Micro-based estimates of heterogeneous pricing rules: The United States vs. the Euro Area[*]

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** Banco de España,

March 2010

Abstract

This paper presents US and euro area estimates for a fully heterogeneous model, in which there is a continuum of firms setting prices with a constant probability of adjustment, which may differ from firm to firm. The estimated model accurately matches the empirical distribution function of individual price durations for the US and the euro area. Incorporating these micro based pricing rules into a DSGE model, we find that nominal shocks have a greater real impact in the fully heterogeneous economy than in the standard Calvo model. We also find that nominal and real shocks bring about a reallocation of resources among sectors. Monetary policy is found to have a greater real impact in the euro area than in the United States.

JEL Codes: C40, D40, E30.

Key words: price setting, heterogeneity, DSGE, Calvo model.

1 Introduction

Recent years have seen an explosion of empirical papers documenting individual pricing behaviour.[1] Prices in euro area countries have been consistently found to be stickier than those in the US.[2] Indeed, Dhyne et al. (2006), find that the average monthly frequency of price adjustment in the euro area is 15.1%, well below that in the US (24.8%).

Figure 1: Cross sectional distribution of monthly frequencies of price change
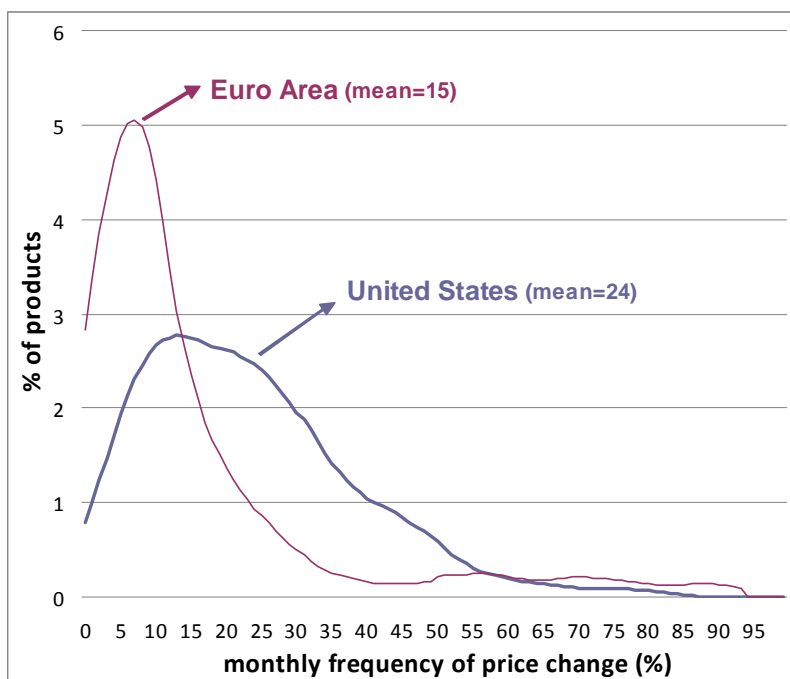
Note: 350 products for the US, 50 products for each of the 10 euro area countries. Epanechnikov kernel estimates.

Another key finding in this literature is that there are dramatic differences in the frequency of price adjustment across products, which reflect heterogeneity in underlying primitives (the different features characterizing the cost and revenue sides of firms).[3] Differences in price adjustment

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[1] Bils and Klenow (2004) is the seminal paper. Álvarez (2008) surveys world-wide evidence.

[2] This is also observed in terms of producer prices, survey data and macroeconomic estimates of the new Keynesian Phillips Curve (Álvarez et al. (2006), Galí et al. (2001)).

[3] A number of papers analyze the determining factors of the frequency of price change. It is found that this frequency is inversely related to the share of labour costs in variable costs and positively depends on the intensity of use of material inputs (Cornille and Dossche (2008) and Hoffmann and Kurz-Kim (2010)). Higher market competition and less market regulation are associated with higher frequencies of adjustment ((Lünnemann and

are observed for broad consumption categories (e.g. services prices typically are very sticky, whereas energy prices adjust very frequently) and also within these broad sectors (e.g. airline fares continuously change, whereas hairdresser prices are very sticky). In this regard, figure 1 presents the cross sectional distribution of monthly frequencies of price change using Bils and Klenow (2004) US data and Dhyne et al. (2006) euro area data. It is clearly seen that there is tremendous heterogeneity in the frequency of price adjustment across products and that the share of products with very low frequencies of price change is much higher in the euro area than in the US. But even this figure may be substantially underestimating the true extent of heterogeneity, reflecting differences in the demand and costs that individual firms face.[4]

Despite the marked heterogeneity found in micro data, most pricing models used in the macro literature make the convenient assumption of the existence of either a representative firm or of many homogeneous firms. A growing strand of research points out that this is not an innocuous assumption, but rather has important macro implications.[5]

In this paper, we allow for heterogeneity by assuming that the US and euro area economies are characterized by a continuum of firms that differ in their frequency of price adjustment. We follow Álvarez and Burriel (2010) and adopt a parsimonious setup[6] that involves estimating a few parameters, but is able to accommodate interesting features of the data, such as heterogeneity in the frequency of price adjustment, the distribution function of price durations and a declining population hazard rate. We also make the assumption that the probability that each individual firm keeps its price unchanged is constant and independent of when the last adjustment took place. This is consistent with the state dependent model of Danziger (1999) in which the probability of price change is a function of structural parameters[7] and the time dependent model by Calvo

Mathä (2010) and Álvarez and Hernando (2007))

[4]In fact, heterogeneity in the frequency of price change is also found in terms of narrowly defined products (Campbell and Eden (2007) and types of outlet (Fougère et al.(2007)).

