be defined in our economy. The first is an increase in γ_t, namely a shock which generates an exogenous reduction in the level of internal funds. The second one is a shock to the standard deviation of idiosyncratic technology shocks, which amounts to an increase in the uncertainty of the economic environment. The third shock is an increase in the monitoring cost parameter μ_t.

Gamma shocks Figure 2 illustrates the impulse responses to a shock to γ_t. This shock is interesting because it generates at the same time a reduction in output and an increase in leverage – leverage can be defined as the ratio of external to internal funds used in production, i.e. as $1/b_t - 1$, and it is therefore negatively related to b_t. To highlight the different persistence of the effects of the shock, depending on the prevailing policy rule, we focus on a serially uncorrelated shock.

The higher γ does not have an effect on funds on impact because of the predetermination of financing decisions, but it represents a fall in internal funds at $t+1$, which leads to an increase in firms' leverage. We will see below that under a Taylor rule this shock brings about a period of deflation, which would be quite persistent if the original shock were also persistent.

Figure 2: Impulse responses to a fall in the value of internal assets under optimal policy

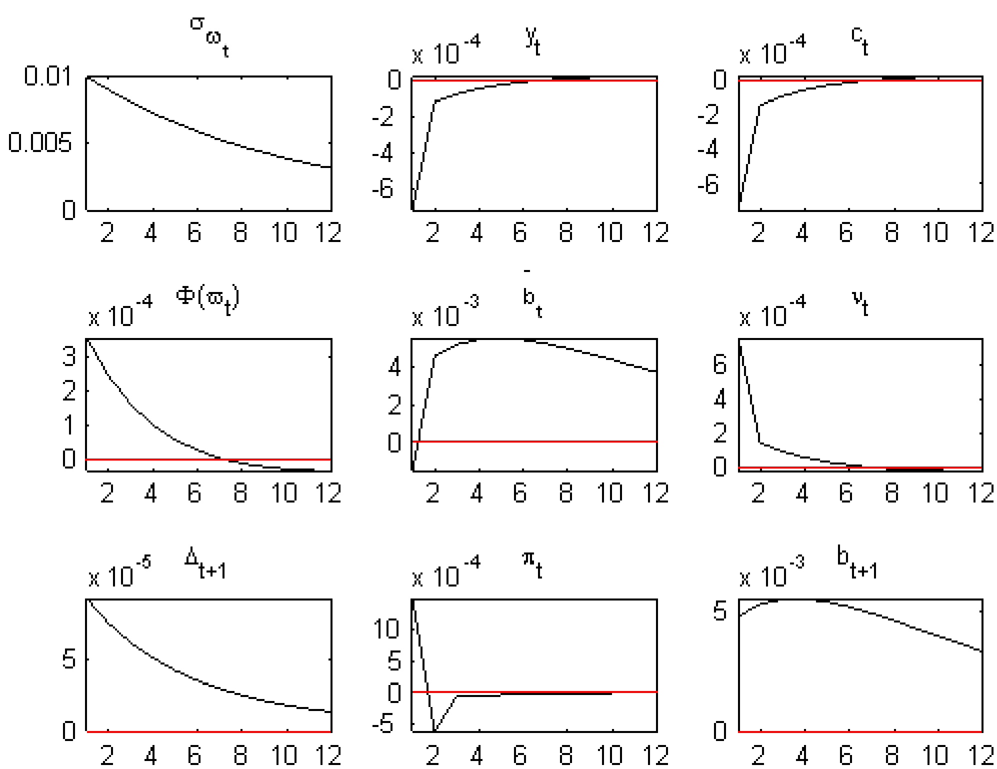


Note: Logarithmic deviations from the non-stochastic steady state. Serially uncorrelated shock.

The optimal policy response, on the contrary, is to create a short-lived period of inflation. The impact increase in the price level lowers the real value of funds, so as to decrease labor and production to levels better consistent with the reduction in the nominal amount of internal funds. This mitigates the increases in the bankruptcy rate, $\Phi(\varpi_t)$, and leads to an increase in the financial mark-up ν_t. When, at $t+1$, leverage and the credit spread increase, the higher profits allow firms to quickly start rebuilding their internal funds. The adjustment process is essentially complete after 3 years. When consumption starts growing towards the steady state, the real rate must increase. For given nominal interest rate, there must be a period of mild deflation.

Standard deviation shocks Figure 3 shows the impulse responses to a persistent increase in the riskiness of the economy, i.e. to an increase in the standard deviation of the idiosyncratic shocks $\omega_{i,t}$.

