Appendix to: Evaluating An Estimated New Keynesian Small Open Economy Model

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Abstract

In this appendix, we provide additional estimation results for the model with the modified UIP condition in Adolfson et al. (2006), in order to provide the interested reader with more information to judge the robustness of our econometric results.

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1. Overview

1.1. Prior vs posterior plots

Figures 1a to 1c report the prior and posterior distributions for all estimated parameters in the model including the modified UIP condition and using a flexible Taylor-type instrument rule to model the pre-inflation targeting regime (i.e., the last model in Table 1). In accordance with the results in Table 1, the figures show that the data is generally informative about the parameters, as the posterior distributions differ from the priors, with a few exceptions (i.e., ξ_w and r_π). The results for these parameters are discussed in greater detail in the main text.

1.2. Raw Metropolis chains

In Figures 2a to 2c we report the 500,000 post burn-in Metropolis draws for each parameter. The Markov chain in Figure 2 show that there are no clear trends or correlations between the parameters in the model.

The CUSUM plots of the Metropolis draws are depicted in Figures 3a to 3c. The solid line (labeled posterior) is the accumulated mean of the draws, while the dashed line (labeled average) is a window average of the last 50,000 draws. As expected from the raw Metropolis draws, the CUSUM plots show no sign of a trending mean in the parameters throughout the whole chain. Even if the window averages are varying slightly, there do not appear to be any high correlation between the different parameters. In particular, we notice from Figures 2 and 3 that the risk premium parameters $\tilde{\phi}_s$, $\rho_{\tilde{\phi}}$ and $\sigma_{\tilde{\phi}}$ do not show any clear sign of cross-correlation or time-varying means, supporting the discussion in the main text that these parameters are well identified.

1.3. Sequential log marginal likelihoods

In Figure 4, we plot the log marginal likelihood sequentially for each 50,000th draw, using all the preceding iterations in the computation of the marginal likelihood with the modified harmonic estimator in Geweke (1999).[1] The lines pertain to eight different probabilities of the truncated elipsodes of the joint posterior distribution. As can be seen from Figure 4, the spread between the largest and smallest elipsodes is not very large after a couple of 100,000 draws. It is also clear from the figure that the marginal likelihood converges after about 200,000 draws. To obtain convergence, we found it to be of critical importance to obtain a good estimate of the Hessian matrix. We start by maximizing the posterior density and evaluating the Hessian matrix at the posterior mode using standard numerical optimization routines. Prior to the optimization we transform all the parameters to the unconstrained domain (we use logit transformations on parameters restricted to the unit interval, and a log transformation for strictly positive parameters). Second, draws from the posterior distribution are generated using the Metropolis-Hastings algorithm.

[1] The marginal likelihood of a model i is defined as $m_i = \int L_i(\theta_i; x)p_i(\theta_i)d\theta_i$, where $L_i(\theta_i; x)$ is the usual likelihood function of the model's parameter vector conditional on the observed data x. $p_i(\theta_i)$ is the prior distribution of the model's parameters. m_i is the unconditional probability of the observed data, under the assumed prior distribution, and is therefore a measure of model fit. The marginal likelihood is a relative measure and should be compared across competing models. The Bayes factor comparing two models i and j is defined as $B_{ij} = m_i/m_j$.

1.4. Multivariate ANOVA

In Figure 5, we report the multivariate potential scale reduction factor (MPSRF) based on four independent Metropolis chains with different starting values consisting of 500,000 draws each. The analysis is based on subsampling every 5th draw for computational reasons. Gelman et al. (1995) argue that by rule-of-thumb, the MPSRF should be less than 1.1 to have a satisfactory convergence. As is evident from Figure 5, this requirement is fulfilled in our chains after about $100,000$ draws (i.e, after 20,000 iterations in the plot), and after about 250,000 draws it is down to less than 1.05. Figure 5 also depicts the total variance in all the four Markov chains together with the variance within each each parallel chain. As the number of draws increases we see that the difference betwen the total and within variation in the parallel chains decreases.

1.5. Contour plots

In the main text, we argue from using uniform priors and comparing Bayesian model probabilities that the parameters associated with the UIP condition, $\tilde{\phi}_s$ and $\rho_{\tilde{\phi}}$, are well identified given our set of observable variables in the measurement equation. In Figure 6 we show contour plots of the likelihood function in the $\left\{\tilde{\phi}_s, \rho_{\tilde{\phi}}\right\}-$space conditional on the posterior median of the other parameters. Figure 6a shows the likelihood for different values of $\tilde{\phi}_s$ and $\rho_{\tilde{\phi}}$ using the entire set of 15 observables. The data is relatively informative about the intrinsic persistence $\tilde{\phi}_s$ but has less to say about the exogenous autocorrelation $\rho_{\tilde{\phi}}$. Even if the likelihood in Figure 6a is nearly constant in the neighborhood of the posterior mode, the likelihood function appear to be well shaped when all the other parameters are not kept fixed. When plotting the likelihood contours using *only* the real exchange rate in the measurement equation, we see that the data is even less informative about $\rho_{\tilde{\phi}}$ (see Figure 6b). $\rho_{\tilde{\phi}}$ can take on almost any value for small values of $\tilde{\phi}_s$, and the likelihood function in this case suggest a crner solution of ($\tilde{\phi}_s = 1$, $\rho_{\tilde{\phi}} = 0$).

References

Adolfson, M., S. Laséen, J. Lindé and M. Villani (2006), "Evaluating An Estimated New Keynesian Small Open Economy Model", Manuscript, Sveriges Riksbank.

Gelman, A., J. Carlin, H. Stern and D. Rubin (1995), *Bayesian Data Analysis*, Chapman & Hall, New York.

Geweke, J. (1999), "Using Simulation Methods for Bayesian Econometrics Models: Inference, Development and Communication", *Econometric Reviews* 18(1), 1-73.