[5]Aoki (2001) and Benigno (2004) show that heterogeneity in price durations has important implications for the design of optimal monetary policy. Carvalho (2006) and Nakamura and Steinsson (2007) show that the impact of nominal shocks is considerably higher in heterogeneous economies than in a homogeneous economy with the same average frequency of price change.

[6]Ecochard and Clayton (2000) introduced this model in the biometric literature on women fecundability.

[7]In Danziger (1999) the probability of price change increases with the uncertainty of idiosyncratic shocks and

(1983).[8]

Our assumption of a flat hazard rate at the individual level is motivated by available empirical evidence. Klenow and Krvstov (2008) find that, allowing for heterogeneity, individual hazard rates are flat[9] and Álvarez et al. (2005) reject the hypothesis of increasing hazard rates at the individual level. In contrast to Danziger (1999) and Calvo (1983), other well known price setting models present counterfactual implications in terms of individual hazard rates.[10] For instance, state dependent models, such as Dotsey et al. (1999), imply an upward sloping hazard.[11] In Taylor (1980) time dependent model firms set prices for a fixed number of periods, so that the hazard rate is one for the duration of the contract and zero for smaller durations, at odds with the data.[12] Finally, in sticky information models, as in Mankiw and Reis (2002), firms set price paths, so that in general prices change every period. This has the counterfactual implication that the frequency of price change is 1.[13]

The main results of this paper are the following: First, our parsimonious setup is able to accurately match interesting features of United States and euro area data, such as heterogeneity in the frequency of price adjustment, the distribution of price durations and a declining population hazard rate. While we assume that each individual firm has a constant hazard rate, the population hazard depends on the share of the different frequency of adjustment firms at each horizon. As the price age increases, the composition of price adjusters shifts towards the low frequency price

the trend in the money supply. It decreases with the size of menu costs and the discount rate.

[8]Woodford (2009) finds that the Calvo model can provide a fairly accurate approximation to the solution of his state dependent model.

[9]Campbell and Eden (2007) using scanner data find downward sloping hazard rates, probably reflecting brand heterogeneity.

[10]Evaluations of pricing models in terms of their implications in terms of frequencies and sizes of price change are presented in Angeloni et al. (2006), Álvarez (2008) or Klenow and Kryvtsov (2008).

[11]In Nakamura and Steinsson (2008), the hazard function is also increasing when there are no idiosyncratic shocks, while it remains steeply upward sloping in the first few periods, even with a high variance of idiosyncratic shocks.

[12]Moreover, this model is clearly inconsistent with the large observed variation in the duration of price spells for individual items (e.g. Aucremanne and Dhyne (2005) or Klenow and Kryvtsov (2008)). Taylor (1993) allows for heterogeneity, but all prices by a given price setter have the same duration. The time dependent model by Sheedy (2007) delivers an increasing hazard rate.

[13]This is also the case for extensions of this model that take into account heterogeneity in the frequency of updating the information set, as in Carvalho (2005).

adjusters, motivating the decline in the population hazard rate.[14] Second, we find that nominal and real shocks bring about a sectoral reallocation of resources, reflecting the heterogeneity in price stickiness. Flexible price setters benefit from their frequent price optimization. Third, we find that in a DSGE model nominal shocks have a greater real impact in our fully heterogeneous economy than in the standard Calvo model. Relative to the standard model, our fully heterogeneous Calvo model initially involves a faster initial response driven by high frequency price adjusters -which tends to limit the impact of nominal shocks-, but a slower subsequent response, driven by stickier price adjusters, which tends to lead to a higher impact of monetary policy. Fourth, we also find that calibrations based on sectoral frequencies of price change substantially overestimate the real impact of monetary shocks. This reflects that the sectoral Calvo model does not allow for within-sector heterogeneity. Finally, monetary policy is found to have a greater real impact in the euro area than in the United States, consistent with the higher share of sticky price setters in the euro area.

After this introduction, the structure of the paper is the following. Section 2 presents the fully heterogeneous model, which is estimated with US and euro area data in section 3. In section 4 we build a Dynamic Stochastic General Equilibrium (DSGE) model, in which we allow for the existence of a continuum of price setters and assess the impact of a number of nominal and real shocks. Conclusions are presented in section 5.

2   The fully heterogeneous model

Heterogeneity in the demand and cost conditions that firms face suggests building models with an infinite number of price setters. We make the assumption that every firm keeps its price unchanged with a probability that is independent of when the last adjustment took place. This is consistent with the menu cost model by Danziger (1999) in which the probability of price change is a function

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[14]This is a well known result in the failure time literature: the mixture of distributions with non-increasing failure rates has a decreasing failure rate (Proschan (1963). See Álvarez et al. (2005) for an application in a price setting context.

of structural parameters and also with the time dependent model by Calvo (1983). For simplicity, we term our model fully heterogeneous.

The model includes a continuum of firms that set prices with a flat hazard rate and where each firm changes its prices with a possibly different probability. The distribution across the population of the price adjustment parameter is characterised by a parsimonius density. To this end, we employ a discrete time model developed in the biometric literature on women fecundability (Ecochard and Clayton (2000)) and used in a price settting context in Álvarez and Burriel (2010). More specifically, we assume that:

(1) each individual sets prices with a constant probability of adjustment, so that the individual survival and hazard functions are given by

$$S\left(k\right) = \Pr(X > k/\theta) = \theta^{k-1} \quad k = 1, 2, 3, ...$$

$$h\left(k\right) = \left(1 - \theta\right)$$

(2) there is an infinite number of price setters, each with a different probability of no price change parameter (θ). To obtain a closed-form expression that allows estimation we assume that the cross sectional distribution of the probability of price change parameter follows a log Hougaard distribution[15].