Figure 3: Impulse responses to an increase in σ_{ω_t} under optimal policy

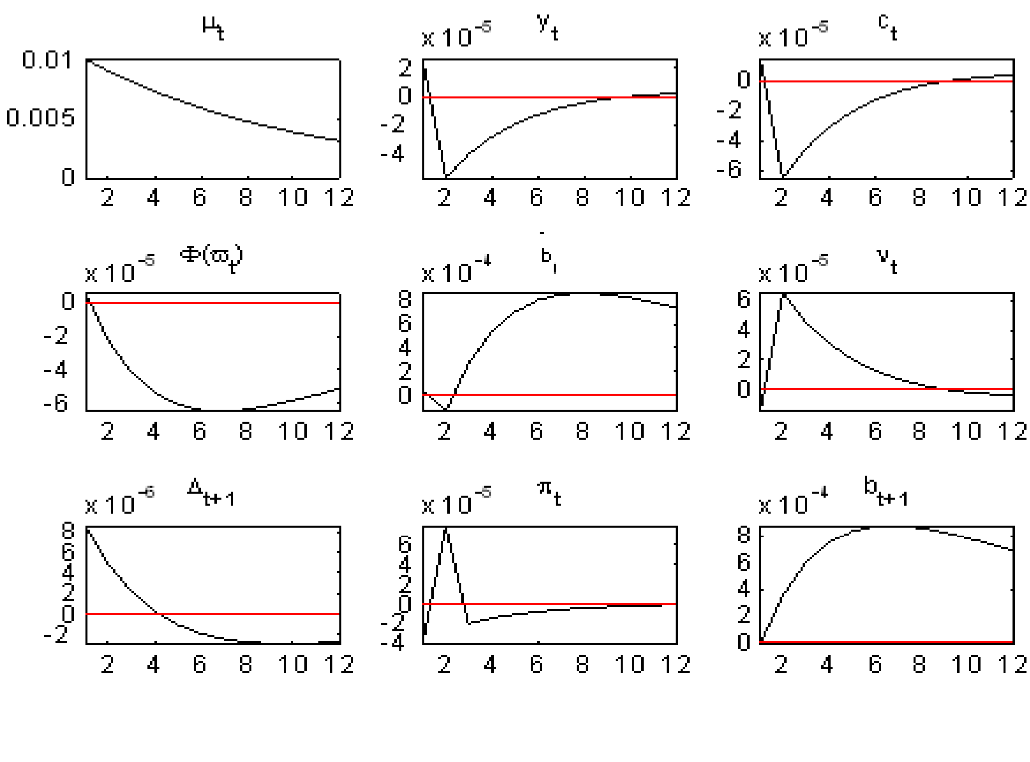


Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9.

This shock is associated with a prospective worsening of credit conditions and an increase in the bankruptcy rate. On impact, as in the case of the negative technology shock, policy engineers an increase in the price level to reduce output. The financing conditions stipulated before the shock are ex-post favorable to firms: on impact, the output contraction enables them to make higher profits, so that they will accumulate more internal funds in the following period. This increase in internal funds allows for a fast economic recovery, in spite of the contemporaneous increase in credit spreads. Even if the shock is serially correlated, output and consumption are back at the steady state after 2 years.

Mu shocks An exogenous increase in the proportion of total funds lost in monitoring activities, μ_t, is different from the shock previously analysed because it mechanically implies a higher waste of resources per unit of output. The optimal policy response is to reduce output in order to minimise the resource loss. If the shock were serially uncorrelated, this would once again be achieved through an impact increase in the price level.

Figure 4: Impulse responses to an increase in μ_t under optimal policy



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9.

Since the shock is persistent, however, policy needs to manage a trade-off between immediate and future resource losses. An impact increase in the price level would not only immediately reduce output, but it would also lead to more profits and a faster accumulation of internal funds. As in the case of an increase in the volatility of idyosincratic shocks, this would imply a quick recovery, hence large future losses in monitoring activity as long as μ_t remains high. Compared to this scenario, future losses would be minimized if the price level were instead cut on impact, so that firms' leverage would increase and the accumulation of internal funds would be especially slow. At the same time, however, an impact fall in the price level would increase the real value of firms' funds which, in turn, would allow them to expand production with an ensuing amplification of the impact resource loss due to the higher μ_t.

It turns out that the optimal response is to do almost nothing on impact, allowing for a very mild fall in the price level (see Figure 4). As a result, output does not fall – it actually increases slightly – and the bankruptcy rate stays almost unchanged. It is only after one period that production falls, due to an increase in both the credit spreads and the price level. Firms start from scratch their slow process of accumulation of internal funds and the shock is reabsorbed very slowly.

3.2 Taylor rule policy

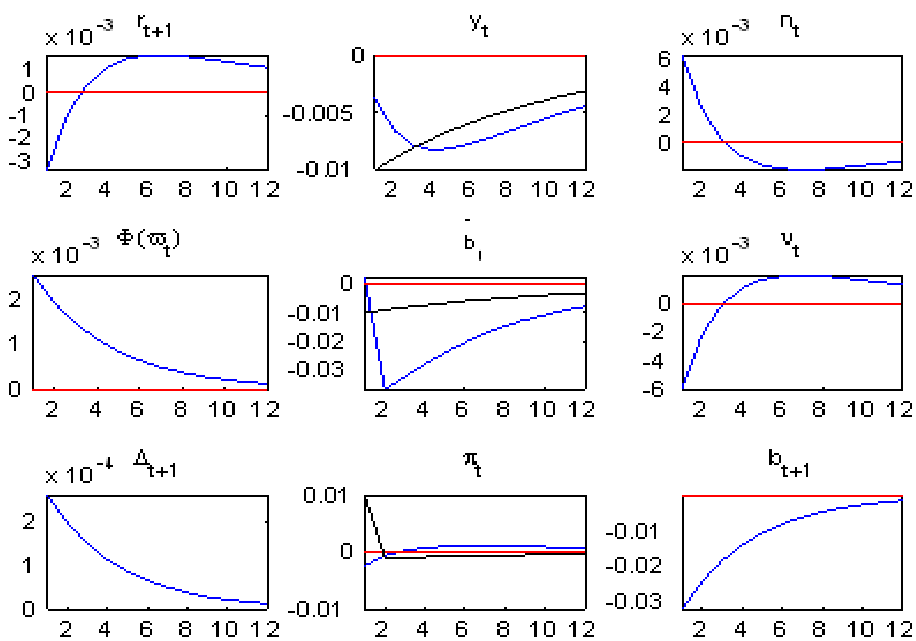
In this section, we compare the impulse responses under optimal policy with those in which policy follows the simple Taylor rule in equation (40).

3.2.1 Technology shocks and the cyclicality of bankruptcies

In response to a negative technology shock, the simple Taylor rule tries to stabilize inflation (see Figure 5). The large amount of nominal funds that firms carry over from the previous period, therefore, has high real value. Given the available funds, firms hire more labor and the output contraction is relatively small, compared to what would be optimal at the new productivity level. As a result, the wage share increases and firms make lower profits, hence they must sharply reduce their internal funds. Leverage goes up, and so do the credit spread and the bankruptcy rate. In the period after the shock, firms start accumulating funds again, but accumulation is slow and output keeps falling for a whole year after the shock. It is only in the second year after the shock that the recovery begins.

Figure 5 illustrates how our model is able to generate realistic, cyclical properties for the credit spread and the bankruptcy ratio. An increase in bankruptcies is almost a definition of recession in the general perception, while the fact that credit spreads are higher during NBER recession dates is documented, for example, in Levin et al. (2004). Generating the correct cyclical relationship between credit spreads, bankruptcies and output is not straightforward in models with financial frictions. For example, spreads are unrealistically procyclical in the Carlstrom and Fuerst (1997, 2000) framework. The reason is that firms' financing decisions are state contingent in those papers. Firms can choose how much to borrow from the banks *after* observing aggregate shocks. Should a negative technology shock occur, they would immediately borrow less and try to cut production. This would avoid large drops in their profits and internal funds, so that their leverage would not increase. As a result, bankruptcy rates and credit spreads could remain constant or decrease during the recession.