Figure 1a: Prior and posterior distributions, friction parameters

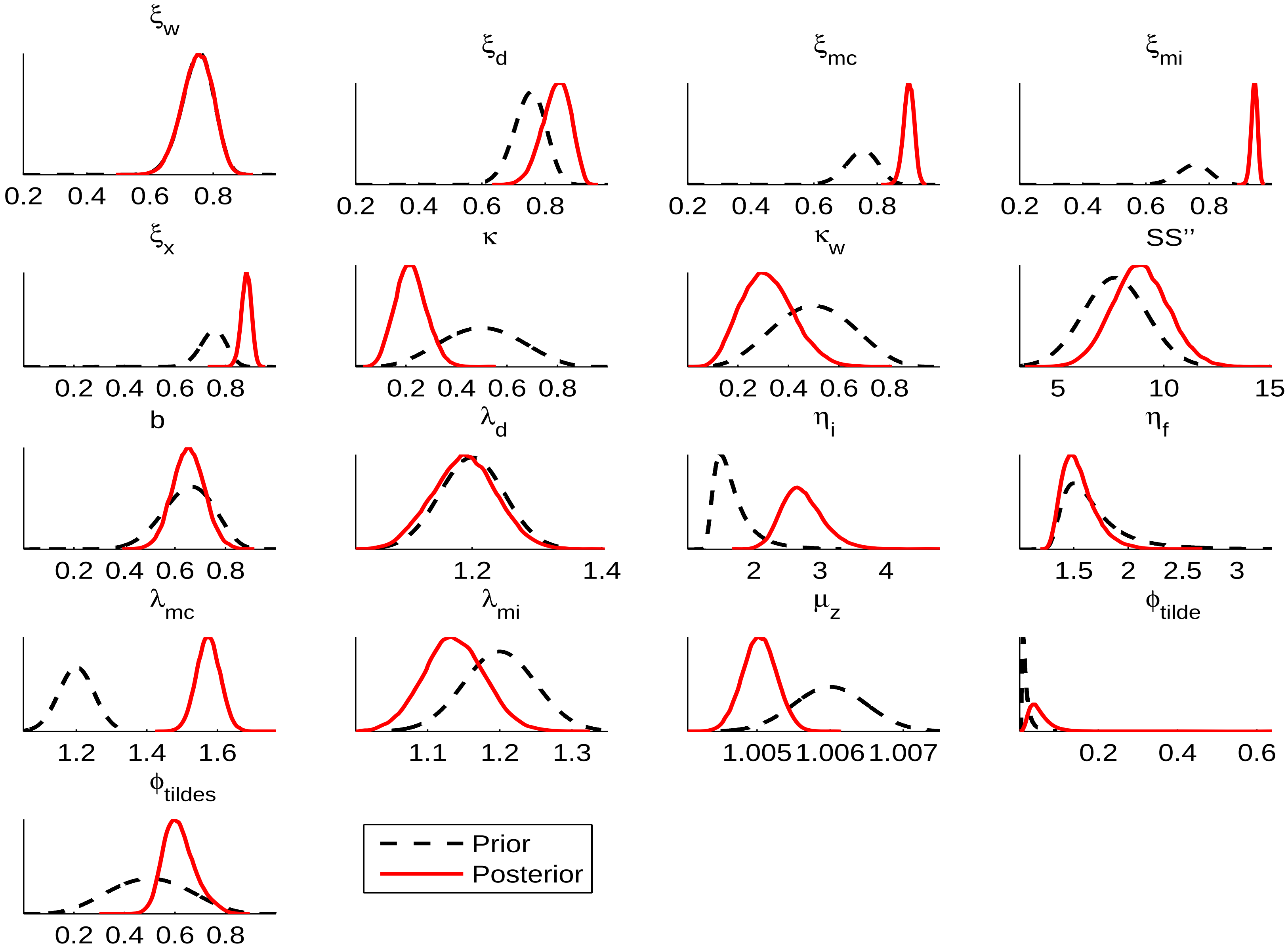


Figure 1b: Prior and posterior distributions, shock parameters

