Capital Regulation and Tail Risk*

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Abstract

The paper studies risk mitigation associated with capital regulation, in a context where banks may choose tail risk assets. We show that this undermines the traditional result that higher capital reduces excess risk-taking driven by limited liability. Moreover, higher capital may have an unintended effect of enabling banks to take more tail risk without the fear of breaching the minimal capital ratio in non-tail risky project realizations. The results are consistent with stylized facts about pre-crisis bank behavior, and suggest implications for the optimal design of capital regulation.

Keywords: Banking, Capital regulation, Risk-taking, Tail risk, Systemic risk

JEL Classifications: G21, G28

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1 Introduction

Regulatory reform in the wake of the recent financial crisis has focused on an increase in capital cushions of financial intermediaries. Basel III rules have doubled the minimal capital ratio, and directed banks to hold excess capital as conservation and countercyclical buffers above the minimum (BIS, 2010). These arrangements complement traditional moral suasion and individual targets used by regulators to ensure adequate capital cushions.

There are two key arguments in favor of higher capital. The first is an ex post argument: capital can be seen as a buffer that absorbs losses and hence reduces the risk of insolvency. This risk absorption role also mitigates systemic risk factors, such as collective uncertainty over counterparty risk, which had a devastating propagation effect during the recent crisis. The second considers the ex ante effects of buffers: capital reduces limited liability-driven incentives of bank shareholders to take excessive risk, by increasing their "skin in the game" (potential loss in case of bank failure; Jensen and Meckling 1976, Holmstrom and Tirole 1997).

Yet some recent experience calls for caution. First, banks are increasingly exposed to tail risk, which causes losses only rarely, but when those materialize they often exceed any plausible initial capital. Such risks can result from a number of strategies. A first example are carry trades reliant on short term wholesale funding, which in 2007-2008 produced highly correlated distressed sales (Gorton, 2010). A second example is the reckless underwriting of contingent liabilities on systemic risk, callable at times of collective distress (Acharya and Richardson, 2009). Finally, the combination of higher profits in normal times and massive losses occasionally arises in undiversified industry exposures to inflated housing markets (Shin, 2009). A useful review of such strategies is provided in Acharya et al. (2009); IMF (2010) highlights the importance of recognizing tail risk in financial stability analysis. Since under tail risk banks do not internalize losses independently of the level of initial capital, the buffer and incentive effects of capital diminish. Higher capital may become a less effective way of controlling bank

risk.

Second, a number of major banks, particularly in the United States, appeared highly capitalized just a couple of years prior to the crisis. Yet these very intermediaries took excessive risks (often tail risk, or highly negatively skewed gambles). In fact, anecdotal evidence suggests that highly capitalized banks were looking for ways to put at risk their capital in order to produce returns for shareholders (Berger et al. 2008, Huang and Ratnovski 2009). Therefore, higher capital may create incentives for risk-taking instead of mitigating them.

This paper seeks to study these concerns by reviewing the effectiveness of capital regulation, and in particular of excess capital buffers (that is, above minimum ratios), in dealing with tail risk events. We reach two key results.

First, we show that the traditional buffer and incentives effects of capital become less powerful when banks have access to tail risk projects. The reason is that tail risk realizations can wipe out almost any level of capital. Left tails limit the effectiveness of capital as the absorbing buffer and restrict "skin in the game" because a part of the losses is never borne by shareholders. Hence, under tail risk, excess risk-shifting incentives of bank shareholders may exist almost independently of the level of initial or required capital.

Second, having established that under tail risk the benefits of higher capital are limited, we consider its possible unintended effects. We note that capital regulation also affects bank risk choices through the threat of capital adjustment costs when banks have to raise equity to comply with minimum capital ratios. (These costs are most commonly associated with equity dilution under asymmetric information on the value of illiquid bank assets, Myers and Majluf, 1984, or reduced managerial incentives for efficiency, Jensen, 1986).[1] Similar to "skin in the game", capital adjustment costs make banks averse to risk, and may discourage risky bank strategies. However, unlike "skin in the game", the incentive effects of capital adjustment costs fall with higher bank capital

[1] The fact that adjustment costs of bank equity raising are significant was highlighted, for example, in the Basel Committee-FSF (2010) assessment of the impact of the transition to stronger capital requirements.

because the probability of breaching the minimal capital ratio decreases.

Of course, if highly capitalized banks internalized all losses, they would have taken risk only if that was socially optimal (would have offered a higher NPV). Yet this result changes dramatically once we introduce tail risk. Then, even banks with high capital never internalize all losses, and may take excess risk. Moreover, the relationship between capital and risk can become non-monotonic. The reason is interesting. In the first place, tail risk leads to insolvency whatever the initial bank capital, so higher capital does not sufficiently discourage risk-taking for well capitalized banks through "skin in the game". At the same time, higher excess capital allows banks to take the riskier projects without breaching the minimal capital ratio (and incurring large capital adjustment costs) in the case of low (non tail) returns. So under tail risk, higher capital may create conditions where highly capitalized banks take more excess risk. Further, we show that the negative effect of extra capital on risk-taking becomes stronger when banks get access to projects with even higher tail risk.

To close the model, we derive the bank's choice of initial capital in the presence of tail risk, and the implications for optimal capital regulation. We show that a bank may *choose* to hold higher capital in order to create a cushion over the minimal capital requirement so as to be able to take tail risk without the fear of a corrective action in case of marginally negative project realizations. Then, capital regulation has to implement two bounds on the values of bank capital: a bound from below (a minimal capital ratio) to prevent ordinary risk-shifting and a bound from above (realistically, in the form of special attention devoted to banks with particularly high capital) in order to assure that they are not taking tail risk.

These results are interesting to consider in historic context. Most sources of tail risk that we described are related to recent financial innovations. In the past, tail risk in traditional loan-oriented depository banking was low (both project returns and withdrawals largely satisfied the law of large numbers), hence "skin in the game" effects dominated, and extra capital led to lower risk-taking. Yet now, when banks have access to tail risk projects, the buffer and "skin in the game" effects that are the cornerstone of

the traditional approach to capital regulation became weak, while effects where higher capital enables risk-taking became stronger. Therefore, due to financial innovation, the beneficial effects of higher capital were reduced, while the scope for undesirable effects increased.

The paper has policy implications relevant for the current debate on strengthening capital regulation. The simpler conclusion is that it is impossible to control all aspects of risk-taking using a single instrument. The problem of capital buffers is that they are effective as long as they can minimize not just the chance of default, but also the loss given default. Contractual innovation in finance has enabled intermediaries to manufacture risk profiles which allow them to take maximum advantage of limited liability even with high levels of capital. The key to contain gambles with skewed returns is to either prohibit extreme bets, or to increase their ex ante cost. Leading policy proposals now emerging are to charge prudential levies on strategies exposed to systemic risk (Acharya et al., 2010), such as extremely mismatched strategies (Perotti and Suarez, 2009, 2010), or derivative positions written on highly correlated risks.

A more intricate conclusion relates to implications for capital regulation. The results do not imply that less capital is better: this was not the case in recent years. However, they suggest the following. First, regulators should acknowledge that traditional capital regulation has limitations in dealing with tail risk. This is similar, for example, to an already-accepted understanding that it has limitations in dealing with correlation risk (Acharya, 2009). Second, banks with significant excess capital may be induced to take excess risk (in order to use or put at risk their capital), as amply demonstrated by the crisis experience. Hence, simply relying on higher and "excess" capital of banks as a means of crisis prevention may have ruinous effects if it produces a false sense of comfort. Finally, authorities should introduce complementary measures to target tail risk next to the policy on pro-cyclical and conservation buffers. In this context, enhanced supervision with a focus on capturing tail risk may be essential.

We see our paper as related to two key strands of the banking literature. First are the papers on the unintended effects of bank capital regulation. Early papers (Kahane 1977,

Kim and Santomero 1988, Koehn and Santomero 1980) took a portfolio optimization approach to banking and caution that higher capital requirements can lead to an increase in risk of the risky part of the bank's portfolio. Later studies focus on incentive effects. Boot and Greenbaum (1993) show that capital requirements can negatively affect asset quality due to a reduction in monitoring incentives. Blum (1999), Caminal and Matutes (2002), Flannery (1989) and Hellman et al. (2000) argue that higher capital can make banks take more risk as they attempt to compensate for the cost of capital. Our paper follows this literature, with a distinct and contemporary focus on tail risk.[2] On the empirical front, Angora et al. (2009) and Bichsel and Blum (2004) find a positive correlation between levels of capital and bank risk-taking.

The second strand are the recent papers on the regulatory implications of increased sophistication of financial intermediaries and the recent crisis. These papers generally argue that dealing with new risks (including systemic and tail risk) requires new regulatory tools (Acharya and Yorulmazer 2007, Acharya et al. 2010, Brunnermeier and Pedersen 2008, Huang and Ratnovski 2011, Perotti and Suarez 2009, 2010).

The structure of the paper is as follows. Section 2 outlines the theoretical model. Section 3 describes the traditional "skin in the game" effect of capital on risk-taking. Section 4 shows how higher capital can enable risk-taking when banks have access to tail risk projects. Section 5 endogenizes bank's choice of initial capital and provides insights for optimal capital regulation. Section 6 concludes. The proofs and extensions are in the Appendix.

2 The Model

The model has three main ingredients. First, the bank is managed by an owner-manager (hereafter, the banker) with limited liability, who can opportunistically engage in asset substitution. Second, the bank operates in a prudential framework based on a minimal

[2]Recent studies develop different measures for banks' tail risk. Acharya and Richardson (2009), Adrian and Brunnermeier (2009), and De Jonghe (2010) compute realized tail risk exposure over a certain period by using historical evidence of tail risk events, while Knaup and Wagner (2010) propose a forward-looking measure for bank tail risk.

capital ratio, with a capital adjustment cost if the bank fails to meet the ratio and has to raise extra equity. Finally, the bank has access to tail risk projects. Such a setup is a stylized representation of the key relevant features of the modern banking system. There are three dates (0, $\frac{1}{2}$, 1), no discounting, and everyone is risk-neutral.

The bank At date 0, the bank has capital C and deposits D. For convenience, we normalize $C + D = 1$. Deposits are fully insured at no cost to the bank; they carry a 0 interest rate and need to be repaid at date 1.[3]

The bank has access to two alternative investment projects. Both require an outlay of 1 at date 0 (all resources available to the bank), and produce return at date 1. The return of the *safe* project is certain: $R_S > 1$. The return of the *risky* project is probabilistic: high, $R_H > R_S$, with probability p; low, $0 < R_L < 1$, with probability $1 - p - \mu$; or extremely low, $R_0 = 0$, with probability μ. We consider the risky project with three outcomes in order to capture both the second (variance) and the third (skewness, or "left tail", driven by the R_0 realization) moments of the project's payoff.