Figure 5: Impulse responses to a negative technology shock under a Taylor rule



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9. The blue lines indicate impulse responses under the Taylor rule; the black lines report the impulse responses under optimal policy already shown in Figure 1.

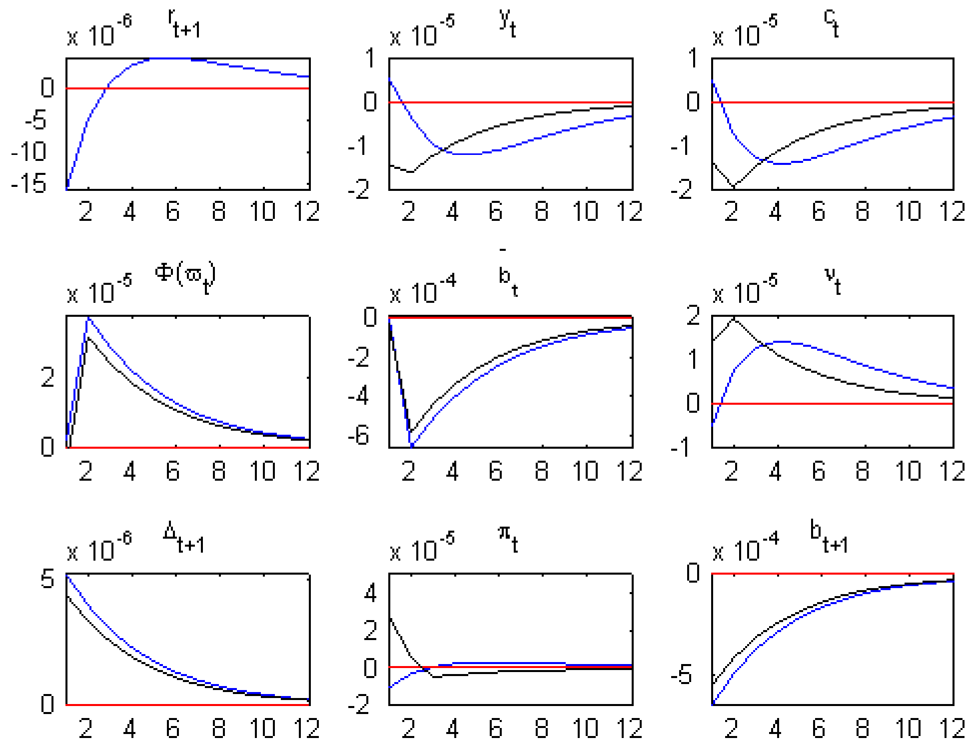
In our simpler model, economic outcomes are reversed because of the pre-determination in financial decisions. Firms' loans are no-longer state contingent, hence they cannot be changed after observing aggregate shocks. This assumption implies that firms are constrained in their impact response to disturbances. After a negative technology shock, firms find themselves with too much funds and their profits will fall because production levels do not fall enough. The reverse would happen during an expansionary shock, when production would initially increase too little and profits would be high.

Our model also generates a realistically hump-shaped impulse response of output and consumption without the need for additional assumptions, such as habit persistence in households' preferences. Once a shock creates the need for changes in internal funds, these changes can only take place slowly. Compared to the habit persistence assumption, our model implies that the hump-shape in impulse responses is policy-dependent. After a technology shock, optimal policy keeps internal funds at their optimal level at any point in time. Firms do not need to accumulate, or decumulate, internal funds, and, as a result, the hump in the response of output and consumption disappears.

3.2.2 Financial shocks

Gamma shocks Contrary to the optimal policy case, under a Taylor rule this shock leads to a fall, rather than an increase, in the price level.

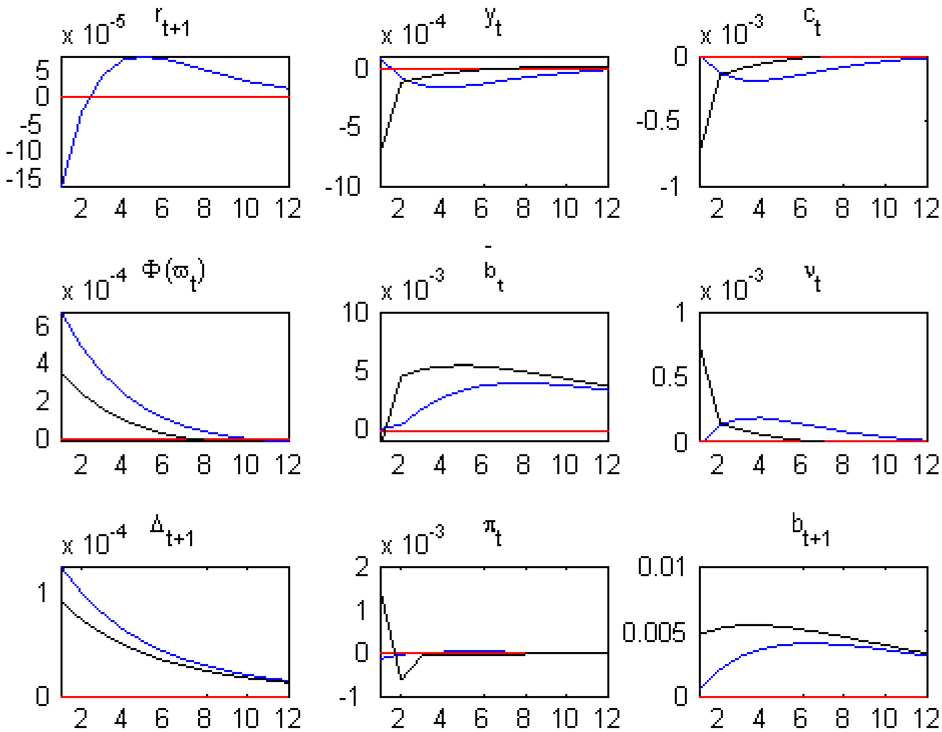
Figure 6: Impulse responses to a fall in the value of internal assets under a Taylor rule



Note: Logarithmic deviations from the non-stochastic steady state. The shock is serially uncorrelated. The blue lines indicate impulse responses under the Taylor rule; the black lines report the impulse responses under optimal policy already shown in Figure 2.