The family of distributions proposed by Hougaard (1984, 1986) $H\left(k, \alpha, \beta, \gamma\right)$ has only 3 parameters and has the desirable property of having a simple moment generating function

$$mgf\left(k\right) = \exp\left\{-\frac{\beta}{\alpha}\left[(\gamma - k)^{\alpha} - \gamma^{\alpha}\right]\right\}$$

We assume that $\mu = -\log\theta$ follows a Hougaard distribution and accordingly denote the distribution over the price adjustment parameter θ as log Hougaard. The Hougaard family of

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[15] Álvarez and Burriel (2010) present expressions for a general density function and for a beta distribution.

distributions nests other distributions used in the literature. The positive stable distribution is obtained if $\gamma = 0$, the gamma distribution if $\alpha = 0$ and the inverse Gaussian distribution if $\alpha = 0.5$.

Taking into account that $\theta = e^{-\mu}$ yields $S(k) = \Pr(X > k/\mu) = e^{-\mu(k-1)}$, so that

$$S(k) = \int_0^1 \Pr(X > k/\mu) g(\mu) d\mu = \int_0^1 e^{-\mu(k-1)} g(\mu) d\mu = mgf[-(k-1)]$$

where mgf is the moment generating function of the distribution of μ. Substituting the moment generating function of the Hougaard family of distributions, we have the following simple expression:

$$S(k) = \exp\left\{-\frac{\beta}{\alpha}\left[(\gamma + (k-1))^\alpha - \gamma^\alpha\right]\right\}$$

3   Data and estimation

The underlying data we use correspond to the individual prices that are used by national statistical agencies to compute consumer price indices. There are several reasons which make these data particularly useful. First, the number of considered goods and services is large and samples are highly representative, since they are based on very detailed household budget surveys. Importantly, services prices -which are typically quite sticky- are included. Second, prices refer to actual transaction prices at the retail level, including indirect taxes, instead of list prices. Third, prices are collected in different types of outlets, which may follow different pricing strategies. Fourth, prices are collected in a large number of cities, thus ensuring high geographical representativity. Fifth, databases contain monthly observations tracking individual items for several years. Finally, these data sets of individual prices typically contain a huge number of price quotes, that may add up to several millions.

Other sources of microeconomic evidence on consumer pricing come from scanner or online data. Scanner data are typically collected from supermarkets, drugstores, and mass merchandisers.

These data cover a narrower set of goods than CPI datasets and exlude services, but sometimes contain additional information on quantities sold and costs and data are usually collected at a weekly frequency. Internet data have also been studied (Lünnemann and Wintr (2006), Mizuno et al. (2009)). In these datasets, products with low frequencies of price adjustment are hardly covered, but sometimes there is detailed information and the frequency of analysis may be daily.

Although CPI datasets are highly valuable, statistical confidentiality reasons place important restrictions on the issues that can be addressed by researchers. In our case, we do not have access to individual data for the US or euro area countries, excluding Spain, but only have information for the distribution function of price durations, a sufficient function to apply our methodology[16]. Our estimates are based on the distribution function of price durations for the euro area and the US. Euro area data refer to the aggregate of Austria, Belgium, Italy, France, Germany, Luxembourg and Spain[17]. All data were kindly provided by the researchers mentioned in the acknowledgements, which had access to individual price data. To obtain the distribution of the euro area, we have aggregated national distributions using Harmonized Index of Consumer Prices country weights, which reflect household consumption expenditure.

We estimate by standard maximum-likelihood methods the fully heterogeneous model for the US and the euro area.The log-likelihood function is a simple function of the survival function of price durations. Specifically, this is given by:

$$L(y; \alpha, \beta, \gamma) = \sum_{k=1}^{K} n_k \log \left[S(k; \alpha, \beta, \gamma) - S(k+1; \alpha, \beta, \gamma) \right]$$

where $S(k; \Psi)$ represents the survival funcion that depends on the parameters (α, β, γ), n_k is the number of prices that are changed after k months and K is the maximum duration.

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[16]Theoretical models have also implications in terms of sizes of price changes, so that estimation of models that simultaneously account for the observed distributions of frequency and size of price change for the US and the euro area is an interesting area for future research.

[17]There are cross country differences in terms of factors such as products covered, sample periods or geographical coverage. However, Dhyne et al. (2006) use a harmonized sample of 50 productsfind and find similar results in terms of frequencies and sizes to those found in country papers.

Table 1 presents parameter estimates for the models considered,[18] as well as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and an statistic measuring the quadratic distance between the empirical and the fitted probability mass funcion (qd):

$$qd = \sum_{k=1}^{\max k} \frac{\left(f_k^{\text{empirical}} - f_k^{\text{fitted}}\right)^2}{f_k^{\text{empirical}}}$$

Table 1: Estimation of price setting models

Model	United States						Euro area					
	a	b	g	AIC	BIC	qd	a	b	g	AIC	BIC	qd
Fully Heterogeneous [FH]	0.46	0.47	0.00	2,723,646	2,723,669	0.008	0.49	0.33	0.00	439,707	439,735	0.019
	'(0,00)	'(0,00)	'(0,00)				(0,00)	'(0,00)	(0,00)			
	theta			AIC	BIC	qd	theta			AIC	BIC	qd
Standard Calvo [C]	0.71			3,142,891	3,142,895	0.414	0.81			507,032	507,035	0.588
	(0,00)						(0,00)					
Sectoral Calvo [SC]	---			---	---	0.57	---			---	---	0.81

Note: Standard errors of estimates are reported within parentheses.