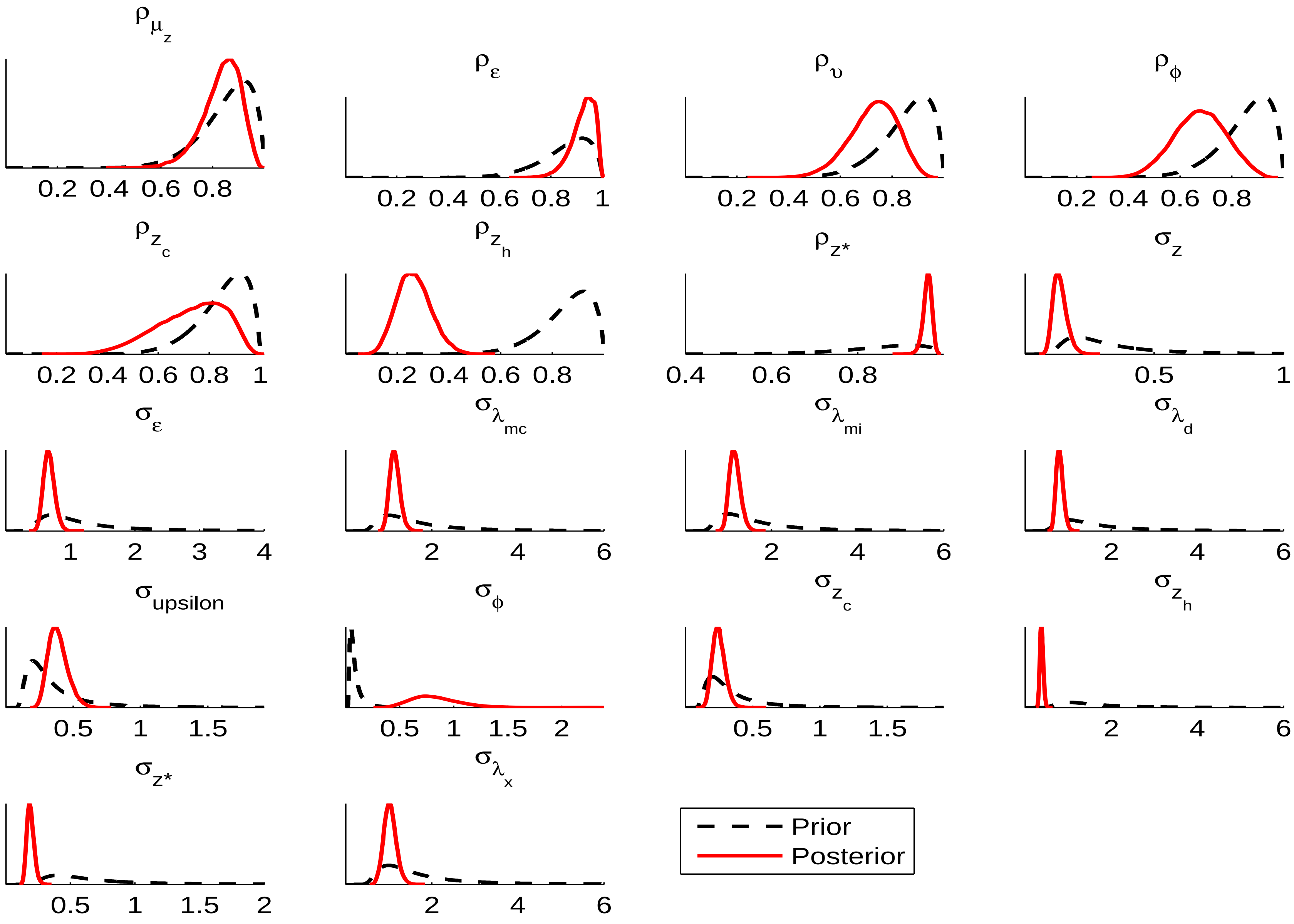


Figure 1c: Prior and posterior distributions, policy parameters

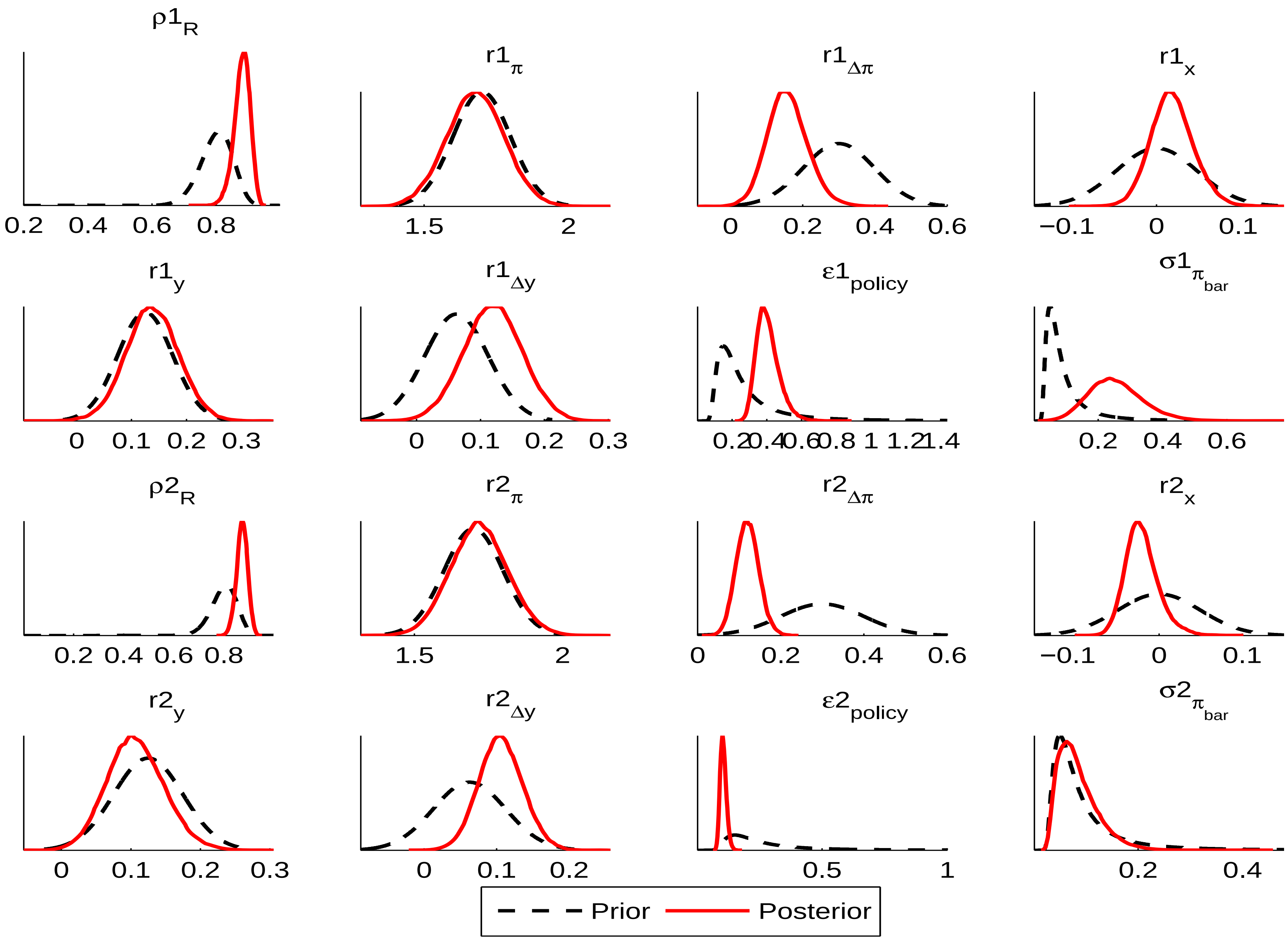


Figure 2a: Plots of the raw Metropolis draws, friction parameters