The situation in which firms' leverage increase and deflation ensues is akin to the "initial state of over-indebtedness" described in Fisher (1933). In Fisher's theory, firms try to deleverage through a fast debt liquidation and the selling tends to drive down prices. If monetary policy accommodates this trend, the price level also falls and the real value of firms liabilities increase further, leading to even higher leverage and further selling.

Figure 7: Impulse responses to an increase in σ_{ω_t} under a Taylor rule

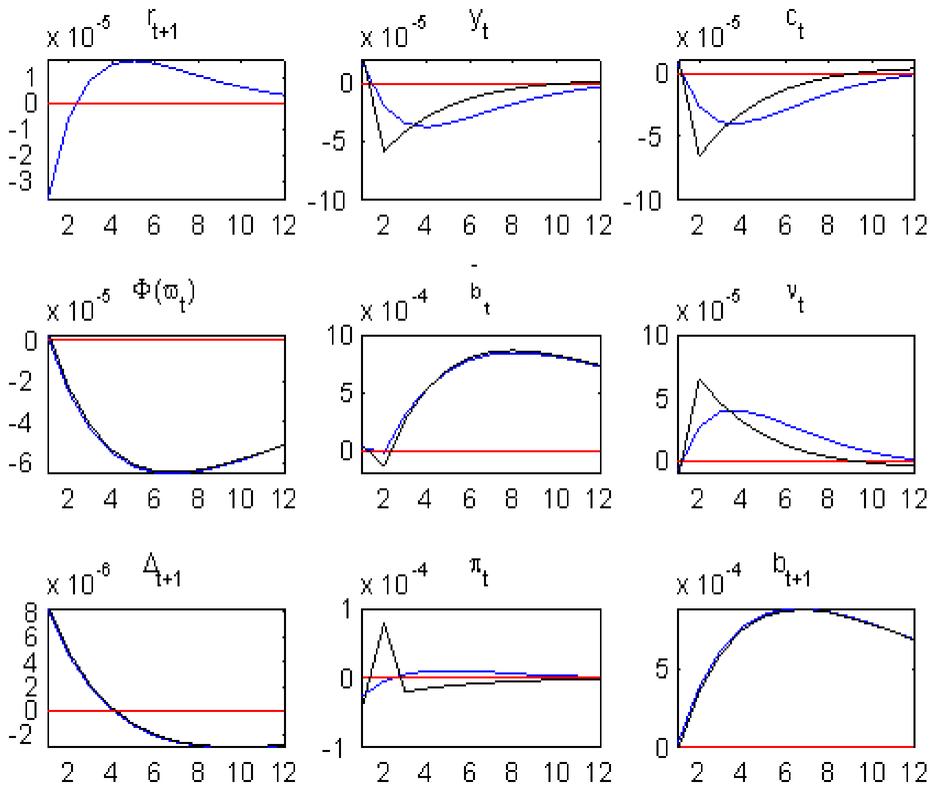


Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9. The blue lines indicate impulse responses under the Taylor rule; the black lines report the impulse responses under optimal policy already shown in Figure 3.

In our model, over-indebtedness and leverage are also exacerbated by deflation, but the mechanics of the model are different (see Figure 6). The progressive increase in leverage leads to an increase in the economy's bankruptcy rate, and a protracted fall in consumption. This, in turn, is associated with a fall in the real interest rate which, given the policy rule, is implemented through a cut in the nominal rate and a small deflationary period. De-leveraging occurs through an accumulation of assets, rather than a liquidation of debt. However, the de-leveraging process is very slow and consumption is still away from the steady state three years after the shock. Compared to the optimal policy case, the recession is more persistent and it comes at the cost of a higher bankruptcy rate ($\Phi\left(\overline{\omega}_t\right)$ increases) and a higher credit spread.

Standard deviation shocks Impulse responses to a persistent increase in the standard deviation of idiosyncratic shocks are displayed in Figure 7.

Figure 8: Impulse responses to an increase in μ_t under a Taylor rule



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9. The blue lines indicate impulse responses under the Taylor rule; the black lines report the impulse responses under optimal policy already shown in Figure 4.

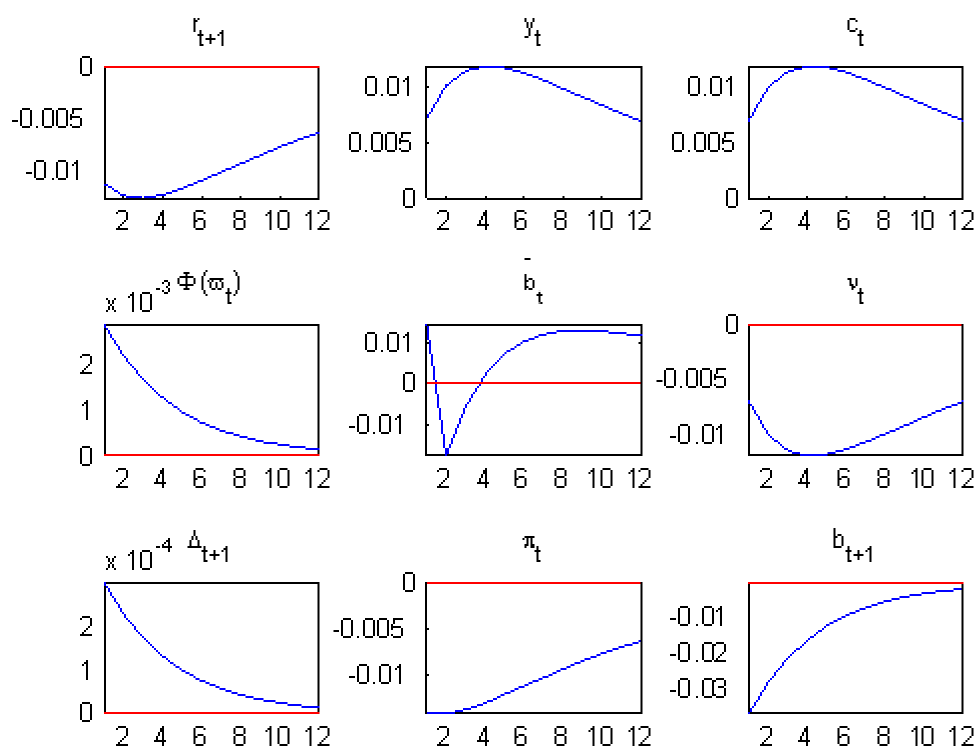
As in the case of optimal policy, credit spreads increase and bank loans tend to fall, pushing down output and consumption. The expected decrease in consumption implies that real interest rates must fall. This happens through a marked decrease in nominal rates and a smaller reduction in the price level that, on impact, boosts the real value of firms' funds and brings about a one-period increase in output. Consequently, the reduction in firms leverage is small and the accumulation of internal funds takes much longer than under optimal policy. The dynamic responses of output and consumption are smoother than under optimal policy, but the recession lasts longer.