Figure 2 shows that the fully heterogeneous model very accurately matches the distribution functions of price durations in the US and the euro area, which results in low qd statistics. Moreover, it seen that the share of very flexible price setters is considerably higher in the US than in the euro area. In contrast with the fully heterogeneous model, the standard Calvo model provides a bad fit of the distribution in terms of qd and substantially underestimates the share of very flexible price setters, particularly so in the US. Figure 3 presents the hazard rates of both models. Heterogeneity in the frequency of price adjustment explains the declining population hazad rate: the population hazard depends on the share of the different frequency of adjustment firms at each horizon. As the price age increases, the composition of price adjusters shifts towards the low frequency price adjusters, motivating the decline in the population hazard rate. Hazard rates cor-

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[18] As a robustness check, we considered a functional form that allowed for individual increasing hazard rates. We did not find evidence in favour of upward sloping hazard functions. We also considered a beta distribution for the distribution of the price adjustment parameter, which lead to a worse fit. Both sets of results are available from the authors upon request.

responding to estimated fully hetereogeneous models closely match empirical ones both for price spells with short and long durations and for both economies. As expected, the standard Calvo model, which implies a flat hazard rate, has a disappointing performance. The Calvo parameter is lower in the US than in the euro area, in line with the higher average frequency of adjustment.

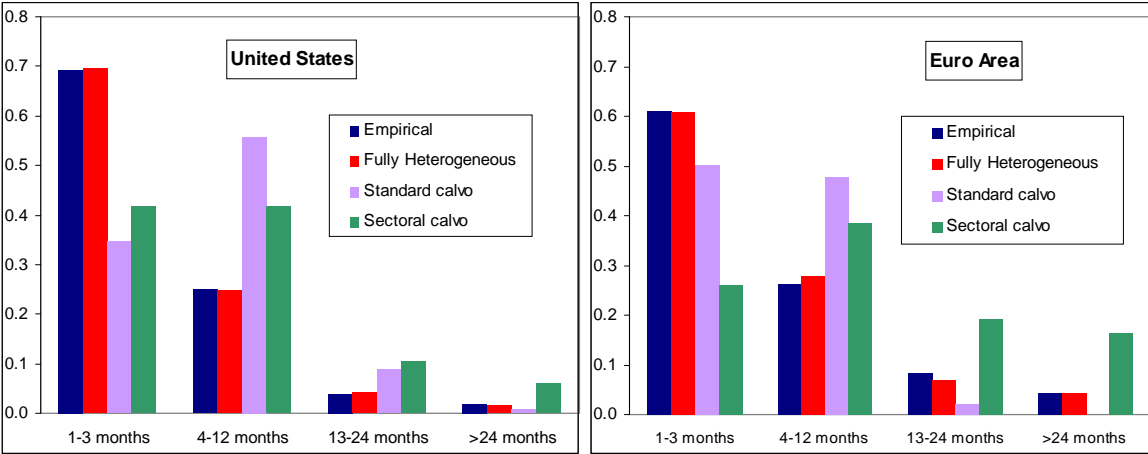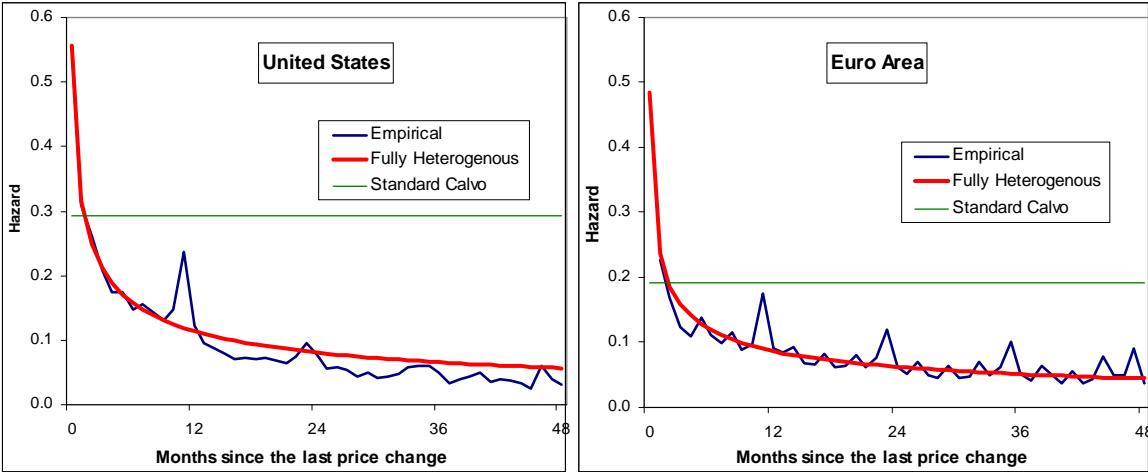**Figure 2: Empirical and model based distributions of price durations**

Figure 3: Empirical versus fitted hazard

3.1   An alternative calibration of heterogeneus pricing rules

An alternative approach to calibrate heterogeneous pricing rules is to assume that each sector follows a Calvo pricing rule with a duration equal to the sectoral empirical average duration (Carvalho (2006) or Klenow and Kryvtsov (2008)). We term this model *sectoral Calvo*. We have calibrated sectoral Calvo models using the 350 sectors in the Bils and Klenow (2004) data set and the 50 products per country in the Dhyne *et al.* (2006) database. Measures of quadratic distance of these models reported in table 1 and figure 2 clearly show that the sectoral Calvo approach implies a severely distorted distribution of price durations, thus casting doubts on its validity, in sharp contrast with the fully heterogeneous model. Indeed, the sectoral Calvo model considerably underestimates the share of prices with very short durations and clearly overestimates the fraction of prices for longer price durations[19]. These distortions reflect the existence of marked within sector heterogeneity in terms of primitives, in line with available empirical evidence.

4   The macroeconomic impact of pricing heterogeneity

In this section, we analyse the macroeconomic implications of the fully heterogeneous model by incorporating its price setting rules into an otherwise standard DSGE model[20]. We first describe the theoretical model and then evaluate the quantitative importance of appropriately dealing with heterogeneity. We find that accurately capturing heterogeneity has important macroeconomic implications and allows for sectoral reallocation of resources effects. We also compare the impact of heterogeneity in the United States and the euro area.