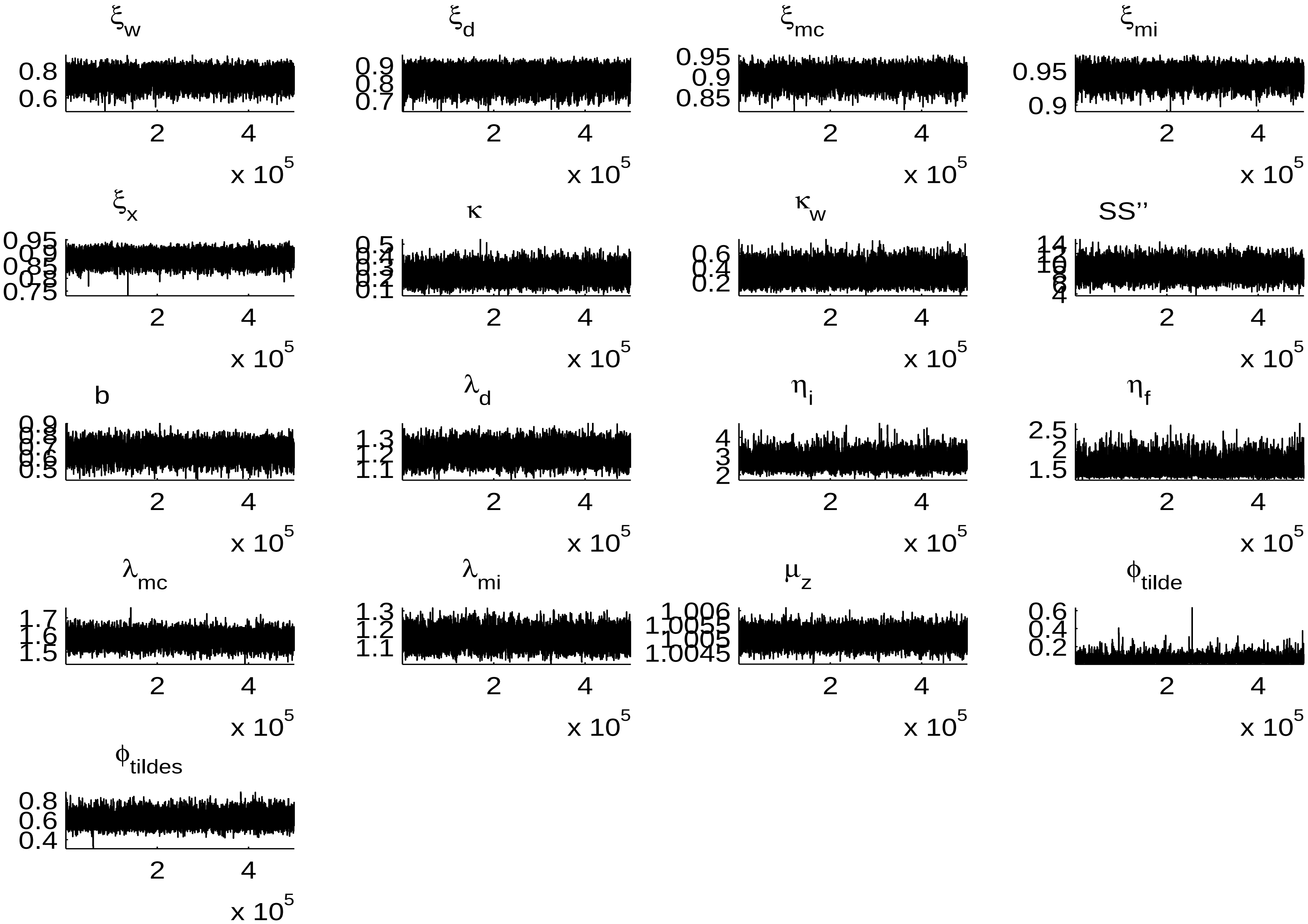


Figure 2b: Plots of the raw Metropolis draws, shock parameters

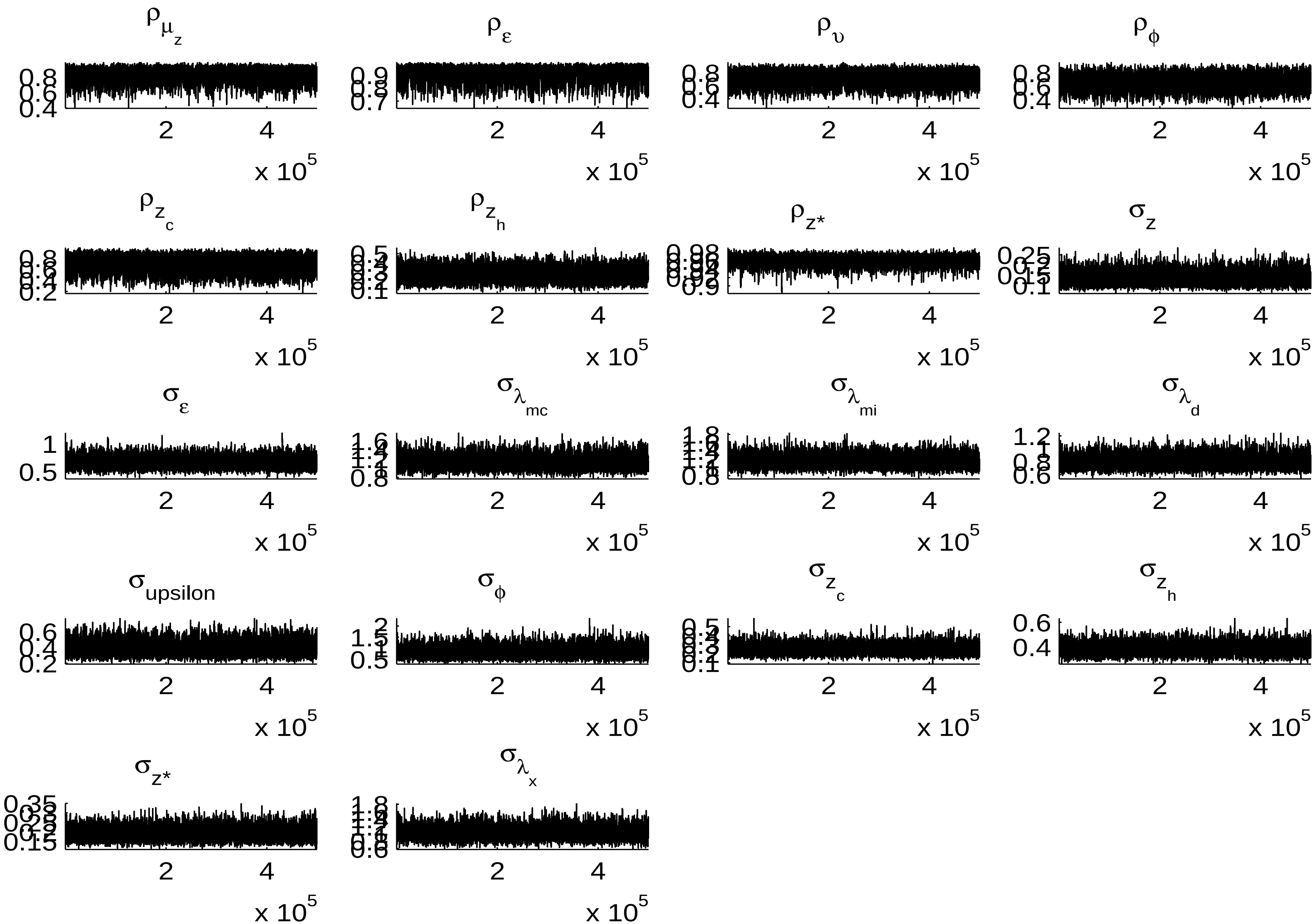