In the spirit of the asset substitution literature, we assume that the net present value (NPV) of the safe project is higher than that of the risky project:

$$R_S > pR_H + (1 - p - \mu)R_L, \tag{1}$$

and yet the return on the safe asset, R_S, is not too high, so that the banker has incentives to choose the risky project at least for low levels of initial capital:

$$R_S - 1 < p(R_H - 1). \tag{2}$$

The left-hand side of (2) is the banker's expected payoff from investing in the safe project, and the right-hand side is the expected payoff from shifting to the risky project, conditional on bank having no initial capital, $C = 0$ and $D = 1$. We consequently

[3] The presence of not fully risk-based deposit insurance is an inherent feature of most contemporary banking systems, and one of main rationales for bank regulation (Dewatripont and Tirole, 1994).

study conditions under which the bank's leverage creates incentives to opportunistically choose the suboptimal, risky project.

For definiteness, the bank chooses the safe project when indifferent. The bank has no continuation value beyond date 1. (We discuss the impact of a positive continuation value in the Appendix; it reduces bank risk-taking but does not affect our qualitative results.)

The bank's project choice is unobservable and unverifiable. However, the return of the project chosen by the bank becomes observable and verifiable before final returns are realized, at date $\frac{1}{2}$.[4] This allows the regulator to impose corrective action on an undercapitalized bank.

Capital regulation Capital regulation is based on the minimal capital (leverage) ratio. We take this regulatory design as exogenous, since it is the key feature of Basel regulation. We define the bank's capital ratio, $c = (A - D)/A$, where A is the value of bank assets, D is the face value of deposits, and $A - D$ is its economic capital. At date 0, before the investment is undertaken, the capital ratio is $c = C/(C + D) = C$. At dates $\frac{1}{2}$ and 1 the capital ratio is $c_i = (R_i - D)/R_i$, with $i \in \{S, H, L, 0\}$ reflecting project choice and realization. The fact that the date $\frac{1}{2}$ capital position is defined in a forward-looking way is consistent with the practice of banks recognizing known future gains or losses.

At any point in time, the bank's capital ratio c must exceed a minimum $c_{\min}$, $c_{\min} > 0$. We assume that the minimal ratio is satisfied at date 0: $c > c_{\min}$. Consequently, the minimal ratio is also satisfied for realizations R_S (when the bank chooses the safe project) and R_H (when the bank chooses the risky project and is successful): $c_H > c_S > c > c_{\min}$, since $R_H > R_S > 1$. The minimal capital ratio is never satisfied for R_0 (in the extreme low outcome of the risky project), since the bank's capital is negative, $c_0 = -\infty < 0 < c_{\min}$. In a low realization of the risky project R_L the banks' capital

[4] The assumption that project choice is unobservable while project returns are, is a standard approach to modelling (Hellman et al. 2000, Rochet 2004).

sufficiency is ambiguous. As we will show below, depending on the bank's initial capital, it can be positive and sufficient, $c_L > c_{\min}$, or positive but insufficient, $0 < c_L < c_{\min}$, or negative, $c_L < 0$.

The regulator imposes corrective action at date $\frac{1}{2}$ if a bank fails to satisfy the minimal capital ratio. The banker is given two options. One is to surrender the bank to the regulator. Then, the bank's equity value is wiped out and the banker receives a zero payoff. Alternatively, the bank can attract additional capital to bring its capital ratio to the regulatory minimum, $c_{\min}$. We assume that attracting capital carries a cost for the existing bank shareholder. The cost reflects the dilution when equity issues are viewed by new investors as negative signals, or when there is a downward sloping demand for bank's shares. Both factors may be especially strong when the offering is performed with urgency. The presence of such costs is well established in the literature (Asquith and Mullins, 1986). In the main model, we treat the cost of recapitalization as fixed at T. In the Appendix we discuss a specification with concave cost (i.e., the cost of recapitalization falls with higher bank capital) and show that it does not affect our results. The banker chooses to abandon rather than recapitalize the bank when indifferent.

Timeline The model outcomes and the sequence of events are depicted in Figure 1.

<< Figure 1 here >>

Intuition Figure 2 provides a simple, illustrative intuition for the effects that we intend to capture in our formal analysis. Consider a bank that chooses between a safe and a risky project, and note how the bank's level of initial capital affects that choice. The classic Myers and Majluf (1984) channel focuses on the consequences of limited liability, which subsidizes risk-taking and tilts the bank's incentives towards a risky project. Then, higher capital discourages risk-taking by making shareholders internalize more of the bank's losses in the bad outcome ("skin in the game", the left panel). We introduce an additional effect associated with the minimal capital requirements. A bank with

positive but insufficient capital is subject to costly corrective action: shareholders have to recapitalize or abandon the bank. This penalizes risky projects. Then, a bank with higher capital may choose *more* risk, because higher capital reduces the probability of breaching the minimal capital ratio (the capital adjustment cost effect, the center panel). Of course, if a highly capitalized bank internalized all the costs of risk-taking, it would choose the risky project only if that was socially optimal (offered a higher NPV). To formalize the excess risk-taking of highly capitalized banks, we combine the two effects in a framework where a bank's risky project can both marginally breach the minimal capital ratio (triggering a capital adjustment cost) and result in an extremely negative outcome (tail risk, which falls under the limited liability constraint, the right panel). We find that, as a result of the combination of the two effects, the relationship between bank capital and risk-taking may become non-linear. In particular – the key result that we will emphasize – banks with higher capital may choose inefficiently high risk, when such risk has a significant tail component.

<< Figure 2 here >>

The return function with three outcomes is the simplest form that supports the insights of this model. The distinction between the marginal bad (low) and the extreme bad (tail) realization is necessary to simultaneously capture the effects of aversion to recapitalization and risk-shifting. Our results can also arise in more general distributions, including continuous, risky return distributions having similar features: a mass in the left tail and a possibility of marginally negative outcomes.

3 "Skin in the game" and tail risk

In this section we show that the traditional "skin in the game" incentive effects of higher capital on risk-taking become weaker when the bank has access to tail risk projects. This brings us to the first policy result, that capital regulation may have limited effectiveness in dealing with tail risk. Throughout the section, we abstract from the effects of capital

adjustment costs (we assume no minimum capital ratio), which we introduce in the next section. We solve the model backwards: first deriving the payoffs depending on bank project choice, and then the project choice itself. The solution is followed by comparative statics and a calibration exercise.

3.1 Payoff and project choice

Consider the bank with access to a tail risk project ($\mu \geq 0$), in a setup with no capital adjustment costs ($T = 0$). The banker's payoff from choosing the safe project is:

$$\Pi_S^{T=0} = R_S - D = R_S - (1 - c). \tag{3}$$

The banker's payoff from choosing the risky project is:

$$\Pi_R^{T=0} = p \cdot [R_H - (1 - c)] + (1 - p - \mu) \cdot \max\{R_L - (1 - c); 0\}, \tag{4}$$

where on the right hand side of (4) the first term is the expected payoff in R_H realization, and the second term is the expected payoff in R_L realization. The third realization, R_0, occurs with probability μ and carries a zero payoff.

The bank chooses the safe project over the risky project when:

$$\Pi_S^{T=0} \geq \Pi_R^{T=0},$$

which is equivalent to:

$$R_S - (1 - c) \geq p \cdot [R_H - (1 - c)] + (1 - p - \mu) \cdot \max\{[R_L - (1 - c)]; 0\}. \tag{5}$$

The following Proposition describes the bank's investment decision.

Proposition 1 *The bank's project choice depends on its initial capital c:*

(a) For

$$R_S < pR_H + (1-p)R_L, \tag{6}$$

the bank chooses the safe project if

$$c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$$

and the risky project otherwise;

(b) For

$$R_S \geq pR_H + (1-p)R_L, \tag{7}$$

the bank chooses the safe project if

$$c \geq 1 - \frac{R_S - pR_H}{1-p}$$

and the risky project otherwise.

Proof. See Appendix. ■

The intuition for Case (b) of the above proposition is that when R_S is high enough, the bank's risk-shifting incentives are so low that the bank will only take a risky project when it has negative capital under the R_L realization, allowing it to shift more of the downside to the creditors. Then, the bank gets the same zero payoff in the R_0 and R_L realizations and its project choice is not affected by the tail risk probability μ. Case (b) therefore represents the case of negligible tail risk. We therefore further focus on Case (a), which allows us to study the impact of tail risk on bank's project choice. We denote:

$$c^{T=0} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}, \tag{8}$$

with $c^{T=0}$ being the threshold for risk-shifting incentives under (6).

3.2 Comparative statics

We study how the threshold $c^{T=0}$, the initial capital necessary to prevent the bank from risk-shifting, is affected by the project's tail risk μ. To maintain comparability, we consider transformations of the risky project that increase μ but preserve its expected value, denoted by $E(R)$. There are various ways to alter model parameters to achieve that, but we highlight the two with the best interpretations, which we analyze in turn.

Case 1 Some of the sources of tail risk in the recent crisis were carry trades or undiversified exposures to housing markets. Such activities transform the distribution of the risky project towards extreme outcomes: within the confines of our models we can interpret that as a shift in the probability mass from R_L to R_0 and R_H. Formally, that implies an increase in μ and p, at the expense of $(1-p-\mu)$. To keep $E(R)$ constant, following to an increase in μ by $\Delta\mu$, p should increase by $\frac{R_L}{R_H-R_L}\Delta\mu$.

Using (8), we find that:

$$\frac{\partial c^{T=0}}{\partial \mu}\Big|_{E(R)=constant} = \frac{R_S - E(R)}{\mu^2} > 0. \tag{9}$$

So that the amount of capital necessary to prevent risk-shifting increases in tail risk.

Case 2 Another source of tail risk was the underwriting of contingent liabilities on market risk; in this case the bank obtains ex-ante premia (higher return) in all cases when the tail risk is not realized. Formally, this can be interpreted as a higher μ compensated by higher R_L and R_H, so that $E(R)$ is constant. In order to achieve this, following an increase in μ by $\Delta\mu$, both R_L and R_H should increase by $R_L\frac{\Delta\mu}{1-\mu-\Delta\mu}$.

Similarly to the previous case, using (8), we find that:

$$\frac{\partial c^{T=0}}{\partial \mu}\Big|_{E(R)=constant} = \frac{R_S - E(R)}{\mu^2} > 0.$$

Hence, again, the amount of capital necessary to prevent risk-shifting increases in

tail risk.

In both cases, observe that $c^{T=0}$ grows logarithmically in μ.[5] Therefore, capital becomes progressively a less effective incentive tool for controlling bank risk-taking as tail risk μ increases, with the effect most pronounced for low values of μ. As an implication, tail risk limits the effectiveness of capital regulation in dealing with bank risk-taking incentives.