The theoretical model used corresponds to the canonical New Keynesian sticky price model, extended to allow for heterogeneity in price setting behaviour. In particular, aggregate demand is the result of optimal consumer choice, leading to an intertemporal IS equation, in which output

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[19]For the US, Klenow and Kryvtsov (2008) present graphical comparisons of sectoral Calvo and Golosov and Lucas (2007) models. Both models predict a substantially higher share of price with long durations than the empirical one.

[20]This model is inspired in Woodford (2003) and Carvalho (2006).

inversely depends on the expected real interest rate. The central bank sets short-run nominal interest rates according to a Taylor rule with interest rate smoothing. The price setting behaviour is similar to the standard model, except that there are k groups of intermediate firms setting prices with a flat hazard rate, each with a possibly different probability of price adjustment.

The representative consumer chooses the path of consumption (C_{it}) and hours worked in each sector and firm (L_{ikjt}) that maximizes his/her (expected discounted) utility, subject to a budget constraint

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_{it}^{1-\sigma} - 1}{1-\sigma} - \int_0^1 f(k) \int_0^1 \frac{L_{ikjt}^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} dj dk \right\}$$

$$\text{s. to: } P_t C_{it} + B_{it} = \int_0^1 f(k) \int_0^1 W_{kjt} L_{ikjt} dj dk + I_{t-1} B_{it-1} + T_t$$

where $\mathbb{E}_0$ denotes the mathematical expectation, β is a discount factor, σ is the intertemporal elasticity of substitution of consumption, $f(k)$ is the density of group k in the distribution of firms, $\frac{1}{\varphi}$ is the inverse of Frisch labor supply elasticity, P_t is the aggregate price level, B_{it} are government bonds held by households, which pay a gross nominal interest rate of I_t, W_{kjt} is the nominal wage paid per hour worked by firm j in group k and T_t is a lump-sum transfer. From the first order conditions, we obtain the standard Euler and labour supply equations

$$C_{it}^{-\sigma} = \beta \mathbb{E}_t \{ C_{it+1}^{-\sigma} \frac{I_t}{\Pi_{t+1}} \}$$

$$L_{ikjt}^{\frac{1}{\varphi}} C_{it}^{\sigma} = \frac{W_{kjt}}{P_t} \text{ for all } k, j.$$

Production occurs in 2 stages. At the bottom of the distribution chain, there are k different groups of intermediate goods producers, each composed of j monopolistically competitive firms. At the top, there is a final goods producer that puts together all the intermediate goods (Y_{kjt}) into a final good (Y_t), which is sold to consumers at price P_t. The competitive producer of final goods solves

the following maximization problem

$$\max_{Y_{kjt}} P_t Y_t - \int_0^1 P_{kjt} Y_{kjt} \, dk$$

$$\text{s.t.} \quad : \quad Y_t = \left[\int_0^1 f(k) \int_0^1 Y_{kjt}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \, dk \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where P_{kjt} is the price of intermediate good j of group k , ε is the elasticity of substitution between output varieties and $f(k)$ is the density of firms in group k. Optimal demand for intermediate goods is then a function of aggregate demand and relative prices and the aggregate price level is a weighted average of firms' prices

$$Y_{kjt} = \left(\frac{P_{kjt}}{P_t} \right)^{-\varepsilon} Y_t$$

$$P_t = \left[\int_0^1 f(k) \int_0^1 P_{kjt}^{1-\varepsilon} \, dj \, dk \right]^{\frac{1}{1-\varepsilon}}$$

Intermediate goods firms use a production function linear in the labour input (L_{kjt}) and productivity A_{kt} .[21] These firms set prices with a flat hazard rate, such that in each period a fraction $(1 - \theta_k)$ of producers in group k can change their prices, while all other firms keep unchanged the previous price. This probability of changing prices differs across groups.[22] Optimal prices (X_{kjt}) are set by solving the maximization problem.

$$\max_{X_{kjt}} \mathbb{E}_t \sum_{s=0}^{\infty} [\beta \theta_k]^s d_{t+s} \frac{Y_{t+s}^{-\sigma}}{P_{t+s}} \{ X_{kjt} - P_{kt+s} mc_{kt+s} \} Y_{kjt+s}$$

$$\text{s.to} \quad : \quad Y_{kjt} = A_{kt} L_{kjt}; \quad Y_{kjt+s} = \left(\frac{X_{kjt}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s}$$

where the real marginal cost equals $mc_{kt} = \frac{W_{kt}}{P_t A_{kt}} = \frac{L_{kt}^{\frac{1}{\varphi}} Y_t^{\sigma}}{A_{kt}}$, using the first order condition of labour supply.

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[21] Note that this problem is different for firms belonging to different sectors since the probability of price change (θ_k) and productivity (A_k) may differ across sectors.

[22] In the empirical application, the distribution over θ_k is obtained from the estimated fully heterogenous model (see Table 1).