Figure 2c: Plots of the raw Metropolis draws, policy parameters

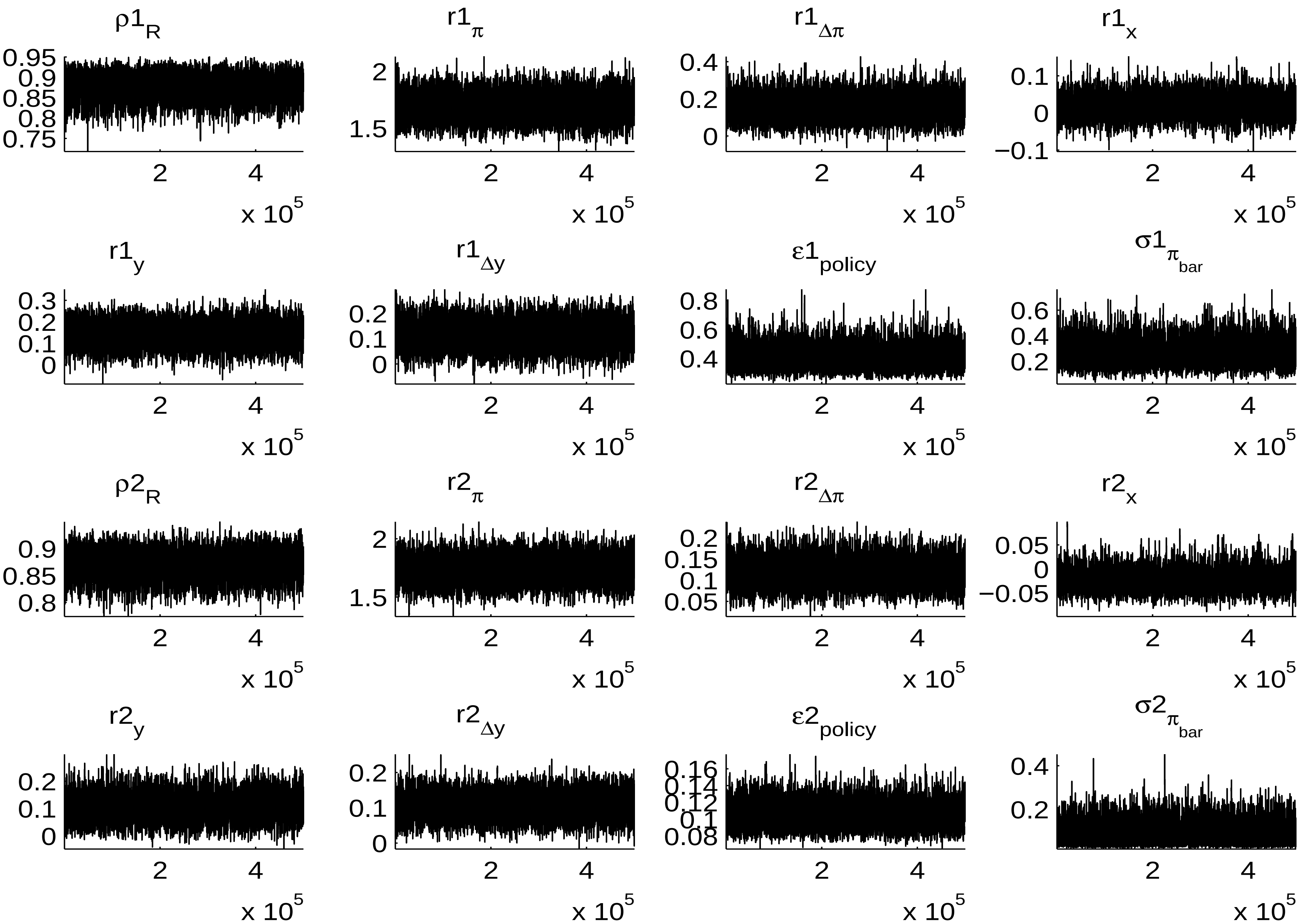


Figure 3a: CUSUM plots of the Metropolis draws, friction parameters