3.3 Economic significance: a quantitative example

The comparative statics exercise verified that, as banks are able to take projects with higher tail risk μ, the buffer and incentive effects of capital diminish. Thus, in order to prevent banks from taking tail risk projects using minimal capital-based ("skin in the game") incentives only, banks will need progressively higher levels of initial capital $c^{T=0}$. This section attempts to highlight the economic significance of these results through a simple calibration exercise.

Consider the following calibration parameters: $R_S = 1.03$, $R_H = 1.14$, $R_L = 0.92$, $p = 50\%$, and $\mu = 1\%$. Then, the expected return on the safe project is 3%, the expected return on the risky project is 2.1%, and the minimal level of capital necessary to prevent risk-shifting is $c^{T=0} = 8\%$. We take these parameter values as representing the case of low (or usual) tail risk.

We ask how $c^{T=0}$ changes if tail risk μ increases, holding the expected value of the risky project fixed, as in the comparative statics exercise, by adjusting p to compensate for higher μ (Case 1 in Section 3.2). The result of the calibration exercise is shown on Figure 3. As μ increases, so does $c^{T=0}$, and the increase in $c^{T=0}$ is economically significant. Indeed, even an increase in μ from 1% to 1.1% increases the initial capital necessary to prevent risk-shifting from 8% to 16.4%. A doubling of μ to 2% percent increases the necessary initial capital to as much as 54%. Such values of initial capital

[5] We re-write $c^{T=0}(\mu) = 1 - \frac{const}{\mu}$, with $const = R_S - E(R)$. The degree of polynomial $c^{T=0}(\mu)$ is given by $\lim_{\mu \to \infty} \frac{\mu \cdot [c^{T=0}(\mu)]'}{c^{T=0}(\mu)}$. This equals 0, the degree of the logarithm function.

are likely implausible in practice.

<< Figure 3 here >>

The calibration confirmed that at least under some plausible circumstances, minimal capital requirements alone cannot prevent banks from taking excess tail risk, because the level of capital necessary for that would need to be implausibly high. In the next section, we study how the costly corrective action on undercapitalized banks may complement the capital requirements in dealing with bank risk-taking.

4 Tail risk and the unintended effects of higher capital

In the previous section, we showed that capital becomes a less effective tool for controlling bank risk-taking in the presence of tail risk. We will now introduce an additional feature – capital adjustment costs – to obtain a stronger result. In addition to being a less powerful tool, higher capital may have unintended effects of *enabling* banks' risk-taking. Specifically, we show that marginally capitalized banks do not take risk because they are averse to breaching the minimal capital ratio in mildly negative realizations of the risky project (R_L). Yet banks with higher capital can take more risk because their chance of breaching the ratio in such realizations is lower. Further, in comparative statics, we demonstrate that the unintended effects of higher capital are stronger when banks get access to projects with higher tail risk.

As before, we solve the model backwards: first we derive the payoffs depending on bank project choice, then the project choice itself. The solution is followed by comparative statics.

4.1 Payoffs and the recapitalization decision

The banker's payoff from choosing the safe project is:

$$\Pi_S = R_S - D = R_S - (1 - c).$$

Now consider the banker's payoff from the risky project. When the project returns R_H, the banker obtains $R_H - (1-c)$. When the project returns R_0, the banker obtains zero.

The case when the risky project returns R_L is more complex, because depending on the relative values of c and R_L, the bank's capital may be positive and sufficient, positive but insufficient, or negative. Consider these in turn.

Under R_L, the bank has positive and sufficient capital ($c_L \geq c_{\min}$) when:

$$\frac{R_L - (1-c)}{R_L} \geq c_{\min},$$

which gives:

$$c \geq c^{Sufficient} = 1 - (1 - c_{\min})R_L. \tag{10}$$

Then, the bank continues to date 1, repays depositors, and obtains $R_L - (1-c)$.

When $c < c^{Sufficient}$, R_L leaves the bank with insufficient capital, $c_L < c_{\min}$, so it has to be abandoned or recapitalized at cost T. The banker chooses to recapitalize the bank for:

$$R_L - (1-c) - T > 0, \tag{11}$$

where the left-hand side is the banker's return after repaying depositors net off the recapitalization cost, and the right hand side is the zero return in case the bank is abandoned. Expression (11) can be re-written as:

$$c > c^{Recapitalize} = 1 + T - R_L. \tag{12}$$

We focus our analysis on the case when $c^{Recapitalize} < c^{Sufficient}$, corresponding to:

$$T < c_{\min} R_L, \tag{13}$$

so that there exist values of c: $c^{Recapitalize} < c < c^{Sufficient}$, where the banker chooses to recapitalize the bank in the R_L realization, instead of abandoning it. When T is larger

than $c_{\min} R_L$, the banker always abandons a bank with insufficient capital. Note that both thresholds $c^{Recapitalize}$ and $c^{Sufficient}$ are in the $(0, 1)$ interval.

Figure 4 illustrates the bank's recapitalization decision.

<< Figure 4 here >>

Overall, the banker's payoff in the R_L realization of the risky project is:

$$\Pi_L = \begin{cases} R_L - (1-c), \text{if } c \geq c^{Sufficient} \\ R_L - (1-c) - T, \text{ if } c^{Recapitalize} < c < c^{Sufficient} \\ 0, \text{ if } c \leq c^{Recapitalize} \end{cases}, \tag{14}$$

and the overall payoff to the risky project is:

$$\Pi_R = p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot \Pi_L. \tag{15}$$

4.2 Project choice

We now consider the bank's project choice at date 0, depending on its initial capital c. The bank chooses the safe project over the risky one for:

$$\Pi_S \geq \Pi_R,$$

which is equivalent to:

$$R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot \Pi_L. \tag{16}$$

To describe the results we introduce two thresholds:

$$W = pR_H + (1-p)R_L - \mu c_{\min} R_L, \tag{17}$$

and

$$Z = pR_H + (1-p)(R_L - T) + \mu(T - c_{\min}R_L). \tag{18}$$

W is a threshold point for the presence of risk-shifting in bank with high capital. For $R_S < W$ there exist values of initial capital such that even a well-capitalized bank with $c \geq c^{Sufficient}$ may still engage in risk-shifting. Z is a threshold point for the presence of the capital adjustment cost effect. For $R_S \geq Z$ there exist values of initial capital such that a less capitalized bank ($c^{Recapitalize} < c < c^{Sufficient}$) may choose a safe project to prevent recapitalization costs upon the R_L realization of the risky project. The derivation of the thresholds is in the Appendix; the Appendix also verifies that $Z < W$.

Then, the risk-shifting and capital adjustment effect of bank project choice interact with each other as follows:

Proposition 2 *The bank's project choice is characterized by thresholds c^* and c^{**}:*

(a) For $Z \leq R_S < W$, there exist $c^* < c^{Sufficient}$, and $c^{**} > c^{Sufficient}$, such that

- *For $c < c^*$ the bank chooses the risky project and may abandon or recapitalize it upon the R_L realization;*
- *For $c^* \leq c < c^{Sufficient}$ the bank chooses the safe project to avoid abandonment or recapitalization upon the R_L realization; the choice of the safe project here represents the capital adjustment cost effect;*
- *For $c^{Sufficient} \leq c < c^{**}$ the bank chooses the risky project because its capital is high enough to avoid breaching the minimal capital ratio in the R_L realization; the choice of the risky project here represents risk-shifting enabled by higher capital;*
- *For $c \geq c^{**}$ the bank chooses the safe project because its capital is high enough to prevent risk-shifting;*

(b) For $R_S < Z$, there exists $c^{**} > c^{Sufficient}$ such that for $c < c^{**}$ the bank chooses

the risky project and for $c \geq c^{**}$ the safe project; there is only a risk-shifting effect: a bank with $c < c^{Sufficient}$ never chooses a safe project to avoid recapitalization cost;

(c) for $R_S \geq W$, there exists $c^* < c^{Sufficient}$ such that for initial capital $c < c^*$ the bank chooses a risky project, and for $c \geq c^*$ the safe project; there is only a capital adjustment cost effect: a bank with $c > c^{Sufficient}$ never engages in risk-shifting.

Proof. See Appendix. ■

The thresholds:

$$c^* = 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}, \tag{19}$$

and:

$$c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}, \tag{20}$$

are also derived in Appendix.

Case (a) of Proposition 2 contains the main result of our paper: that the relationship between bank capital and risk-taking can be non-monotonic in the presence of tail risk and capital adjustment cost. When capital is very low, $c < c^*$, the banker faces strong risk-shifting incentives and a low cost of abandoning the bank, hence chooses high risk. For intermediate initial capital, $c^* \leq c < c^{Sufficient}$, the banker's equity value is higher, and the banker chooses a safe project to avoid abandoning or recapitalizing the bank in the R_L realization. The choice of the safe project is driven by capital adjustment cost - a novel effect highlighted in this paper. Yet as soon as the bank has initial capital high enough to satisfy the minimal ratio in the R_L realization, for $c^{Sufficient} \leq c < c^{**}$, the capital adjustment cost stops being binding and the banker again switches to the risky project, driven by the risk-shifting effect. Finally, for very high levels of capital, $c \geq c^{**}$, the banker has so much skin in the game that risk-shifting incentives are not binding. This is the traditional effect of capital regulation; recall that under tail risk c^{**} may be prohibitively very high. The bank's project choice is depicted in Figure 5.

<< Figure 5 here >>

4.3 Comparative statics

In this section we repeat the comparative statics exercise of Section 3.2, in the presence of capital adjustment costs – with respect to the Case (a) of Proposition 2. We show that when tail risk increases (the risky project has a heavier left tail), highly capitalized banks get stronger incentives to take excess risk. We use the two transformations of the risky project highlighted in Section 3.2.

Case 1 When a higher μ is compensated by a higher p, keeping $E(R)$ constant, that affects both thresholds c^* and c^{**}. To focus on banker's incentives to take excessive risk, we consider the interval $[c^{Sufficient}, c^{**})$, corresponding to levels of initial bank capital for which the bank undertakes the risky project. Note that $c^{Sufficient}$ is determined only by $c_{\min}$ and R_L (see (10)), and hence is unaffected by a change in the probability distribution of the risky project. The critical threshold for the discussion is therefore c^{**}. From (8), $c^{**} = c^{T=0}$. The impact of the change in probability distribution of the risky project on c^{**} is the same as in (9):

$$\frac{\partial c^{**}}{\partial \mu}\Big|_{E(R)=constant} = \frac{R_S - E(R)}{\mu^2} > 0.$$

This means that when tail risk increases, the interval $[c^{Sufficient}, c^{**})$ on which a well-capitalized bank chooses the risky project expands. Interestingly, the interval expands because banks with *higher* capital start taking more risk. This highlights the relationship between tail risk and the unintended effects of higher bank capital. The intuition is that when investment returns become more polarized, they enable well-capitalized banks to earn higher profits in good time, while at the same time reducing the expected cost of recapitalization since the intermediate low return R_L is less frequent. Unintended effects of bank capital affect specifically the well-capitalized banks. Figure 6a illustrates the case.