Mu shock Figure 8 shows the impulse responses to μ_t. In this case, the differences compared to the optimal policy case are smaller. The dynamics of the credit spread, of internal funds and of the bankruptcy rate are almost identical. The resource loss in monitoring, however, is higher under the Taylor rule, because output falls less in the few quarters after the shock, when μ_t is highest, and more after 1 year, when μ_t is returning to its steady state level.

3.2.3 Policy shocks

Figure 9 shows the impulse responses to a serially correlated shock to the Taylor rule, corresponding to a cut in the policy rate.

Figure 9: Impulse responses to a monetary policy shock



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9.

The shock is useful to illustrate the general features of the "monetary policy transmission mechanism" in our model. These are characterized by the slow mechanism of accumulation of internal funds, which produces very persistent responses in all variables.

The shock generates an immediate fall in the price level which boosts the real value of firms' nominal funds and induces a boom in production and consumption through an increase in employment higher real wages. Since leverage is predetermined in the first period, the higher production level brings about an increase in the bankruptcy rate. Profits fall and, after one period, firms find themselves short of internal funds and start rebuilding them. The adjustment process is very slow. Three years after the shock, output, consumption and employment are still far away from the steady state.

3.3 Optimal policy when a non-zero interest rate is optimal. The relevance of the lower bound.

In this section, we explore to which extent the optimal policy recommendations described above are affected by the fact that the nominal interest rate is kept constant at zero. In the calibration, we keep all other parameters unchanged, but we assume a proportional government spending shock $g = 0.2$ in steady state. As discussed above, the optimal steady state level of the nominal interest rate increases proportionately. As a result, there is also an increase in the steady state level of the credit spread and of the bankruptcy rate.[3]

3.3.1 Technology shocks

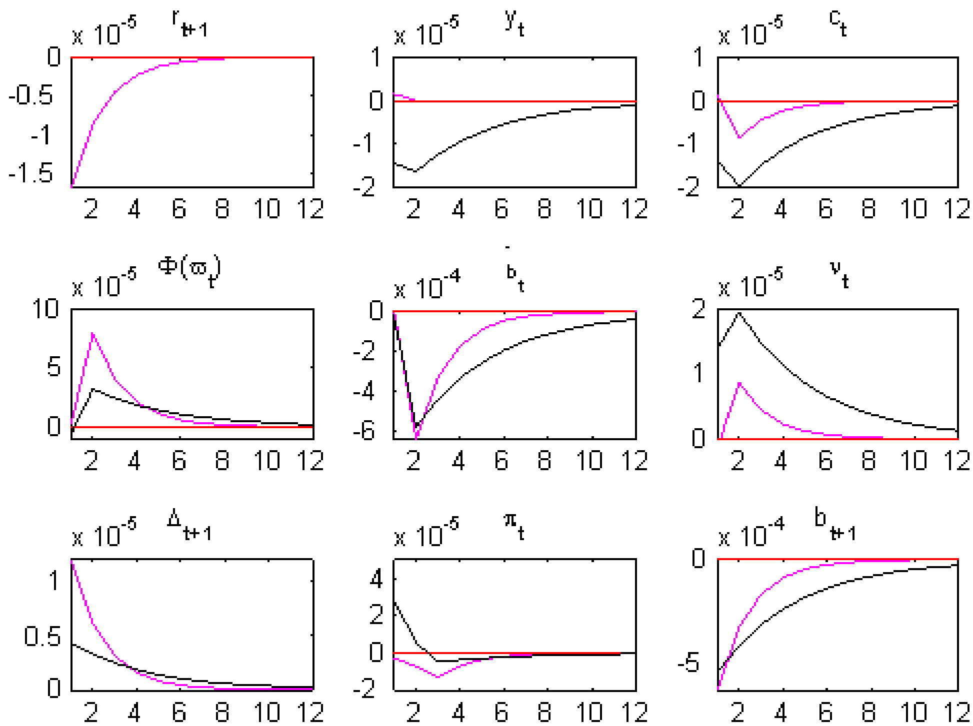
In spite of the availability of the nominal interest rate as a policy instrument, the optimal response to a technology shock is the same as before. As already discussed, policy is able to replicate the allocations which would be attained in a frictionless model even when the nominal interest rate must be kept constant (at zero). There are therefore no reasons to deviate from that policy even if the nominal interest rate can be moved.

3.3.2 Financial shocks

For all financial shocks, the flexibility of using the nominal interest rate allows policy to speed up the adjustment after financial shocks. The effect of these shocks on output are completely reversed. We illustrate this general result with reference to a serially uncorrelated shock to γ.

Figure 10: Impulse responses to a fall in the value of internal assets under optimal policy

[3] In steady state, the credit spread increases to 1.27% and the bankruptcy rate to 6.7%.



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9. The violet lines indicate impulse responses under optimal policy when $g > 0$; the black lines report the impulse responses under optimal policy already shown in Figure 2.

The impulse responses to this shock under optimal policy are shown in figure 10, together with the impulse responses of the case where the Friedman rule is optimal. The most striking result is that the impact of this shock on output, which is persistently contractionary when the short term nominal rate is kept fixed at zero, becomes mildly expansionary and very short-lived in this cases.