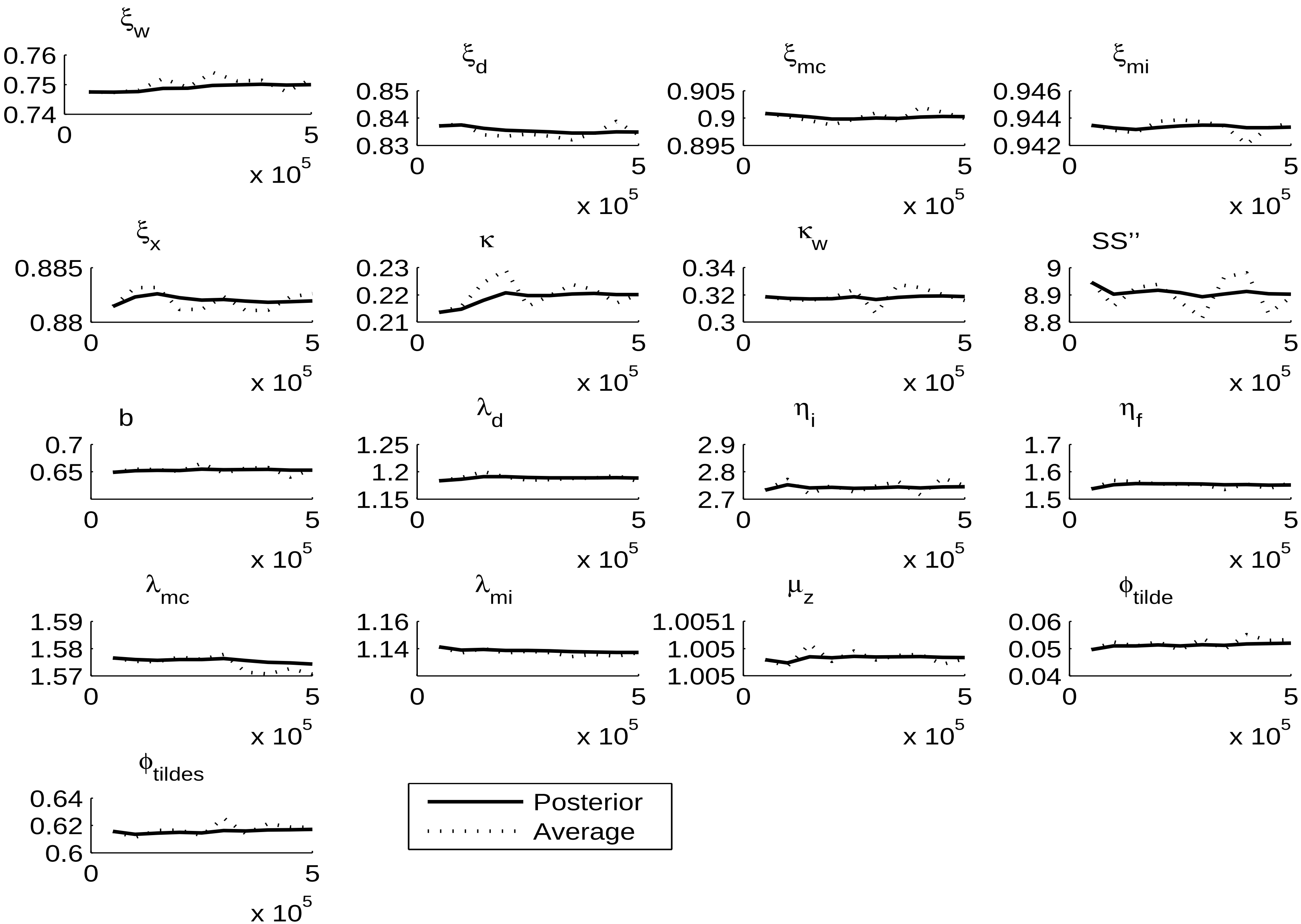


Figure 3b: CUSUM plots of the Metropolis draws, shock parameters

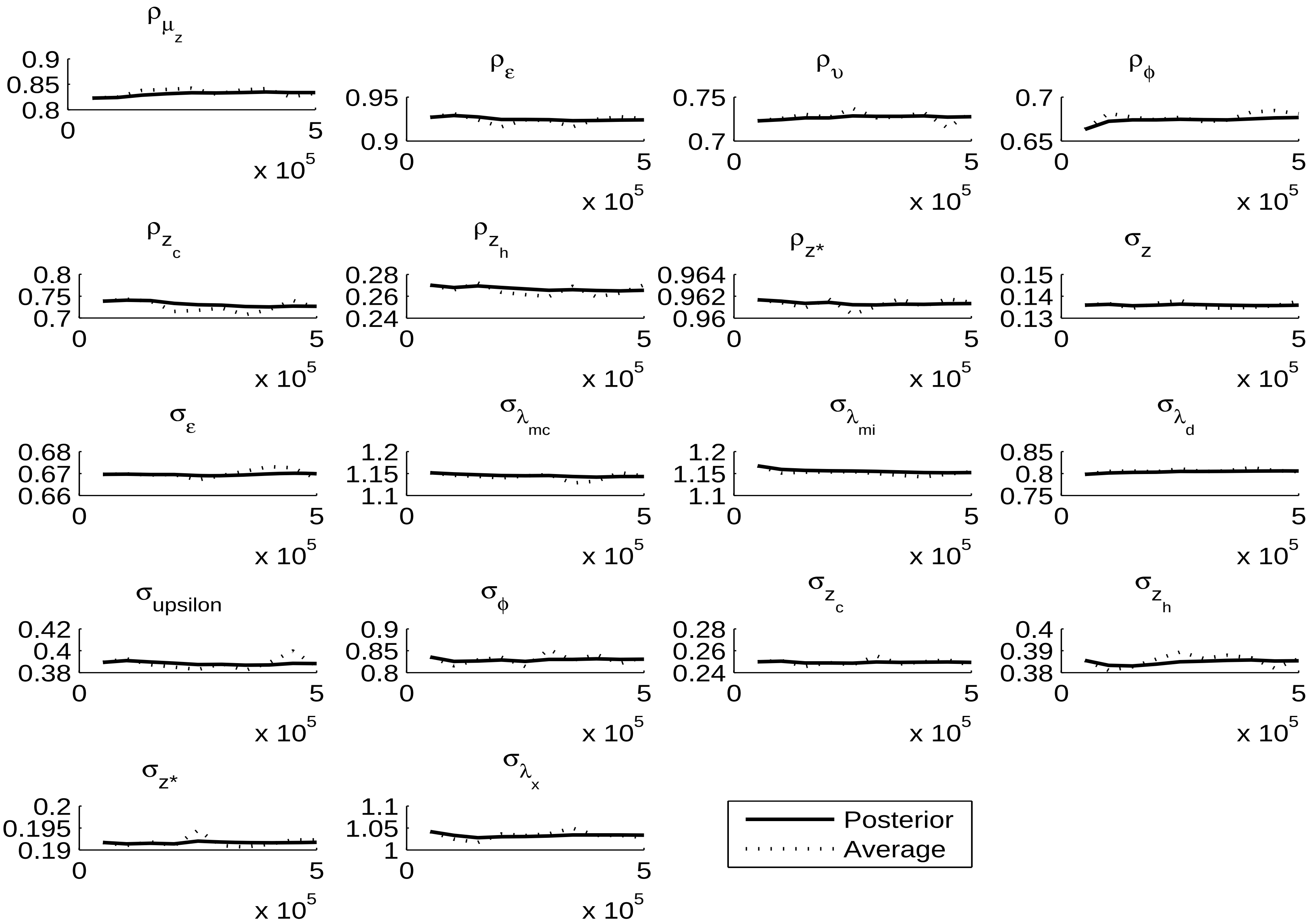


Figure 3c: CUSUM plots of the Metropolis draws, policy parameters