<< Figure 6a here >>

Case 2 When a higher μ is compensated by higher R_L and R_H, this change in the return profile of the risky asset affects thresholds c^*, c^{**}, and $c^{Sufficient}$. To focus on the incentives of well-capitalized banks to take excessive risk, we again consider the interval $[c^{Sufficient}, c^{**})$. Note that $c^{Sufficient}$ is decreasing in R_L (see (10)) and hence in μ. At the same time, from (9), c^{**} is increasing in μ. Hence here the interval $[c^{Sufficient}, c^{**})$ widens even more in μ than in case 1, and both more and less capitalized banks start choosing the risky projects. Figure 6b illustrates the case.

<< Figure 6b here >>

5 Bank capital choices and optimal capital regulation

We have previously identified how bank incentives to take tail risk depend on its initial level of capital. We can now study how the ability of banks to take tail risk affects bank capital choices and what are the implications for the optimal capital regulation. To endogenize bank capital choice, we need to introduce the bank's cost of holding capital. We assume that:

(A) The bank's private cost of holding capital is $c\gamma$, $\gamma > 1$. This cost represents the alternative cost of using banker's own funds elsewhere, see Hellmann et al. (2000). The assumption assures that, all else equal, the bank will want to hold as little capital as possible.

(B) The cost of bank capital becomes prohibitive for high values of capital, making $c_{\min} = c^{**}$ impossible to implement. Recall that, under tail risk, c^{**} (the level of capital necessary to prevent bank risk-taking solely through the "skin-in-the-game" channel) can become very high (Section 3.3).

We can now formulate the result on the bank's endogenous choice of initial capital. We focus on case Case (a) of Proposition 2 where the relationship between bank capital and risk-taking is non-monotonic in bank capital.

Lemma 1 *Setting $c_{\min} < c^*$ is never optimal (because the bank will always choose $c_{\min}$ and a risky project); therefore $c_{\min} \geq c^*$.*

Proof. *It follows from Proposition 2 and Assumption A.* ■

We can now formulate the result on the bank's capital choice:

Proposition 3 *For $c_{\min} \geq c^*$, the bank's private capital choice is either the minimal capital $c_{\min}$ or the level of capital sufficient to avoid recapitalization costs in the R_L realization of the risky project $c^{Sufficient}$. There exists $\gamma^* > 1$:*

$$\gamma^* = 1 + \mu \frac{R_L}{1 - R_L} - \frac{R_S - pR_H - (1 - p - \mu)R_L}{(1 - c_{\min})(1 - R_L)} \tag{21}$$

such that the bank chooses $c^{Sufficient}$ for $\gamma < \gamma^*$ and $c_{\min}$ otherwise.

Proof. See Appendix. ■

The intuition is as follows. Because capital is costly, the bank will choose the lowest capital possible on each of the intervals $[c_{\min}, c^{Sufficient})$ and $[c^{Sufficient}, c^{**})$. Capital below $c_{\min}$ is ruled out by capital regulation under Lemma 1 and capital at or above c^{**} is ruled out by assumption B above. To establish the bank's choice for capital we therefore have to compare profits at two points: $c_{\min}$ and $c^{Sufficient}$. The banker will prefer $c^{Sufficient}$ if the cost of maintaining extra capital, proportional to γ, is not too high.

It is important to understand the economic logic behind the choice between $c_{\min}$ and $c^{Sufficient}$. The point $c_{\min}$ gives lower capital, so the bank saves on its cost. But the bank has to take the safe project (socially optimal but less beneficial to shareholders) to avoid the capital adjustment cost. Should a bank switch to $c^{Sufficient}$, it would voluntarily choose to incur the cost of holding higher capital because that would *enable the bank to take higher risk*. Indeed, the bank will choose the risky project because it is not anymore constrained by the threat of capital adjustment cost. We have therefore established that, in the presence of tail risk, banks may choose to accumulate capital *in order* to be able to take tail risk.

With this is mind, we can now turn to optimal capital regulation. Recall that the level of capital that allows to rule out bank risk-taking through "skin in the game" only, c^{**}, is not plausible. Therefore, the only objective of regulation that is feasible in our setup is to assure that the bank's capital is in the $[c^*, c^{Sufficient})$ interval where a bank takes the safe project due to its aversion to capital adjustment costs. Both below and above this interval, the bank will undertake the risky project. Such a regulatory outcome can be implemented with two instruments. The first is a standard minimal capital requirement, set at $c_{\min} = c^*$ (there is no reason to set $c_{\min} > c^*$ since capital is costly). The second is an effective constraint on the bank's excess capital over the minimal capital requirement. The constraint can be interpreted, in practice, not as a limit, but as special attention that regulators should give to banks with excess capital, anticipating that such banks are more likely than others to take tail risk. We summarize with the conjecture:

Conjecture 1 *In the presence of tail risk, optimal capital regulation combines a minimal capital requirement $c_{\min} = c^*$ and effective constraints on banks with excess capital (above $c^{Sufficient}$) to prevent them from taking tail risk.*

6 Conclusion

This paper examined the relationship between bank capital and risk-taking when banks have access to tail risk projects. We showed that traditional capital regulation becomes less effective in controlling bank risk because banks never internalize the negative realizations of tail risk projects. Moreover, we have suggested novel channels for unintended effects of higher capital: it enables banks to take higher tail risk without the fear of breaching the minimal capital requirement in mildly bad (i.e., non-tail) project realizations. The results are consistent with stylized facts about pre-crisis bank behavior, and have implications for the design of bank regulation.

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A Proofs

A.1 Proof of Proposition 1

Consider first the case when $c > 1 - R_L$. The relevant incentive compatibility condition derived from (5) becomes $R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot [R_L - (1-c)]$, which can be rewritten as $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. We denote $c^{T=0} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold exists and is strictly larger than $1 - R_L$ for values of R_S satisfying (6). Hence, for $R_S < pR_H + (1-p)R_L$, $\exists\ c^{T=0} \in (1 - R_L, 1]$ such that $\forall\ c \in (1 - R_L, c^{T=0})$ the risky project is selected and $\forall\ c \in [c^{T=0}, 1]$ the safe project is chosen. Otherwise, if (6) is not fulfilled, the bank selects the safe project $\forall\ c \in (1 - R_L, 1]$.

Consider now the case $c \leq 1 - R_L$. The relevant incentive compatibility condition derived from (5) is $R_S - (1-c) \geq p \cdot [R_H - (1-c)]$. The condition is equivalent with $c \geq 1 - \frac{R_S - pR_H}{1-p}$. We denote $c^{T=0}_{Traditional} = 1 - \frac{R_S - pR_H}{1-p}$. The threshold exists and is below or equal to $1 - R_L$ for values of R_S satisfying (7). Thus, for $R_S \geq pR_H + (1-p)R_L$, $\exists$ $c^{T=0}_{Traditional} \in [0, 1 - R_L]$ such that $\forall\ c \in [0, c^{T=0}_{Traditional})$ the risky project is selected and $\forall\ c \in [c^{T=0}_{Traditional}, 1 - R_L]$ the safe project is chosen. Otherwise, if (7) is not fulfilled, the bank selects the risky project $\forall\ c \in [0, 1 - R_L]$.

To sum up, when (6) is fulfilled, $\exists\ c^{T=0} \in (1 - R_L, 1]$ such that $\forall\ c \in [0, c^{T=0})$ the risky project is selected and $\forall\ c \in [c^{T=0}, 1]$ the safe project is chosen. Likewise, when (6) is not fulfilled, $\exists\ c^{T=0}_{Traditional} \in [0, 1 - R_L]$ such that $\forall\ c \in [0, c^{T=0}_{Traditional})$ the risky project is selected and $\forall\ c \in [c^{T=0}_{Traditional}, 1]$ the safe project is chosen.

A.2 Proof of Proposition 2

We consider two scenarios in turn. We start by analyzing a scenario in which the cost of recapitalization is such that $\frac{\mu}{1-p} c_{\min} R_L < T < c_{\min} R_L$. Subsequently we show that our results are similar for $T \leq \frac{\mu}{1-p} c_{\min} R_L$. Both cases follow from assumption (13) of low adjustment cost.

A.2.1 $\frac{\mu}{1-p} c_{\min} R_L < T < c_{\min} R_L$

We study bank's behavior for three levels of initial capital: low (i.e., $c \leq c^{Recapitalize}$), intermediate (i.e., $c^{Recapitalize} < c < c^{Sufficient}$) and high (i.e., $c \geq c^{Sufficient}$).

Consider first the case when $c \in [0, c^{Recapitalize}]$. The banker never finds optimal to recapitalize for low realization of the risky project. The relevant incentive compatibility condition derived from (16) is $R_S - (1-c) \geq p \cdot [R_H - (1-c)]$, where the left-hand side is the return on investing in the safe project and the right-hand side is the expected return on selecting the risky project. The condition can be rewritten as $c \geq 1 - \frac{R_S - pR_H}{1-p}$. We denote $c_1^* = 1 - \frac{R_S - pR_H}{1-p}$. The threshold c_1^* exists if and only if the next two constraints are jointly satisfied:

$$R_S < 1 - p + pR_H, \tag{T1a'}$$

$$R_S > pR_H + (1-p)(R_L - T). \tag{T1a}$$

The former condition guarantees a positive c_1^*, while the latter forces the threshold to be lower than $c^{Recapitalize}$, the upper limit for the interval we analyze. If (T1a') is not fulfilled, then $c_1^* < 0$ and $\forall\ c \in [0, c^{Recapitalize}]$, the bank prefers the safe project. If (T1a) is not fulfilled, then $c_1^* > c^{Recapitalize}$ and $\forall\ c \in [0, c^{Recapitalize}]$ the bank invests risky. When both constraints are simultaneously satisfied, $\exists\ c_1^* \in [0, c^{Recapitalize}]$ such that $\forall\ c \in [0, c_1^*)$ the risky project is selected and $\forall\ c \in [c_1^*, c^{Recapitalize}]$ the safe project is chosen. Assumption (2) implies that (T1a') is always fulfilled.