The optimal relative price $x_{kt} = \frac{X_{kt}}{P_{kt}}$ is given by:

$$\frac{X_{kt}}{P_{kt}} = \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} [\beta\theta_k]^s d_{t+s} Y_{t+s}^{1-\sigma} \Pi_{kt,t+s} \Pi_{t,t+s}^{\varepsilon-1} \left(\frac{\varepsilon}{\varepsilon-1}\right) mc_{kt+s}}{\mathbb{E}_t \sum_{\tau=0}^{\infty} [\beta\theta_k]^s d_{t+s} Y_{t+s}^{1-\sigma} \Pi_{t,t+s}^{\varepsilon-1}}$$

where $\Pi_{kt,t+s} = \prod_{h=1}^{s} \Pi_{kt+h}$ and $\Pi_{kt} = \frac{P_{kt}}{P_{kt-1}}$ is the sectoral inflation rate. Finally, the aggregate price index of group k can be expressed as a function of the past aggregate price index and the new optimal price:

$$P_{kt}^{1-\varepsilon} = (1-\theta_k) X_{kt}^{1-\varepsilon} + \theta_k P_{kt-1}^{1-\varepsilon}$$

The economy wide price index is a frequency weighted average of the prices of all k groups

$$P_t^{1-\varepsilon} = \int_0^1 f(k) P_{kt}^{1-\varepsilon} dk$$

The equilibrium of this model, in log-linearized terms, is composed of $3k+3$ equations. That is, the 3 equations of the standard model -aggregate NKPC, the IS curve and the Taylor rule-

Aggregate NKPC: $\widehat{\Pi}_t = \beta\mathbb{E}_t\widehat{\Pi}_{t+1} + \sigma\psi\widehat{Y}_t + \frac{1}{\varphi} \int_0^1 f(k) \frac{(1-\theta_k)(1-\beta\theta_k)}{\theta_k} \left(\widehat{Y}_{kt} - (1+\varphi)\widehat{A}_{kt}\right) dk$

IS curve: $\widehat{Y}_t = -\frac{1}{\sigma}\mathbb{E}_t\left(\widehat{I}_t - \widehat{\Pi}_{t+1}\right) + \mathbb{E}_t\widehat{Y}_{t+1}$

Taylor rule: $\widehat{I}_t = \gamma_I\widehat{I}_{t-1} + (1-\gamma_I)\left(\gamma_\Pi\widehat{\Pi}_t + \gamma_y\widehat{Y}_t\right) + \widehat{m}_t$

plus 3 equations for each of the k groups determining their inflation and output gaps[23]

k Sectoral NKPCs: $\widehat{\Pi}_{kt} = \beta\mathbb{E}_t\widehat{\Pi}_{kt+1} + \frac{(1-\theta_k)(1-\beta\theta_k)}{\theta_k}\left[\sigma\widehat{Y}_t + \frac{1}{\varphi}\left(\widehat{Y}_{kt} - (1+\varphi)\widehat{A}_{kt}\right)\right]$

where: $\widehat{\Pi}_{kt} = \widehat{\Pi}_t + \widehat{p}_{kt} - \widehat{p}_{kt-1}$

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[23]We assume a zero steady state level of inflation, as is standard in the NKPC literature. An appendix with the full derivation of the non-linear model is available from the authors upon request.

$$\text{k sectoral output demands: } \widehat{Y}_{kt} = \widehat{Y}_t - \varepsilon \widehat{p}_{kt}$$

and the shocks processes

$$\text{monetary:} \quad \widehat{m}_t = \rho_m \widehat{m}_{t-1} + \sigma_m \varepsilon_{m,t} \text{ where } \varepsilon_{m,t} \sim \mathcal{N}(0,1),$$

$$\text{productivity} \quad : \quad \widehat{A}_{kt} = \rho_A \widehat{A}_{kt-1} + \sigma_A \varepsilon_{Ak,t} \text{ where } \varepsilon_{A,t} \sim \mathcal{N}(0,1)$$

where $\psi = \int_0^1 f(k) \frac{(1-\theta_k)[1-\beta\theta_k]}{\theta_k} dk$, I_t is the nominal interest rate, m_t represents a monetary shock and $p_{kt} = \frac{p_{kt}}{P_{kt}}$ is the relative price index of group k. We redefine the variables as log-deviations from steady state, i.e. $\widehat{var}_t = \log var_t - \log var$, where var is the steady-state value for the variable var_t.

The calibration used is very standard (see Carvalho (2006)), except for the price setting rules, which correspond to the estimated fully heterogeneous model (see first row of table 1). Note that the time unit in the model is 1 month. Thus, the Taylor rule coefficient on lagged nominal interest rate is γ_I=0.91, on the inflation rate is γ_Π=1.53 and on the output gap is γ_y=0.93/12[24], the consumer's discount rate β=0.9975, to have an (annualized) steady state nominal interest rate of 3%, the elasticity of labour supply φ=0.5, the elasticity of substitution between intermediate varieties ε=11 and the intertemporal elasticity of consumption σ=1.[25] Except for price setting rules, which are estimated, we use the same vector of parameter values for the euro area and the US. Finally, the persistence of shocks is calibrated considering: $\rho_m = 0$, $\rho_A = 0.9$.

4.1 Impulse responses to aggregate and sectoral shocks

To better understand the fully heterogeneous model, we start by analyzing the impulse response functions of the US and euro area model economies after a transitory productivity shock and a temporary rise in interest rates. Moreover, we also consider sectoral productivity shocks to specific

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[24]This is divided by 12 to correct for the fact that the estimates in the literature are based on annualized inflation and interest rates (Rudebusch (2002)).

[25]We have also considered an alternative calibration with lower real rigidities ($\varphi = 0.5$ and $\varepsilon = 5$). Results are qualitatively similar.

groups of price setters. To highlight the importance of price setting heterogeneity, we decompose the responses of inflation and output (black lines in figures 4 and 5) into the contributions of three types of firms: flexible price setters - those with average price durations less than 3 months (light purple bars)-, intermediate price setters -those with average price durations between 3 months and a year (red bars)- and sticky price setters -those with average price durations over a year (blue bars).[26]

The aggregate behaviour of US and euro area economies after an increase in aggregate productivity is fairly standard (left hand side panel of figure 4). Higher productivity reduces production costs. Lower expected real marginal costs trigger price falls, which result in lower inflation. Price developments lead the central bank to reduce interest rates, despite the increase in output, given the relative weights of inflation and output in the Taylor rule. On impact, the expected real interest rate is positive, which helps further expand output. The shock also brings about interesting sectoral resource reallocation effects. The most flexible price setters are able to quickly reoptimize their prices, adjusting them sizably downwards and expanding significantly their production. In contrast, firms with stickiest prices can hardly change them on impact and, as a result, see their relative price increase, suffering substantial output losses. That is, the most flexible firms adjust prices aggressively, so as to gain market share at the expense of the more rigid ones. This shock also exemplifies what Carvalho (2006) termed the *heterogeneity effect*. After a heterogeneous economy is hit by a shock, aggregate variables are initially mostly driven by the more flexible firms, which carry out most of the price changes. As time passes by, aggregate variables are mostly driven by stickier firms, so the speed of adjustment slows down through time.