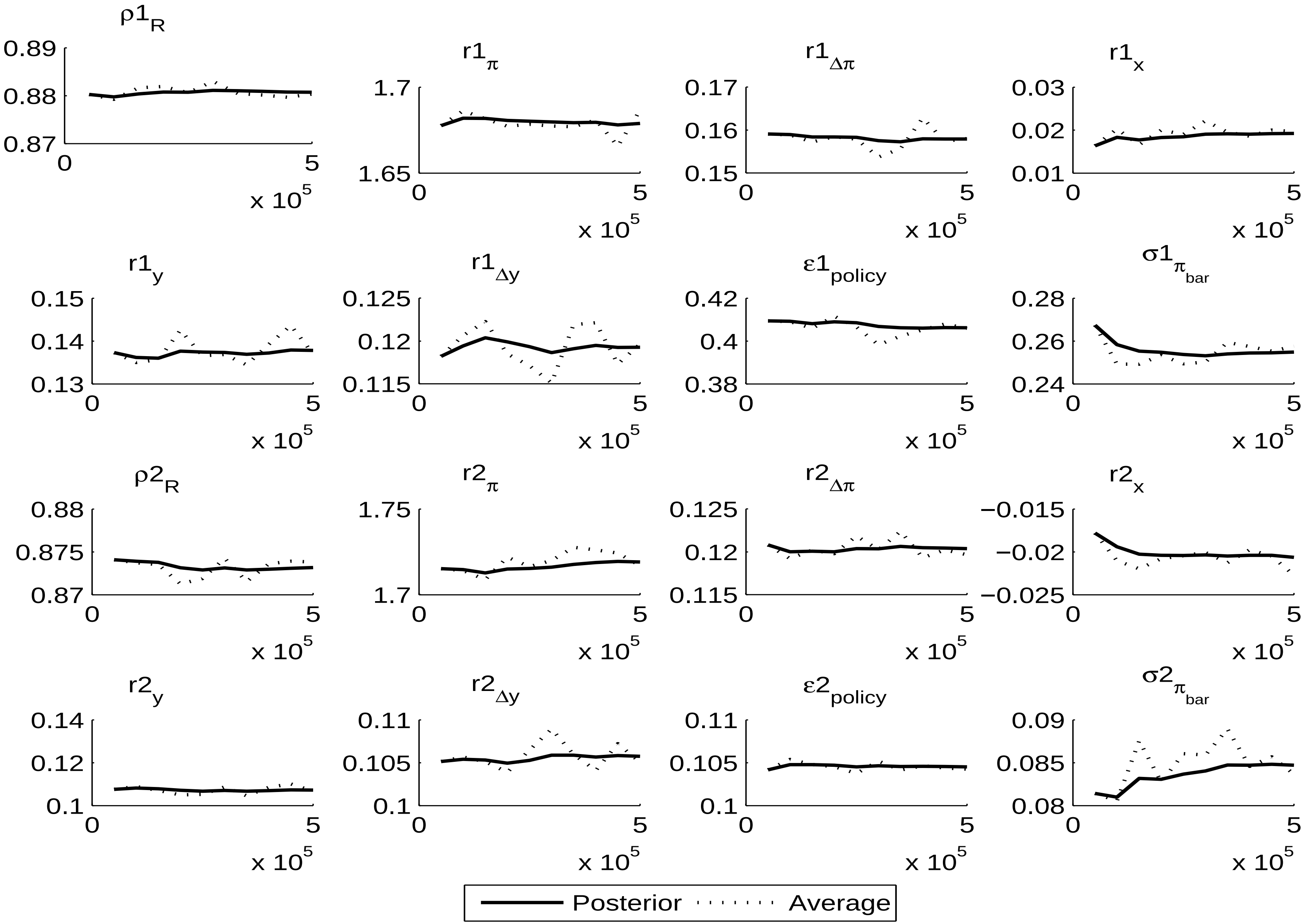


Figure 4: Sequential marginal likelihoods

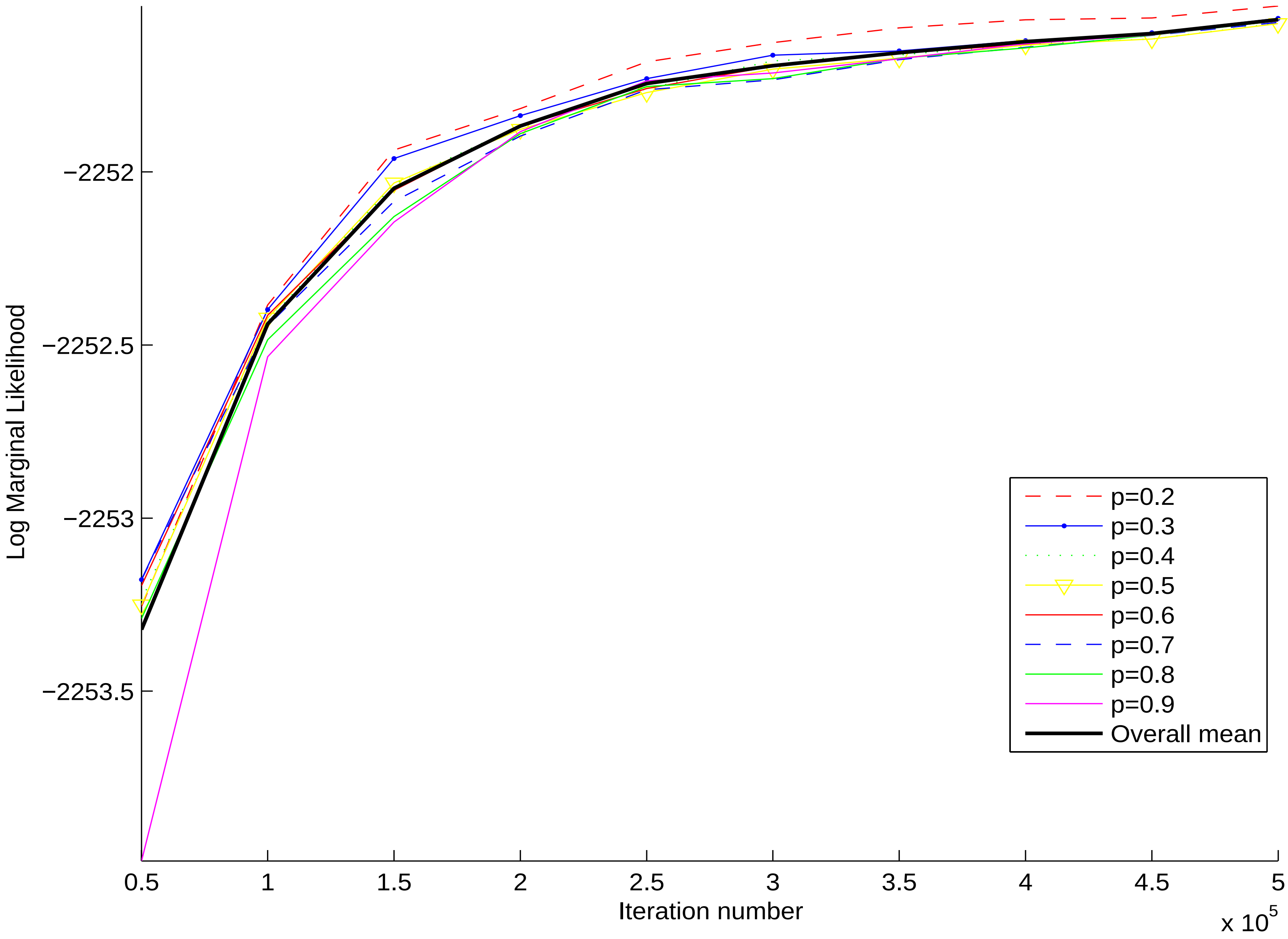
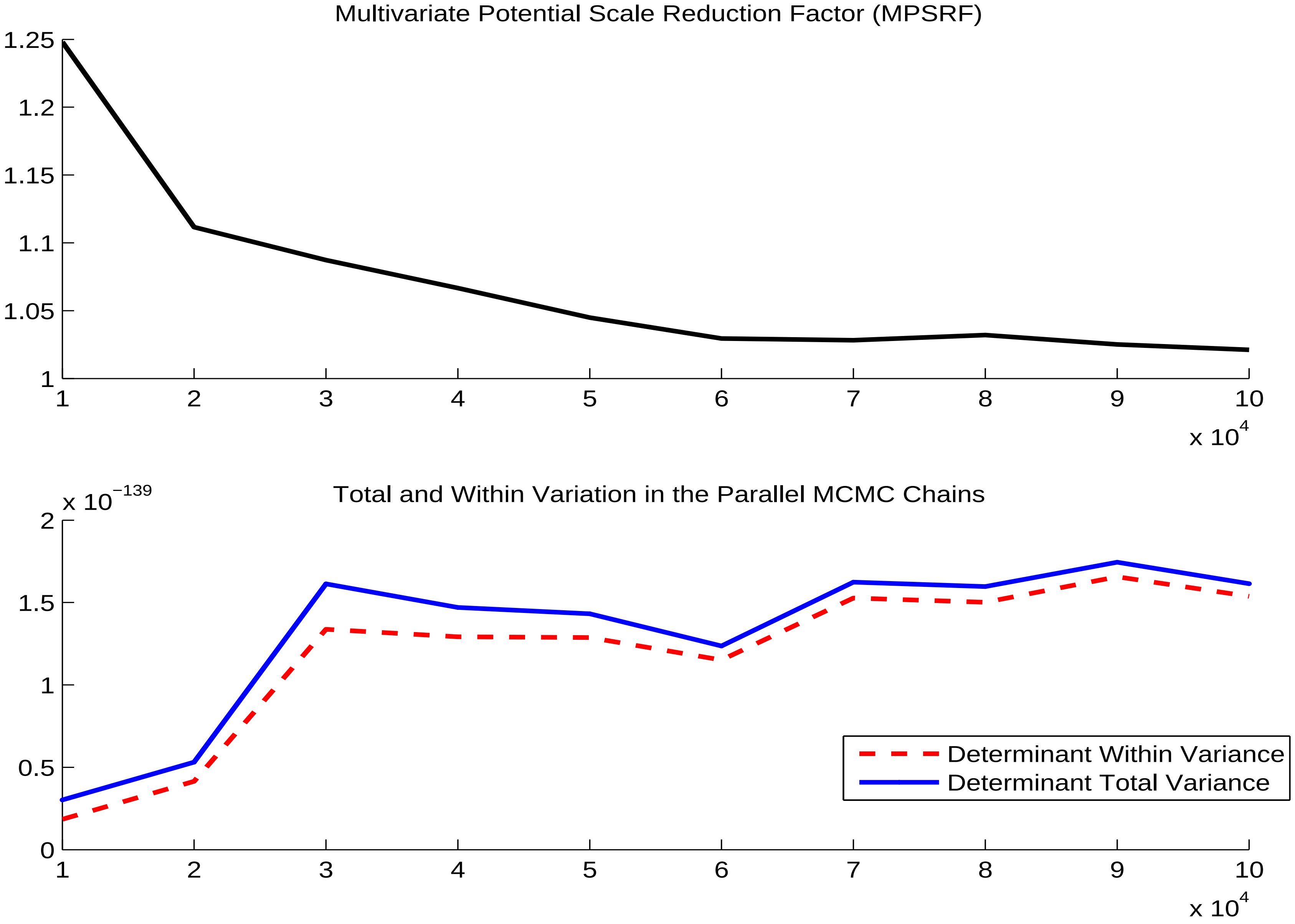