Consider now the case when $c \in (c^{Recapitalize}, c^{Sufficient})$. The banker finds optimal to recapitalize for low realization R_L. The relevant incentive compatibility condition is $R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot [R_L - (1-c) - T]$, with the certain return on choosing the safe project depicted on the left-hand side, and expected return on investing in the risky project depicted on the right-hand side. Rearranging terms the condition can be rewritten as $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}$. We denote $c_2^* = 1 - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}$. Similarly with the previous case, the threshold c_2^* exists if

and only if it is simultaneously higher and lower than the lower and the higher boundary of the analyzed interval, respectively. The conditions are as follows:

$$R_S < pR_H + (1-p)(R_L - T), \tag{T2a}$$

$$R_S > pR_H + (1-p)(R_L - T) + \mu(T - c_{\min} R_L). \tag{T2b}$$

Condition (T2a) is the opposite of (T1a). Thus, a satisfied condition (T1a) implies that (T2a) is not fulfilled. Condition (T2a) not satisfied implies that $c_2^* < c^{Recapitalize}$ and $\forall\ c \in (c^{Recapitalize}, c^{Sufficient})$, the bank prefers the safe project. If the second condition is not fulfilled, then $c_2^* > c^{Sufficient}$ and $\forall\ c \in (c^{Recapitalize}, c^{Sufficient})$ the bank invests risky. When both constraints are simultaneously satisfied, $\exists\ c_2^* \in (c^{Recapitalize}, c^{Sufficient})$ such that $\forall\ c \in (c^{Recapitalize}, c_2^*)$ the risky project is selected. The safe project is preferred $\forall\ c \in [c_2^*, c^{Sufficient})$.

Consider now the final interval, when $c \in [c^{Sufficient}, 1]$. For low realization R_L the bank always complies with the regulatory requirements. No additional capital is needed. The relevant incentive compatibility condition is $R_S - (1-c) \geq p \cdot [R_H - (1-c)] + (1-p-\mu) \cdot [R_L - (1-c)]$. Rearranging terms the condition becomes $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. We denote $c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold c^{**} exists if and only if $c^{**} > c^{Sufficient}$ and $c^{**} < 1$. The later is always fulfilled following from the assumption (1) of higher NPV for the safe project. The former condition is depicted in (T3a) below. When (T3a) is not satisfied, the bank prefers the safe project for any level of initial capital larger than $c^{Sufficient}$. Otherwise, $\forall\ c \in [c^{Sufficient}, c^{**})$ the risky project is selected, while the safe project is preferred $\forall\ c \in [c^{**}, 1]$.

$$R_S < pR_H + (1-p)R_L - \mu c_{\min} R_L. \tag{T3a}$$

Next we discuss the process of project selection. Recall that $Z = pR_H + (1-p)(R_L - T) + \mu(T - c_{\min} R_L)$ and $W = pR_H + (1-p)R_L - \mu c_{\min} R_L$, from (18) and (17), respectively.

We also denote $B = pR_H + (1-p)(R_L - T)$. Under assumption (13), $Z < B < W$. We distinguish among four possible scenarios.

Scenario S1: $R_S < Z$. As a consequence, condition (T2b) is not satisfied and $\forall$ $c \in (c^{Recapitalize}, c^{Sufficient})$ the bank selects the risky project. $R_S < Z$ also implies that $R_S < B$ and $R_S < W$. Condition (T1a) is not satisfied but (T3a) is. As a result, the bank invests risky $\forall$ $c \in [0, c^{Recapitalize}] \cup [c^{Sufficient}, c^{**})$, and the bank invests safe $\forall$ $c \in [c^{**}, 1]$.

Scenario S2: $Z \leq R_S \leq B$. The right-hand side implies that condition (T1a) is not satisfied. For initial capital c lower than $c^{Recapitalize}$ the bank prefers the risky project. The left hand side implies that condition (T2b) is fulfilled. Also condition (T2a) is satisfied being the opposite of (T1a). Hence, we can conclude that $\exists$ $c^* \in (c^{Recapitalize}, c^{Sufficient})$ with $c^* = c_2^*$, such that $\forall$ $c \in (c^{Recapitalize}, c^*)$ the risky project is selected, while the safe project is preferred $\forall$ $c \in [c^*, c^{Sufficient})$. Condition (T3a) is also satisfied. Similarly with the previous scenario, the bank invests risky $\forall$ $c \in [c^{Sufficient}, c^{**})$, and safe $\forall$ $c \in [c^{**}, 1]$.

Scenario S3: $B < R_S < W$. The left hand-side implies that condition (T1a) is satisfied. We can argue that $\exists$ $c^* \in (0, c^{Recapitalize})$ with $c^* = c_1^*$, such that $\forall$ $c \in [0, c^*)$ the risky project is selected, while the safe project is preferred $\forall$ $c \in [c^*, c^{Recapitalize}]$. Condition (T1a) implies that (T2a) is not satisfied. Thus, $\forall$ $c \in (c^{Recapitalize}, c^{Sufficient})$ the safe project will be selected. The bank investment decision is identical with the one from previous scenarios when the level of capital is high enough (i.e., c larger than $c^{Sufficient}$).

Scenario S4: $R_S \geq W$. Neither condition (T3a) nor condition (T2a) are satisfied anymore. The bank selects the safe project $\forall$ $c \in (c^{Recapitalize}, 1]$. However, condition (T1a) is fulfilled. Hence, $\exists$ $c^* \in [0, c^{Recapitalize}]$ with $c^* = c_1^*$, such that $\forall$ $c \in [0, c^*)$ the risky project is selected, while the safe project is preferred $\forall$ $c \in [c^*, c^{Recapitalize}]$.

The values for thresholds c^* and c^{**} for Case (a) of Proposition 2, are derived under Scenario S2 above, for $Z \leq R_S \leq B$.

A.2.2 $T \leq \frac{\mu}{1-p} c_{\min} R_L$

We consider now the scenario under which the cost of recapitalization is very low. Lowering T has no quantitative impact on $c^{Sufficient}$ and $c^{Recapitalize}$, the thresholds in initial capital which trigger bank's decision between raising additional capital or letting the regulator to overtake the bank. Their relative position is unchanged: $c^{Sufficient}$ is larger than $c^{Recapitalize}$ following from easily verifiable identity $\frac{\mu}{1-p} c_{\min} R_L < c_{\min} R_L$ combined with our restriction on T. However, the process of project selection under assumption (13) is marginally affected. In this scenario $Z < W < B$, as a consequence of lower T. As discussed before, we distinguish among four possible scenarios (S1') $R_S < Z$, (S2') $Z \leq R_S < W$, (S3') $W \leq R_S < B$ and (S4') $R_S > B$. Discussions for scenarios S1', S2' and S4' are identical with our previous discussion for scenarios S1, S2, and S4. We discuss scenario S3' next. $W \leq R_S$ implies that condition (T3a) is not satisfied. Hence, the bank prefers the safe project for any level of initial capital larger than $c^{Sufficient}$. $R_S < B$ implies that condition (T1a) is not satisfied. For initial capital c lower than $c^{Recapitalize}$ the bank prefers the risky project. However, condition (T2a) is satisfied being the opposite of (T1a), and also condition (T2b) is implied by the fact that $W > Z$. Hence, we can conclude that $\exists\ c^* \in (c^{Recapitalize}, c^{Sufficient})$ with $c^* = c_2^*$, such that $\forall\ c \in (c^{Recapitalize}, c^*)$ the risky project is selected, while the safe project is preferred $\forall\ c \in [c^*, c^{Sufficient})$.

A.3 Analysis of the robustness of results of Proposition 2

We offer here a discussion for the results of Proposition 2. We analyze bank's project choice for the case of high cost of recapitalization: $T > c_{\min} R_L$; we show that our results are robust to this specification. Recall that under assumption (13), there exist values of c such that $c^{Recapitalize} < c < c^{Sufficient}$ where the banker chooses to recapitalize the bank following the R_L realization instead of abandoning it. We show next that when T is larger than $c_{\min} R_L$, the banker always abandons a bank with insufficient capital. Although the main results from Proposition 2 are not qualitatively affected, higher

recapitalization cost has a quantitative impact on our results. Therefore, we start by deriving the new conditions which drive these results. It is optimal for the bank to raise additional capital (if this was demanded by the regulator) when conditions $c_L < c_{\min}$ and (11) are simultaneously satisfied. The former condition implies that $c < 1-R_L(1-c_{\min})$, while from the latter $c > 1 + T - R_L$. Under our modified assumption of high cost of recapitalization T, these conditions can not be satisfied simultaneously. For any levels of initial capital c below $1-R_L(1-c_{\min})$, the bank receives a request for adding extra capital but she never finds optimal to do so because such an action will not generate positive payoffs. As a result, the bank is closed and the shareholder expropriated. Conversely, when the level of initial capital is above $1 - R_L(1 - c_{\min})$ the banking authority doesn't take any corrective action against the bank since returns R_L are above the critical level $R_{\min}$. We denote:

$$c_{NEW}^{Recapitalize} = 1 - R_L(1 - c_{\min}), \tag{22}$$

where $c_{NEW}^{Recapitalize} \in (0, 1)$. Next, we explore the bank's project choice for levels of initial capital below and above this critical threshold.

Consider first the case when $c \in (0, c_{NEW}^{Recapitalize})$.The bank never recapitalizes for the low realization of the risky project. The bank would have incentive to select the safe project when $R_S - (1 - c) \geq p[R_H - (1 - c)]$, which implies that initial capital c to be larger than $1 - \frac{R_S - pR_H}{1-p}$. We previously denoted $c_1^* = 1 - \frac{R_S - pR_H}{1-p}$. This threshold exists if and only if (T1a') and the following condition are jointly satisfied:

$$R_S > pR_H + (1 - p)R_L(1 - c_{\min}). \tag{T1a NEW}$$

The second condition guarantees that c_1^* is lower than $c_{NEW}^{Recapitalize}$, the upper boundary for the interval we analyze. For large returns on the safe project (i.e., condition (T1a') is not fulfilled), $\forall\ c \in (0, c_{NEW}^{Recapitalize})$, the bank prefers the safe project. If (T1a NEW) is not fulfilled, then $\forall\ c \in (0, c_{NEW}^{Recapitalize})$ the bank invests risky. Otherwise, when

both constraints are simultaneously satisfied, $\forall\ c \in (0, c_1^*)$ the risky project is selected and $\forall\ c \in (c_1^*, c_{NEW}^{Recapitalize})$ the safe project is chosen. Our assumption (2) implies that (T1a') is always fulfilled.

Consider now the second case when $c \in (c_{NEW}^{Recapitalize}, 1)$. The bank always complies with the regulatory requirements when R_L is obtained due to high initial capital. No additional capital is needed. The bank would have incentive to select the safe project when $R_S - (1-c) \geq p[R_H - (1-c)] + (1-p-\mu)[R_L - (1-c)]$, which implies $c \geq 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. We previously denoted $c^{**} = 1 - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}$. The threshold c^{**} exists if and only if condition (T3a) is satisfied. The safe project is preferred for any level of initial capital larger than $c_{NEW}^{Recapitalize}$ whenever (T3a) is not satisfied. Otherwise, $\forall\ c \in (c_{NEW}^{Recapitalize}, c^{**})$ the risky project is selected, while the safe project is preferred $\forall$ $c \in (c^{**}, 1)$.