Given that output is at the steady state after a slight impact increase, policy does not need to generate inflation to kick-start the processes of accumulation of nominal funds. It can improve credit conditions directly, but reducing the policy interest rate and therefore, *ceteris paribus*, loan rates. While the increase in the credit spread is even larger here than in the case when the Friedman rule is optimal, the increase is offset by a slightly larger than one-to-one reduction in the policy rate.

The effect on the other variables is comparable to the case where the Friedman rule is optimal, but the adjustment process is much faster. It is literally complete after two years, compared to three years or more in the benchmark case.

4 Two benchmark models

In order to understand the mechanisms responsible for these results, we analyze two benchmark models: i) a model where assets are predetermined, but where internal and external funds are perfect substitutes (i.e. monitoring costs are zero); and ii) a model with real exogenous internal funds and no predetermined assets, as in De Fiore and Tristani (2008).

4.1 A model where internal and external funds are perfect substitutes

When $\mu_t = 0$, for all t, internal and external funds are perfect substitutes. We use this first benchmark model to show that even in the absence of asymmetric information and costly state verification, price stability is not optimal. Hence, the predetermination of assets and the nominal denomination of funds are key for the result obtained in our general model that price stability is not optimal.

We also use this benchmark model to assess the role played by asymmetric information and monitoring costs in explaining business cycle fluctuations. We find that, although these imperfections play a quantitatively minor role in determining the cyclical behavior of non-financial variables, they tend to amplify the reaction of the economy to shocks.

4.1.1 Price stability is not optimal

Define $x_t = \frac{X_{t-1}}{P_t}$. The equilibrium conditions in this economy are given by (3)-(6), together with

$$R^l_{t-1} = R^d_{t-1} = R_{t-1}$$

$$E_{t-1}\left[v_t\right] = R_{t-1}$$

$$N_t = \frac{x_t P_t}{W_t} \tag{41}$$

$$v_t = \frac{A_t P_t}{W_t} \tag{42}$$

$$c_t = \left(1 - g_t\right) A_t N_t. \tag{43}$$

The implementability conditions are for $t \geq 1$:

$$E_{t-1}\left[\frac{u_c\left(t\right) A_t}{\alpha}\right] = R_{t-1} \tag{44}$$

$$c_t = \left(1 - g_t\right) A_t N_t \tag{45}$$

Every allocation in this set can be implemented. The other conditions determine other variables:

$$R^l_{t-1} = R^d_{t-1} = R_{t-1}$$

determine R^l_{t-1} and R^d_{t-1}.

For $t = 0$, given a value X_{-1} and an allocation C_0 and N_0,

$$N_0 = \frac{X_{-1}}{W_0}$$

and

$$\frac{u_c(0)}{\alpha} = \frac{P_0}{W_0}, \tag{46}$$

are satisfied by the choice of W_0 and P_0.

For $t \geq 1$, given an allocation c_t and N_t, condition

$$\frac{u_c(t-1)}{P_{t-1}} = R_{t-1}\beta E_{t-1}\frac{u_c(t)}{P_t} \tag{47}$$

restricts P_t; for a given P_t,

$$N_t = \frac{x_t P_t}{W_t}$$

and

$$\frac{u_c(t)}{\alpha} = \frac{P_t}{W_t}, \tag{48}$$

determine W_t and x_t;

$$v_t = \frac{A_t P_t}{W_t},$$

determine v_t;

$$\frac{u_c(t-1)}{P_{t-1}} = Q^{-1}_{t-1,t}\frac{u_c(t)}{P_t}, \tag{49}$$

determines $Q^{-1}_{t-1,t}$; and

$$\frac{E_{t-1}u_m(t)}{E_{t-1}u_c(t)} = R^d_{t-1} - 1. \tag{50}$$

restricts m_t.

The optimal allocation maximizes utility subject to

$$c_t = (1-g_t)A_t N_t.$$

Optimality requires that

$$\frac{u_c(t)}{\alpha} = \frac{1}{(1-g_t)A_t}. \tag{51}$$

The optimal policy is to pick the interest rate so that condition (44) is satisfied

$$R_{t-1} = E_{t-1}\left[\frac{1}{1-g_t}\right].$$

In this economy, however, monetary policy should do much more. It should move the price level according to

$$P_t = \frac{W_t}{(1-g_t)\,A_t},$$

so that (51) is satisfied.

With $g_t = 0$, or with an exogenous level of government consumption, the optimal interest rate policy would be the Friedman rule, $R_{t-1} = 1$, and the price level policy would guarantee that

$$\frac{u_c(t)}{\alpha} = \frac{1}{A_t}.$$

Under log-linear preferences, labor would not move in reaction to shocks, $N_t = \alpha^{-1}$. Since

$$N_t = \frac{X_{t-1}}{W_t},$$

the wage rate could not move either and, from

$$\frac{u_c(t)}{\alpha} = \frac{P_t}{W_t},$$

the price level would have to move with consumption. In this case the allocations are trivial. There are only technology shocks. The optimal allocations will respond by having labor constant, and consumption adjusts fully to the shocks.