The impact of a 25 basis points rise in interest rates on aggregate inflation and output is also quite standard (right hand side panel of figure 4). On impact, the increase in the nominal cost of borrowing results in an increase in the expected real interest rate, given the sticky nature of prices, which slows down economic activity, helping reduce aggregate inflation. As time passes by, the fall

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[26]The calibrated fully heterogeneous model considers the individual behaviour of 500 types of firms. For expositional purposes only, we aggregate the behaviour of the 500 types into three categories.

in inflation and output diminishes, as nominal and real interest rates return to their steady state. Our model adds to the standard analysis the possibility of resource reallocation among sectors following a shock, due to heterogeneity in price stickiness. The most flexible price setters are able to quickly reoptimise their prices, which leads them to cut them substantially and expand their output, at the expense of the rest of firms in the economy. In sharp contrast, sticky price setters can hardly change their prices on impact, with the subsequent increase in their relative price and loss of output. As time passes by, the relative disadvantage of sticky price setters disappears.

The differences between the impulse responses of the US and euro area are entirely driven by the fact that prices in the United States are more flexible on average, with a larger (smaller) share of more flexible (rigid) price setters, since our calibration for the rest of parameters in the model is identical. On impact, productivity shocks lead to a greater fall of inflation and a larger increase in output in the US relative to the euro area, which forces the monetary authority to lower interest rates to a greater extent in the US. In fact, figure 4 shows that the more flexible price setters in the US cut their prices by a larger amount, contributing to a greater reduction of inflation and to a larger increase in output. However, as time passes by, the larger share of rigid firms in the euro area generate a greater heterogeneity effect and make aggregate variables more persistent. As a consequence, inflation and output in the euro area return at a slower pace to the steady state.

On the other hand, a rise in nominal interest rates also has a stronger impact on US inflation, but a smaller effect on output in comparison with the euro area. First of all, the lower persistence of prices in the US requires a greater shock to the Taylor rule to achieve the same rise of interest rates in both economies. This is because, as before, following the shock a substantially larger share of US firms reset prices and optimally decide to lower them and to a larger extent, reducing inflation by less. As a consequence, the US monetary authority does not need to increase rates as much as the one of the euro area. Again, the stronger heterogeneity effect in the euro area slows down the return to the steady state of inflation and output, which is reflected in the higher persistence of euro area nominal interest rates.

Overall, euro area monetary policy has a greater leverage on the real economy in the short run than US monetary policy, reflecting the fact that the higher degree of price stickiness in the euro area requires smaller, but more persistent interest rate changes.

Figure 4: Impulse response functions to aggregate shocks.

Aggregate productivity shock   25 basis points interest rise
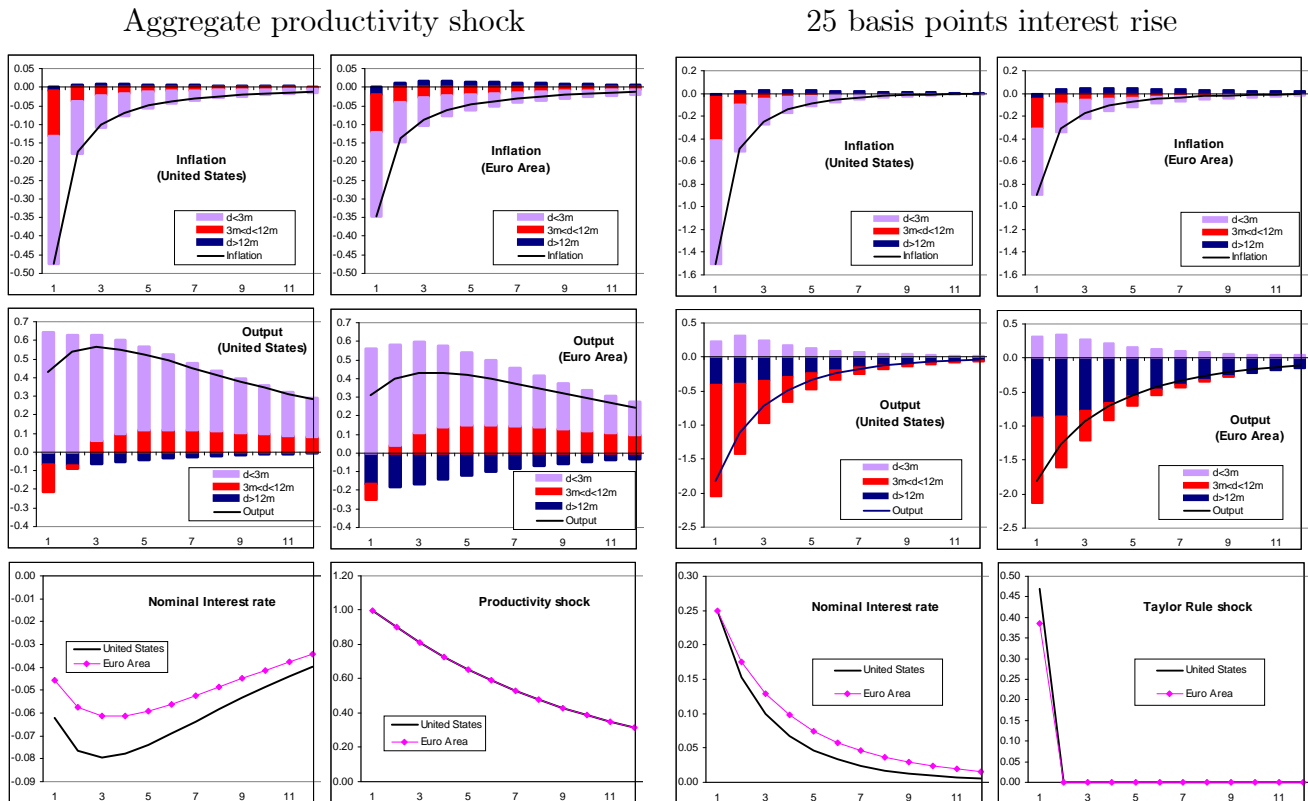
An advantage of our fully heterogeneous model is that it also allows one to study the impact of sectoral shocks affecting different types of price setters. Figure 5 compares a positive shock to the productivity of the most flexible prices setters -those with an average price duration lower than 3 months -, with an equivalent shock to the stickiest ones -those with average price duration higher than a year. Both shocks lead to price falls and an output increase of the firms that experience a transitory improvement in productivity. This impact is greater when the shock affects the most flexible firms for two reasons. First, there are more flexible price setters than