Recall that $W = pR_H + (1-p)R_L - \mu c_{\min} R_L$. We also denote $Q = pR_H + (1-p)R_L(1-c_{\min})$. It is easy to show that $Q < W$ due to the identity $1 - p - \mu > 0$. We distinguish among only three possible scenarios.

Scenario S1": $R_S \leq Q$. As a consequence, condition (T1a NEW) is not satisfied and $\forall\ c \in (0, c_{NEW}^{Recapitalize})$ the bank selects the risky project. $R_S < Q$ implies that $R_S < W$.Condition (T3a) is satisfied. As a result, the bank invests risky $\forall\ c \in (c_{NEW}^{Recapitalize}, c^{**})$, while she prefers the safe project $\forall\ c \in (c^{**}, 1)$.

Scenario S2": $Q < R_S < W$. The left hand-side implies that condition (T1a NEW) is satisfied. This implies that $\exists\ c^* \in (0, c_{NEW}^{Recapitalize})$ with $c^* = c_1^*$, such that $\forall\ c \in (0, c^*)$ the risky project is selected, while the safe project is preferred $\forall\ c \in (c^*, c_{NEW}^{Recapitalize})$. Similarly with the previous scenario, the bank invests risky $\forall\ c \in (c_{NEW}^{Recapitalize}, c^{**})$, and safe $\forall\ c \in (c^{**}, 1)$. This result is implied by R_S being lower than W.

Scenario S3": $R_S \geq W$. Condition (T1a NEW) is satisfied while condition (T3a) is not. Hence, the bank selects the risky projects $\forall\ c \in (0, c^*)$, with $c^* = c_1^*$, and she selects the safe project $\forall\ c \in (c^*, 1)$.

To conclude, we can argue that the qualitative results of Proposition 2 are valid

under the relaxed assumption. Nevertheless, condition (T2b) has to be replaced by the relevant condition (T1a NEW).

A.4 Bank's choice when the return on safe project is large

Let us consider here that the return on the safe asset is large, that is $R_S > 1-p+pR_H$. This drives the following results under Assumption (13): (1) condition (T1a') is not satisfied, implying that $\forall\ c \in [0, c^{Recapitalize}]$, the bank prefers the safe project; (2) condition (T1a) is satisfied, which implies that condition (T2a) is not and as a result $\forall\ c \in (c^{Recapitalize}, c^{Sufficient})$, the bank prefers the safe project; (3) condition (T3a) is not satisfied and as a consequence $\forall\ c \in [c^{Sufficient}, 1]$, the bank invests in the safe project. Summing up, for any levels of initial capital c, the bank prefers the safe project when the certain return R_S is high enough.

A.5 Proof of Proposition 3

From equation (10), $c^{Sufficient} > c_{\min}$. We consider three cases for the level of $c_{\min}$. Consider first the case when $c_{\min} \leq c^{Recapitalize}$. From Case (a) of Proposition 2, the banker finds optimal to select the risky project when the level of initial capital is in this region. The banker's expected payoff is $p \cdot [R_H - (1-c)] - c\gamma$, which is decreasing in initial capital c (the first derivative is negative since $\gamma > 1$ by assumption (A)). Hence, under minimal capital ratio regulation, the banker chooses $c_{\min}$ as initial capital.

Consider now the case when $c_{\min} \in (c^{Recapitalize}, c^*)$. Again, from Case (a) of Proposition 2, the banker finds optimal to select the risky project and recapitalize in the R_L realization. The banker's expected payoff is $p \cdot [R_H - (1-c)] + (1-p-\mu)[R_L - (1-c) - T] - c\gamma$, which is decreasing in initial capital c. Hence, under minimal capital ratio regulation, the banker chooses again $c_{\min}$ as initial capital.

Consider now the last case when $c_{\min} \in [c^*, c^{Sufficient})$. The banker finds optimal to take no risk. The safe project is selected and the expected payoff for the banker is $R_S - (1-c) - c\gamma$. The expected payoff decreases in c. However, the banker can be better

off selecting a higher level of initial capital. Consider that the banker decides to hold $c^{Sufficient}$. This allows risk-taking (see Case (a) from Proposition 2). The expected payoff is $p \cdot [R_H - (1 - c^{Sufficient})] + (1 - p - \mu) \cdot [R_L - (1 - c^{Sufficient})] - c^{Sufficient}\gamma$. The banker is better off selecting a higher level of capital if and only if

$$p \cdot [R_H - (1 - c^{Sufficient})] + (1 - p - \mu) \cdot [R_L - (1 - c^{Sufficient})] - c^{Sufficient}\gamma > R_S - (1 - c_{\min}) - c_{\min}\gamma. \tag{23}$$

Rearranging terms in (23), the condition becomes $\gamma < 1 + \mu \frac{R_L}{1 - R_L} - \frac{R_S - pR_H - (1-p-\mu)R_L}{(1-c_{\min})(1-R_L)}$. We denote $\gamma^* = 1 + \mu \frac{R_L}{1-R_L} - \frac{R_S - pR_H - (1-p-\mu)R_L}{(1-c_{\min})(1-R_L)}$. The threshold γ^* is higher than 1 for $R_S < W$, with W given in (17) - see also Case (a) of Proposition 2 for further details. Therefore, we can conclude that $\exists\ \gamma^* \in (1, \infty)$ such that $\forall\ \gamma \in (1, \gamma^*)$ the banker selects $c = c^{Sufficient}$ and $\forall\ \gamma \in [\gamma^*, \infty)$ the banker selects $c = c_{\min}$.

B Extensions

We offer here two extensions for our model and examine the implications of charter value and of different specification for recapitalization costs. We show that our results are robust to these generalizations.

B.1 Charter value

In Section 2 we have assumed that there is no charter value for the continuation of bank's activity. In this section we introduce a positive charter value $V > 0$ and show that our results are robust to this extension. Our model suggests that low competition in banking, which provides a high charter value, leads to investment in the efficient safe project even by well-capitalized banks.

The role of banks' franchise values have been shown relevant in other studies. Hellmann et al. (2000) and Repullo (2004) argue that prudent behavior can be facilitated

by increasing banks' charter value. They study the links between capital requirements, competition for deposits, charter value and risk-taking incentives, and point out that banks are more likely to gamble and to take more risk in a competitive banking system, since competition erodes profits and implicitly the franchise value. A similar idea is put forward by Matutes and Vives (2000). They argue that capital regulation should be complemented by deposit rate regulation and direct asset restrictions in order to efficiently keep risk-taking under control. Acharya (2003) explores how continuation value affects risk preferences in the context of optimal regulation, and demonstrates the disciplinating effect of charter value on bank risk-taking. Finally, Keeley (1990) and Furlong and Kwan (2005) explore empirically the relation between charter value and different measures of bank risk, and find strong evidence that bank charter value disciplined bank risk-taking.

In the new setting, the banker's payoff to the safe project after repaying depositors becomes $\Pi_S^V = R_S - (1-c) + V$. The banker's payoff to the risky project is as follows: when R_H is realized, the banker gets $\Pi_H^V = R_H - (1-c) + V$, while the payoff is 0 for extremely low realization R_0. When the low return R_L is realized and capital is positive but insufficient ex-post, the banker prefers to recapitalize at a cost T for lower levels of initial capital c. The reason for this is that banker's expected payoff increases by V if bank is not closed by the regulator. Hence, the bank raises additional capital when initial capital c is higher than $c_V^{Recapitalize}$, where:

$$c_V^{Recapitalize} = 1 + T - R_L - V, \tag{24}$$

and $c_V^{Recapitalize} < c^{Recapitalize}$. On the other hand, the threshold point $c^{Sufficient}$ does not change since it is given by the exogenous regulation.

We make the simplifying assumption that the charter value is not larger than a certain threshold:

$$V < 1 + T - R_L. \tag{25}$$

This makes threshold $c_V^{Recapitalize}$ positive and assures the existence for the area [0, $c_V^{Recapitalize}$] where the bank is abandoned for low realization of the risky project. Consider the area $(c_V^{Recapitalize}, c^{Sufficient})$. When initial capital c is in this range, a bank which is subject to regulator's corrective action prefers to raise additional capital. Since $c_V^{Recapitalize} < c^{Recapitalize}$, while right boundary of the interval is left unchanged by any increase in V, we can argue that any reduction in banking competition, which increases bank charter value, makes the decision to raise fresh capital more likely.

We introduce the following two thresholds:

$$Z_V = pR_H + (1-p)(R_L - T) + \mu(T - V - c_{\min}R_L), \tag{26}$$

as the new threshold for the binding impact of the prompt corrective action (with $Z_V < Z$), and

$$B = pR_H + (1-p)(R_L - T). \tag{27}$$

Following a similar proof as for Proposition 2, we can show that there exist two thresholds c_V^* and c_V^{**} for the level of initial bank capital such that under assumption (13) and for levels of return on the safe project satisfying $Z_V < R_S < B$, with Z_v and B defined in (26) and (27), respectively, the bank's investment preference is as follows:

(a) the bank prefers the risky project for $0 \leq c < c_V^*$, while for $c_V^* \leq c < c^{Sufficient}$ the safe project is preferred, with $c_V^* \in (c_V^{Recapitalize}, c^{Sufficient})$, where $c_V^{Recapitalize}$ and $c^{Sufficient}$ are defined in (24) and (10), respectively, and

$$c_V^* = 1 - V - \frac{R_S - pR_H - (1-p-\mu)(R_L - T)}{\mu}, \tag{28}$$

(b) the bank prefers the risky project for $c^{Sufficient} \leq c < c_V^{**}$, and the safe project for $c \geq c_V^{**}$, where $c_V^{**} \in (c^{Sufficient}, 1)$ and

$$c_V^{**} = 1 - V - \frac{R_S - pR_H - (1-p-\mu)R_L}{\mu}. \tag{29}$$

Observe that a positive charter value has a negative impact on all relevant thresholds which drive bank's preferences (i.e., $c_V^{Recapitalize}$, c_V^*, and c_V^{**}), except for $c^{Sufficient}$. Hence, we can argue that higher charter value plays the role of a counterbalancing force to the risk-taking incentives generated by the presence of risky projects with heavy left tails. This means that when the continuation value of bank's activity is high enough, both intervals $(0, c_V^*)$ and $(c^{Sufficient}, c_V^{**})$ shrink. This suggests that low competition in the banking industry induces banks with larger capital buffers to take less risk.

In summary, the results of our basic model are therefore robust to the introduction of charter value, conditional on the fact that this value is not too large. Large values of charter value reduces risk-taking incentives even for well-capitalized banks.