Figure 5: Multivariate ANOVA



Note: The analysis is based on four Metropolis chains with different inital values and 500,000 draws each, subsampling every 5th draw.

Figure 6a: Log likelihood contours in the $\{\widetilde{\phi}_s,\, \rho_\phi\}$-space, using all observable variables

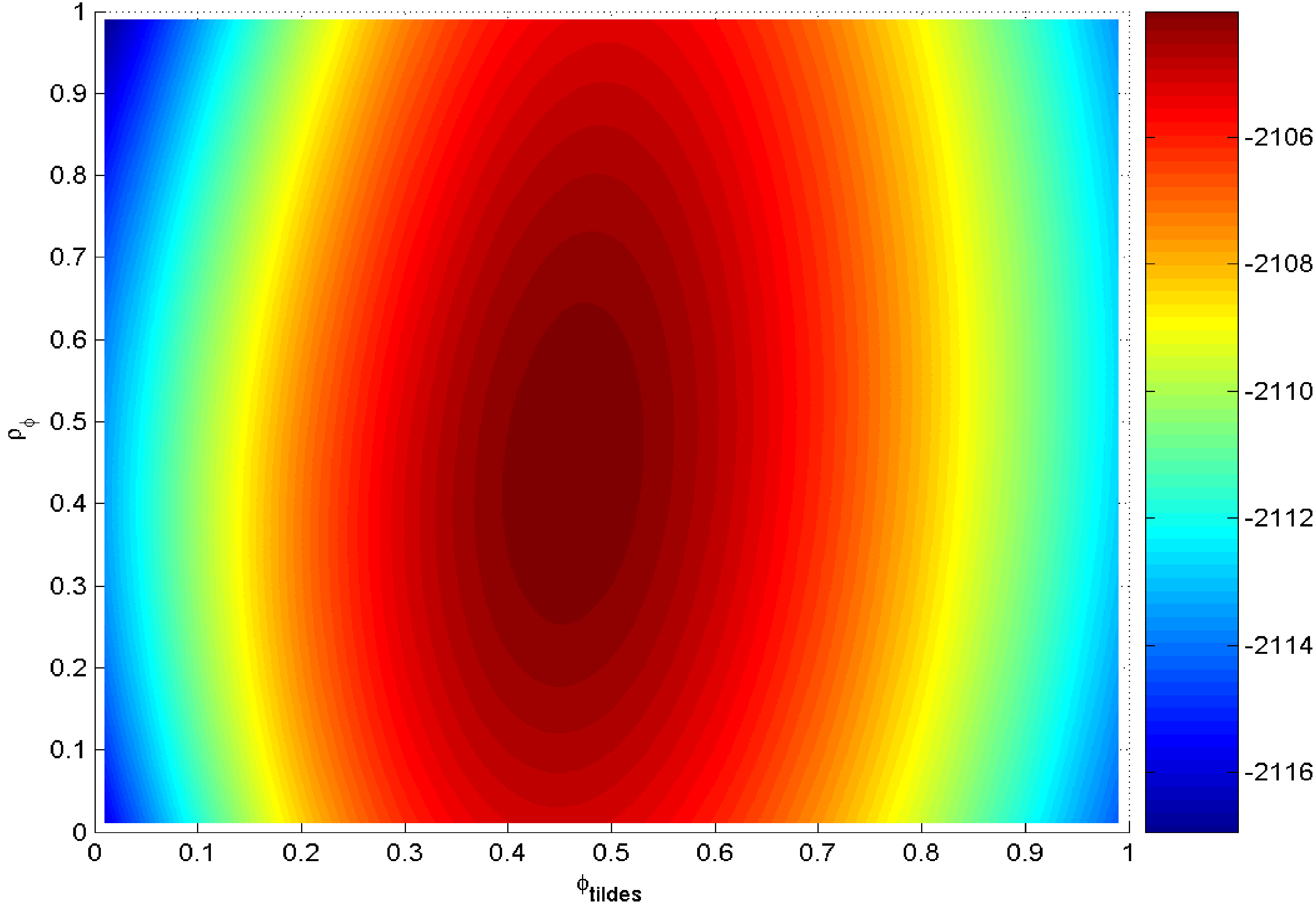


Figure 6b: Log likelihood contours in the $\{\widetilde{\phi}_s, \rho_\phi\}$-space, only using the real exchange rate