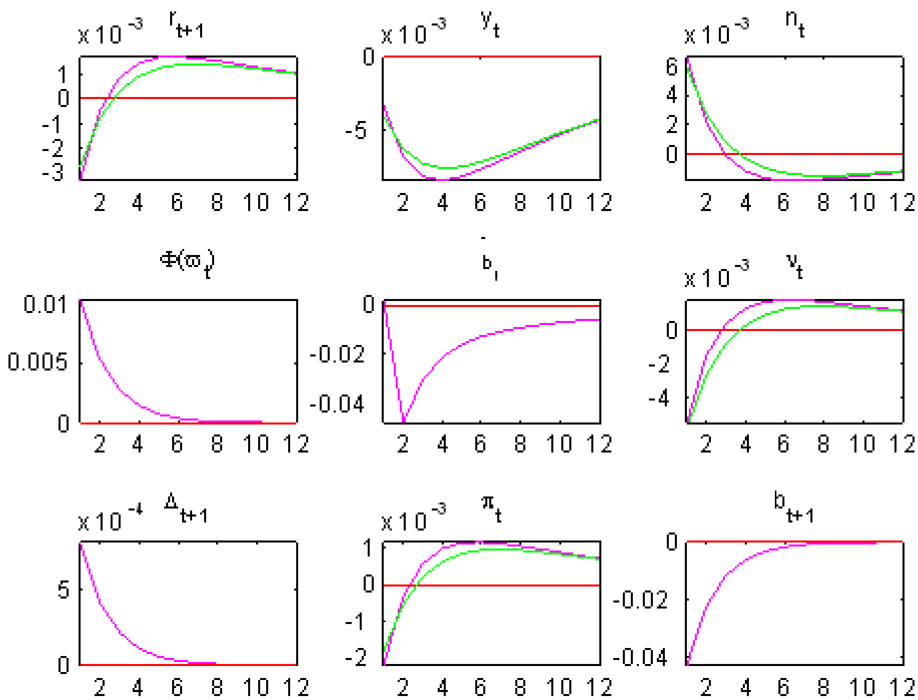
With $g_t \geq 0$, the optimal interest rate policy is away from the Friedman rule, $R_{t-1} > 0$. As with no government expenditures, the price level policy guarantees that (51) is satisfied. In this case, there are only technology and government expenditure shocks. The optimal allocations responds to the shocks by having labor constant, and consumption adjusts fully.

4.1.2 The role of asymmetric information and monitoring costs

Figure 11 compares the reaction to a technology shock under the Taylor rule in the general model of section 2 and in this benchmark model. The figure shows that the differences between the two cases are not overwhelming, but they go in an intuitively appealing direction.

The model with asymmetric information and monitoring costs tends to amplify business cycle fluctuations in response to shocks. Compared to a simpler model (the green lines in figure 11), the recession induced by a negative technology shock is deeper when accompanied by an increase in credit spreads and in the bankruptcy rate (the magenta lines). Employment fluctuations are also more pronounced and so is the volatility of inflation and of the policy interest rate. Similar results apply to a government spending shock.

Figure 11: Impulse responses to a negative techology shock under a Taylor rule



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9. The green lines report the impulse responses in this benchmark model when $g > 0$; the magenta lines indicate impulse responses in our full model when $g > 0$.

4.2 A model with exogenous real internal funds

The second benchmark model we consider builds upon the setup of De Fiore and Tristani (2008). We depart slightly from that setting by assuming that i) prices are flexible; ii) entrepreneurs are very patient, so that they never consume; iii) government expenditures are a share g_t of total output. There are two main differences relative to our general model. First, there is no predetermination. The asset market occurs at the beginning of the period, before the goods market. Thus, assets can be decided optimally at the beginning of each period t, after the occurrence of the shocks. Second, internal funds are exogenous and real.

In this benchmark economy, monetary policy can directly affect leverage, since external funds adjust in reaction to shocks while internal funds are fixed. Differently from the case of our general model, this implies that $\overline{\omega}$ must no longer be taken as given in steady state. Subsidising production is therefore no longer costless, since it also leads to an increase in leverage, in bankruptcy rates and in resource losses in monitoring activities. As a result, optimal policy would set the average nominal interest rate at a level slightly higher than zero.

A second and more obvious difference, compared to the general model, is monetary policy has effects only through the nominal interest rate, since the price level is irrelevant for choosing allocations.

When government consumption is a positive share of output, there is again a strong reason to deviate from the Friedman rule. Optimal policy is to set a positive average interest rate. While optimal policy in the general model keeps the policy rate constant in reaction to a technology shock, it is optimal here to raise nominal interest rates in booms and lower them in recessions.

Finally, under optimal monetary policy, bankruptcy rates are procyclical in response to technology shocks in this model, while they are acyclical (and countercyclical under a Taylor rule) in the general model. The reason is that, in this benchmark economy, firms can choose to borrow less and to cut production in response to a negative technology shock. Since internal funds are given, leverage falls. As a result, and at odds with the data, bankruptcy rates and credit spreads decrease during the recession.

4.2.1 Monetary policy instruments

Given the timing of assets and goods markets, the household budget constraint is given by

$$M_t + E_t Q_{t,t+1}\overline{A}_{t+1} + D_t \leq \overline{A}_t + R^d_{t-1}D_{t-1} + M_{t-1} - P_{t-1}c_{t-1} + W_{t-1}n_{t-1} - T_{t-1}.$$

We denote the amount of exogenous, real internal funds with τ. The equilibrium conditions are given by (3)-(5), (25), (41), (42),

$$c_t = (1 - g_t) A_t n_t \left[1 - \mu_t G(\overline{\omega}_t)\right]. \tag{52}$$

together with

$$\frac{u_m(t)}{u_c(t)} = R_t - 1, \tag{53}$$

$$v_t = \frac{R_t}{1 - \mu_t G(\overline{\omega}_t) - \frac{\mu_t f(\overline{\omega}_t)\overline{\omega}_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}} \tag{54}$$

$$x_t = \left\{ \frac{R_t}{R_t - v_t g(\overline{\omega}_t; \mu_t)} \right\} \tau \tag{55}$$

$$\overline{\omega}_t = \frac{R_t^l \left(1 - \frac{\tau}{x_t}\right)}{v_t}. \tag{56}$$

The equilibrium conditions can be summarized by

$$\frac{u_c(t)}{\alpha} = \frac{R_t}{\left[1 - \mu G(\overline{\omega}_t) - \frac{\mu_t f(\overline{\omega}_t)\overline{\omega}_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}\right] A_t}, \tag{57}$$

$$c_t = (1 - g_t)\left[1 - \mu_t G(\overline{\omega}_t)\right] A_t N_t \tag{58}$$

and

$$\frac{f(\overline{\omega}_t)}{g(\overline{\omega}_t; \mu_t)} = \frac{\frac{\tau}{\frac{N_t}{\frac{u_c(t)}{\alpha}} - \tau}}{1 - \mu \frac{\overline{\omega}_t \phi(\overline{\omega}_t)}{1 - \Phi(\overline{\omega}_t)}}. \tag{59}$$

These are the implementability conditions. They are three equations in three unknowns, given R_t. There is only one solution. The other equilibrium conditions determine other variables: (54) determines v_t;

$$x_t = \frac{A_t N_t}{v_t}$$

determines x_t; (56) determines R_t^l; (4) determines $Q_{t,t+1}^{-1}$; (5) determines π_{t+1}.