B.2 Concave capital adjustment cost

In Section 2 we considered a simple fixed cost of recapitalization. We now show that results are robust to a more general specification of this cost function.

In this section we discuss a variation of the model in which the cost of recapitalization has a fixed and a variable component. The variable component is proportional to the amount of new capital that the bank has to raise in order to comply with the minimal capital ratio. Specifically, the bank has to raise a capital level $R_{\min} - R_L$, where $R_{\min}$ equals:

$$R_{\min} = \frac{1-c}{1-c_{\min}}. \tag{30}$$

The above threshold is derived from the condition of a minimal capital ratio of $c_{\min}$ (i.e., $c \leq c_{\min} = [R_{\min} - (1-c)]/R_{\min}$), by solving for the value of bank's assets (i.e., $R_{\min}$).[6] In this new setting, the recapitalization cost is concave in capital level, and has

[6] Consider the following example. Assume that the bank has to raise δ units of capital to satisfy the

the following specification:

$$Cost(c, R_L) = T + \beta(\frac{1-c}{1-c_{\min}} - R_L). \tag{31}$$

We assume that variable cost of recapitalization (i.e., β) is positive and not as low as to make the banker abandon the bank regardless the level of initial capital:

$$T < R_L(1+\beta). \tag{32}$$

The banker's payoff from the safe project, as well as the realizations of the risky project are the same as in the basic model. However, when the low realization R_L is obtained, the bank is abandoned more often than in the basic model due to higher cost of recapitalization. The bank is closed when $c < c_{CC}^{Recapitalize}$, where:

$$c_{CC}^{Recapitalize} = 1 + \frac{T - R_L(1+\beta)}{1 + \frac{\beta}{1-c_{\min}}}, \tag{33}$$

(with CC for concave cost).

Under assumption (13), $c_{CC}^{Recapitalize} > c^{Recapitalize}$. On the other hand, the level of capital which guarantees that the bank satisfies ex-post the regulatory minimal upon realization of R_L (i.e., $c^{Sufficient}$) remains unchanged. Hence, the interval $(c_{CC}^{Recapitalize}, c^{Sufficient})$ shrinks, suggesting that the bank is less likely to raise additional capital if required to do so.

We denote:

$$B_{CC} = pR_H + (1-p)\frac{R_L(1+\beta) - T}{1 + \frac{\beta}{1-c_{\min}}}. \tag{34}$$

Following the lines of proof for Proposition 2 we can show that exist two thresholds c_{CC}^{*} and c_{CC}^{**} for the level of initial bank capital such that under assumption (13) and for

regulatory minimum when R_L is realized. Hence, $c_{\min} = \frac{R_L-(1-c)+\delta}{R_L+\delta}$.This implies that $R_L + \delta = \frac{1-c}{1-c_{\min}}$, which equals $R_{\min}$ according to (30). We can conclude that $\delta = R_{\min} - R_L$.

level of return on the safe project satisfying $Z < R_S < B_{CC}$, with Z and B_{CC} defined in (18) and (34), respectively, the bank's investment preference is as follows:

(a) the bank prefers the risky project for $0 \leq c < c^*_{CC}$, while for $c^*_{CC} \leq c < c^{Sufficient}$ the safe project is preferred, with $c^*_{CC} \in (c^{Recapitalize}_{CC}, c^{Sufficient})$, where $c^{Recapitalize}_{CC}$and $c^{Sufficient}$ are defined in (33) and (10), respectively, and

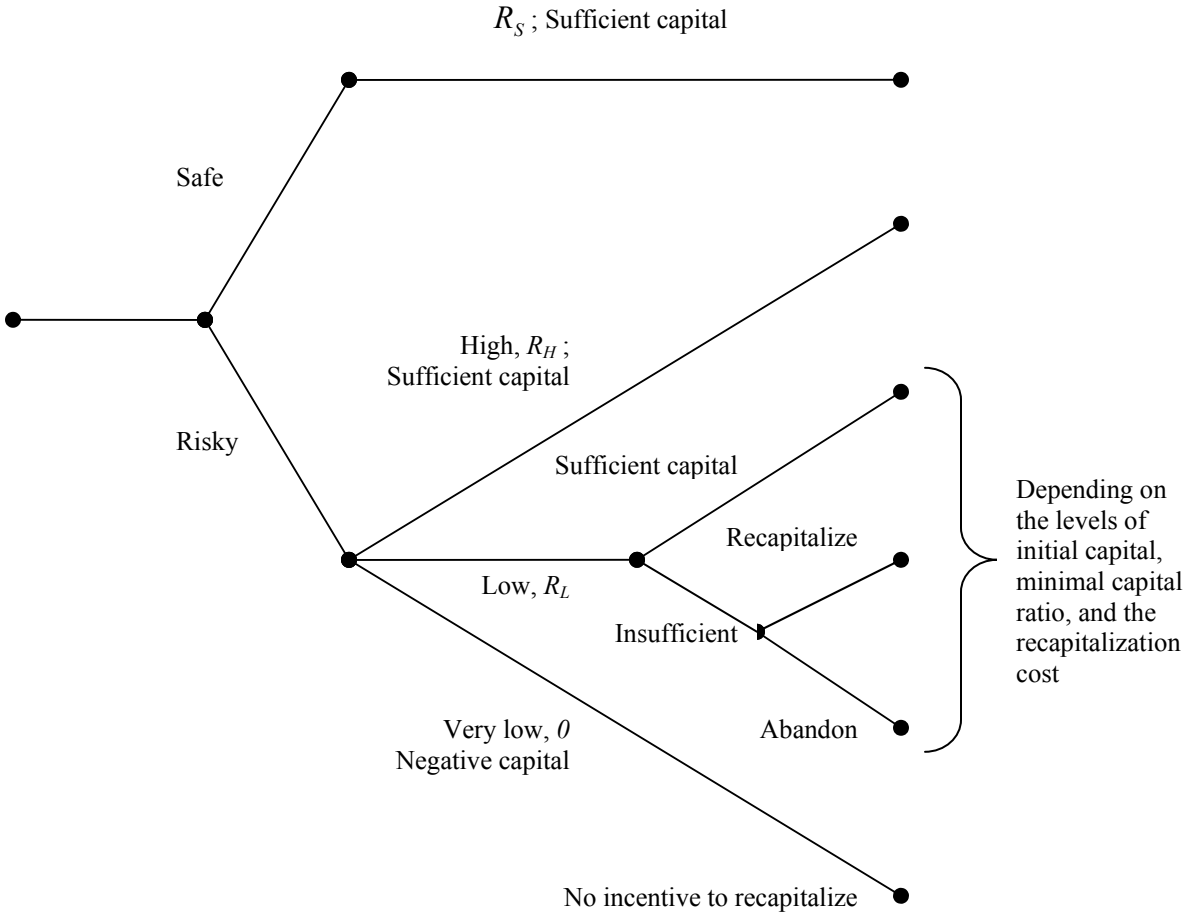
$$c^*_{CC} = 1 - \frac{R_S - pR_H - (1-p-\mu)[R_L(1+\beta) - T]}{\mu - \frac{\beta}{1-c_{\min}}(1-p-\mu)}; \tag{35}$$

(b) the bank prefers the risky project for $c^{Sufficient} \leq c < c^{**}_{CC}$, and the safe project for $c \geq c^{**}_{CC}$, where $c^{**}_{CC} \in (c^{Sufficient}, 1)$ and $c^{**}_{CC} = c^{**}$, with c^{**} defined in (20).

Observe that the introduction of a variable component for recapitalization cost leaves both boundaries of the interval $(c^{Sufficient}, c^{**}_{CC})$ unchanged. Thus, our model is robust to this specification and a concave cost of recapitalization does not affect the risk-taking incentives of well-capitalized banks when projects exhibiting heavier left tails are available for investment.

Figure 1.

The timeline



Date 0

- Bank has capital *C* and deposits *D*
- Bank selects one project

Date 1/2

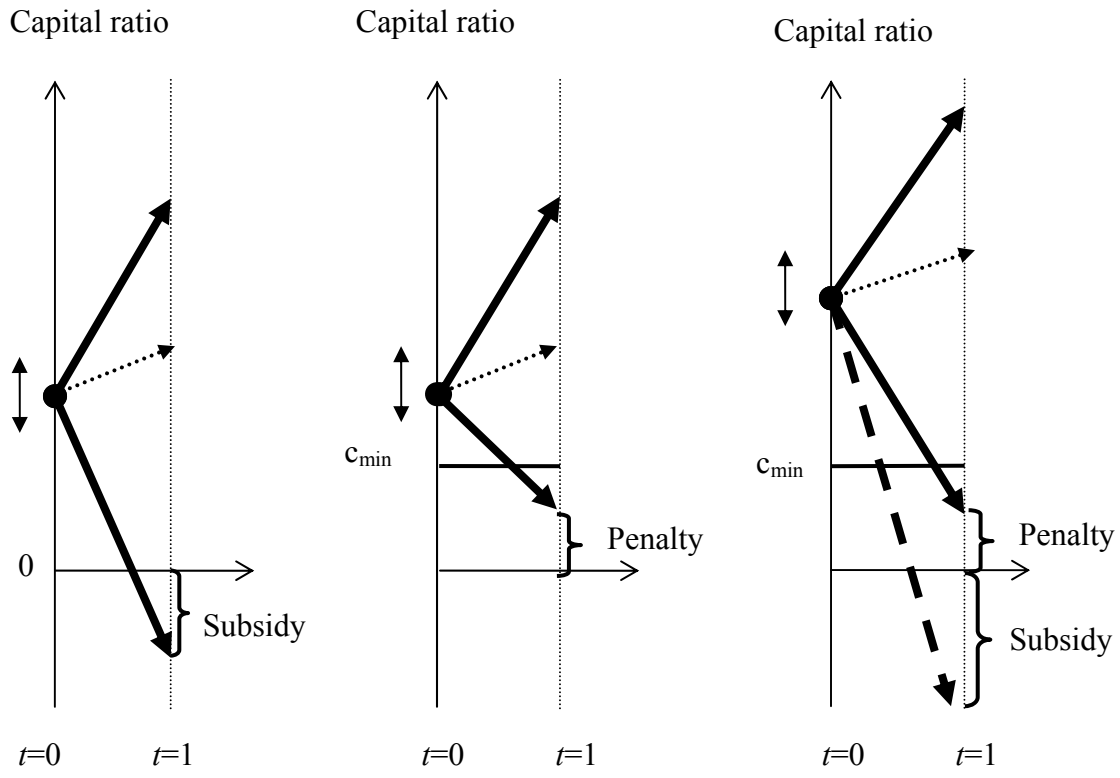
- Information about future returns
- Regulator may take corrective action
- Bank decides whether to recapitalize

Date 1

- Returns are realized and distributed

Figure 2.

The two opposite effects of capital on bank risk-taking



Left panel:
The effect of limited liability

Higher capital reduces risk-taking

Central panel:
The effect of capital adjustment cost

Higher capital may increase risk-taking

Right panel:

The combined effect

Higher capital may enable excess risk-taking

Safe project

Risky project

The left tail of the Risky project

Figure 3.

Tail risk and the initial capital required to prevent risk-shifting

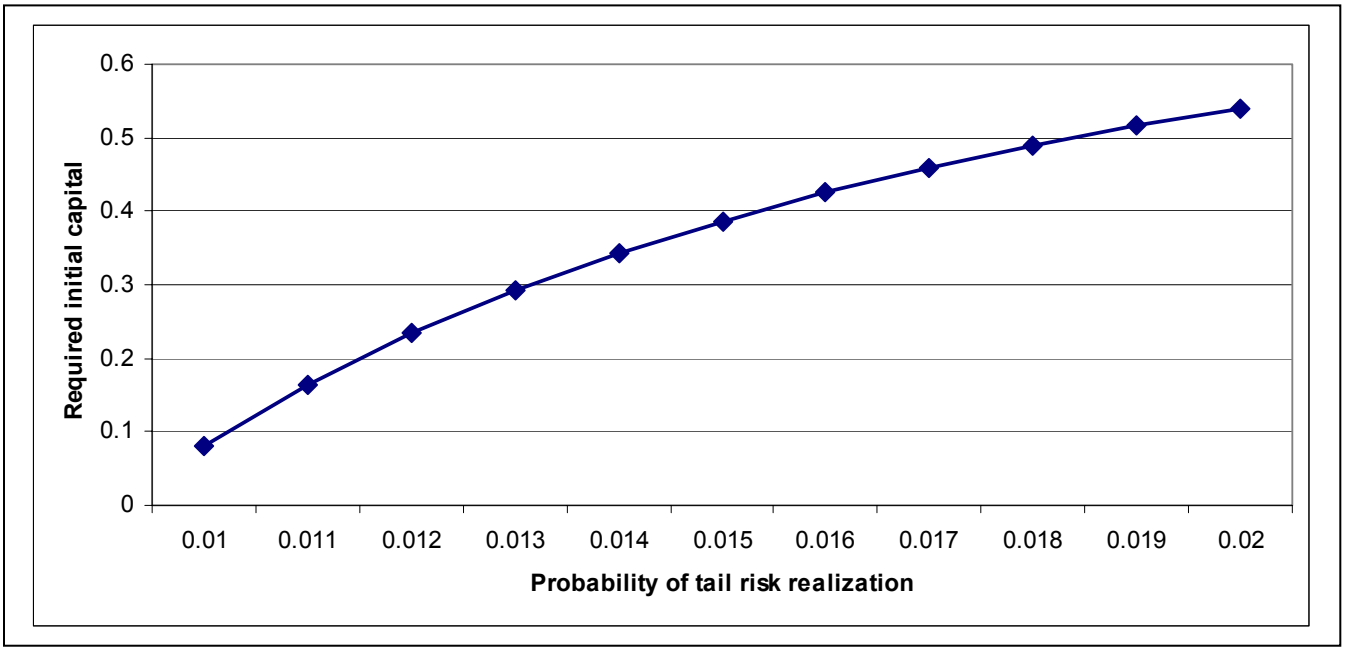


Figure 4.

Bank's recapitalization decision and payoffs

The bank's recapitalization decision and banker's payoffs as a function of initial capital c, upon the realization of low return R_L are as follows. For $c \geq c^{Sufficient}$, the bank has positive and sufficient capital at date ½. The bank continues to date 1, repays depositors and obtains a positive payoff. For $c < c^{Sufficient}$, the bank has positive and insufficient or negative capital. The bank can be either abandoned or recapitalized. The bank is abandoned for $c \leq c^{Recapitalize}$. As a result the bank is closed and the banker gets a zero payoff. The bank is recapitalized at a cost for $c^{Recapitalize} < c < c^{Sufficient}$. The bank continues to date 1, repays depositors, pays the recapitalization cost, and obtains a positive payoff.

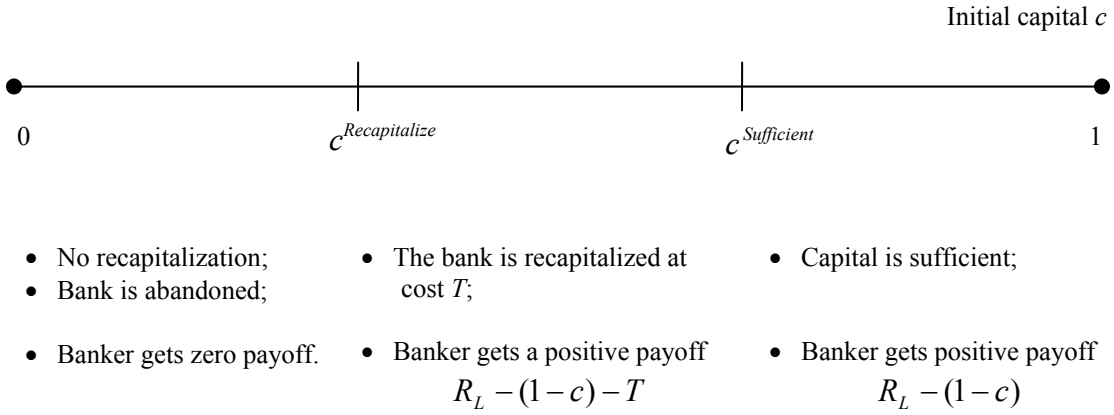


Figure 5.

Bank's project choice

The bank's project choice depending on the level of initial capital, in Case (a) of Proposition 2: The relationship between bank capital and risk-taking is non-linear and is characterized by two thresholds as follows. When the level of capital is low ($c < c^{*}$), the bank prefers the risky project, while for high level of capital ($c \geq c^{**}$) the safe project is chosen. For intermediate level of capital ($c^{*} \leq c < c^{**}$), the bank prefers either the safe project (for $c^{*} \leq c < c^{Sufficient}$) or the risky one (for $c^{Sufficient} \leq c < c^{**}$).

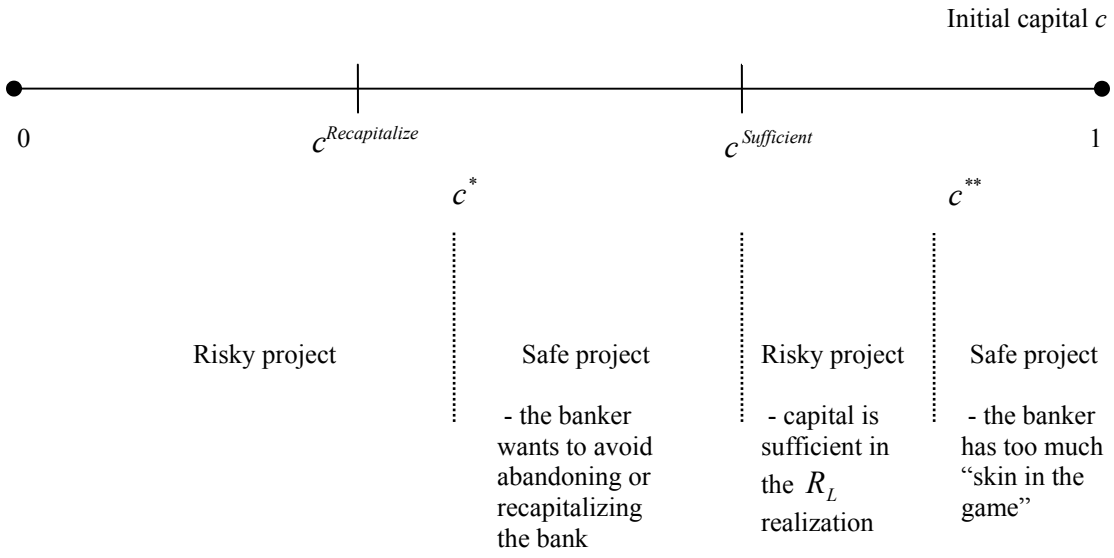


Figure 6a.

Bank's project choice when the risky project has a heavier left tail. Case 1

A heavier left tail is characterized by a higher probability for the extremely low outcome (i.e., a higher μ). A change in the return profile of the risky project following a change in probability distribution (i.e., both p and μ are increased, other else equal, such that the expected value of the risky project remains the same), affects both thresholds c^{*} and c^{**}. The interval $[c^{Sufficient}, c^{**})$ widens, suggesting that well-capitalized banks which behave prudently in absence of tail risk projects, have a strong incentive to undertake more risk, if projects with heavier left tail are available in economy.

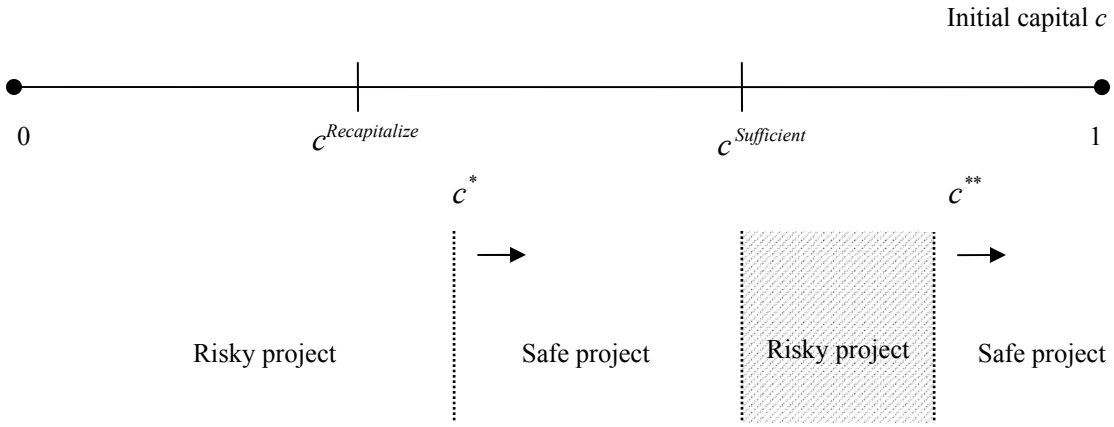


Figure 6b.

Bank's project choice when the risky project has a heavier left tail. Case 2

A heavier left tail is characterized by a higher probability for the extremely low outcome (i.e., a higher μ). A change in the return profile of the risky project following a change in probability distribution (i.e., μ is increased), compensated by higher R_L and R_H, other else equal, such that the expected value of the risky project remains the same, affects thresholds c^* and c^{**}, and $c^{Sufficient}$ as well. The interval $[c^{Sufficient}, c^{**})$ widens even more, suggesting that both well and less capitalized banks will start choosing the risky project.