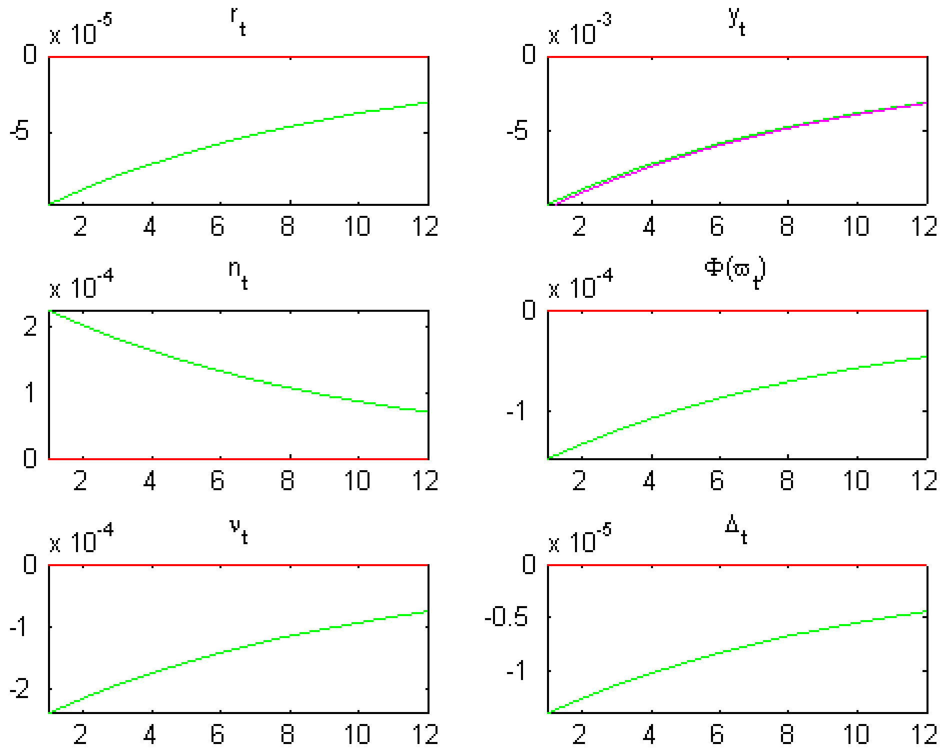
In this economy, policy has effects only through the nominal interest rate R_t.

4.2.2 Optimal monetary policy

The optimal allocation maximizes utility subject to (57)-(59). We only illustrate numerically the $g_t > 0$ case.

Under the optimal policy, the behavior of the nominal interest rate in reaction to shocks is very different from the one observed in the general model. Figure 12 compares the response of this economy to a technology shock under optimal policy to the responses of our general model, when government expenditure is a positive share of output. In this latter case, for instance, the interest rate remains at the Friedman rule following a technology shock, while monetary policy changes the price level in order to affect real wages and the real value of total funds. Instead, in this benchmark case, where there is no room for monetary policy to choose the price level, it is optimal to cut nominal interest rates in order to reduce the negative effect on output of the shock.

Figure 12: Impulse responses to a negative techology shock in the model with real internal funds



Note: Logarithmic deviations from the non-stochastic steady state. Correlation of the shock: 0.9. The green lines report the impulse responses in the model with exogenous real internal funds when $g > 0$; the violet lines indicate impulse responses in our full model when $g > 0$.

4.2.3 Cyclical properties

In the general model, the credit spread and the bankruptcy rate remain constant under the optimal policy in response to a technology shock. This is not the case in the economy where internal funds are real. As shown in figure 12, the credit spread and the bankruptcy rate fall after a negative technology shock.

One important difference between the two cases is that the bankruptcy rate and the spread is procyclical in the economy with real internal funds. The reason is that, following a negative technology shock, firms would choose to borrow less and to cut production. Since internal funds are given, leverage would fall. As a result, and at odds with the data, bankruptcy rates and credit spreads would decrease during a recession.

Under the optimal policy, our general model generates acyclical spreads and bankruptcies. Under a Taylor rule, that model is able to capture the countercyclicality of bankruptcies observed in the data by restricting the ability of households and firms to revise their portfolio decisions after the occurrence of the shocks.[4]

5 Conclusions

The model described in this paper represents an attempt to clarify the policy incentives created by the nominal denomination of firms' debt. Our analysis is based on a number of simplifying assumptions and does not aim to provide quantitative policy prescriptions. Nevertheless, we highlight a few results which may be of relevance also in more general frameworks.

The first result is that maintaning price stability at all times is not optimal when firms' financial positions are denominated in nominal terms and debt contracts are not state-contingent. After a negative technology shock, for example, an impact increase in the price level stabilises firms' leverage and allows for a more efficient economic response to the shock. This ability of monetary policy to influence the real value of firms' assets and liabilities derives from our assumption that, when shocks occur, financial contracts are predetermined. We demonstrate that a policy response through the price level can be so powerful in our model that, in response to technology shocks, there is no need for the central bank to adjust the nominal interest rate.

The second result is that the optimal response to an exogenous reduction in internal funds, which amounts to an increase in firms' leverage, is either to significantly reduce the nominal interest rate or, if the nominal rate is at its zero bound, to engineer a short period of controlled inflation. These policy responses have the advantages of mitigating the adverse consequences of the shock on bankruptcy rates and of allowing firms to de-leverage more quickly.

Finally, we show that a simple Taylor-type rule would produce significantly different economic outcomes from those prevailing if policy is set optimally. For example, under a Taylor rule bankruptcy rates would increase during recessions, as it appears to be the case in the empirical evidence. Bankruptcy rate would instead be acyclical under optimal policy.

[4] Under rule (40), π_t would not be determined in the benchmark model with real, exogenous internal funds. However, the procyclicality ofspreads and bankruptcies arises for instance also under a rule that reacts to π_{t-1} rather than to current inflation.

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