rigid ones in these economies. Second, after a rise in their productivity, flexible firms are able to adjust their prices much quicker and gain more market share from their competitors, than when the sectoral productivity shock affects only the stickiest firms, which need substantial time to reoptimise. However, the impact of the shocks on the rest of firms is quite different. When the sectoral productivity shock affects the most flexible firms, intermediate and sticky price setters increase their prices and incur output losses, whereas when the sectoral shock affects the stickiest firms, the flexible price setters are also able to cut down their prices. The reason is that flexible firms can adjust prices now, so as to reduce output losses, because they know that they will be able to change them again in the near future. In contrast, sticky price setters are more constrained on the frequency with which they carry out price changes, so when they decide to adjust prices the expected gain has to be large enough to compensate them from the fact that they are likely to remain unchanged for a protracted period.

Figure 5: Impulse response functions to sectoral productivity shocks.

Flexible prices (average price duration<3 months)   Stickier prices (average price duration>12 months)
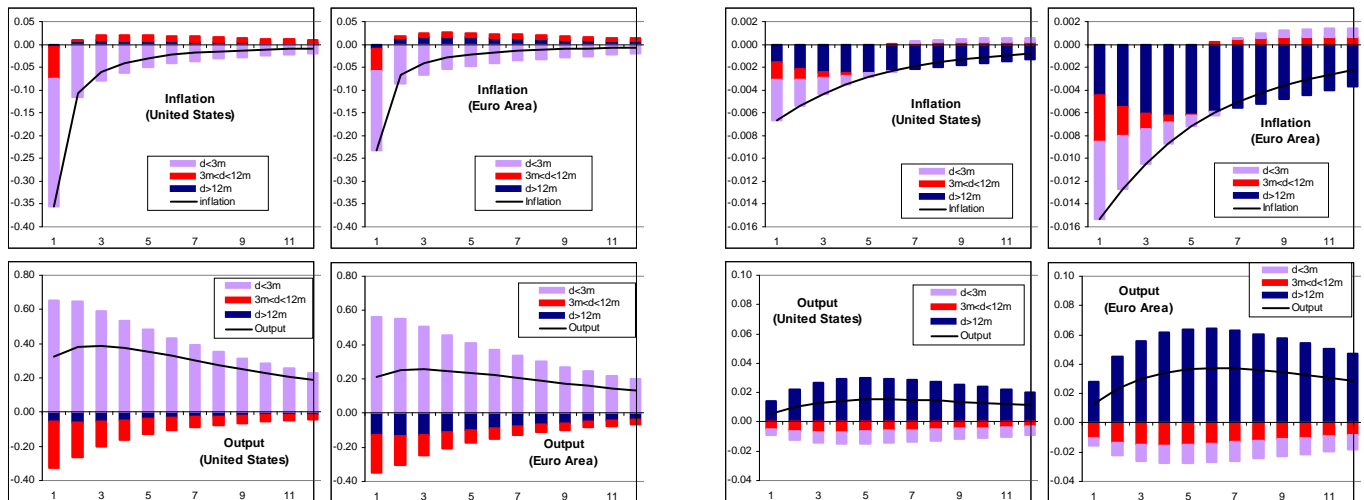
The differences between the responses of the US and euro area after sectoral productivity shocks, as is the case for aggregate shocks, are completely driven by the different sizes of the two groups of firms in each area. The US is a more flexible economy, with a higher share of flexible price

setters and lower of the stickiest ones. Thus, the rise in output and the fall in inflation is higher after the shock to the flexible firms and lower after the one to more rigid firms.

4.2   Comparison with other macro models

Figure 6 compares the impulse response functions of a 25 basis points transitory nominal interest rate rise using the estimated fully heterogeneous [FH] (red thick line) and standard Calvo (green thin line) models and table 3 presents the initial and cumulative impacts on output and the inflation rate. Qualitative responses are the same in the US and the euro area, although there are important quantitative differences.

Impulse responses show that the FH economy has a higher persistence than the standard Calvo one, in the sense that following a shock it takes output longer to return to its steady state than in the Calvo one. On cumulative terms, output falls to a larger extent, while inflation falls by less and returns quicker to its steady state in FH. On impact, however, output decreases to a lower extent in the Calvo economy. This reflects the fact that the heterogenous economy includes a larger number of very flexible price setters than the Calvo one.[27] These very flexible price setters are able to quickly reoptimize prices after the shock, so that the demand for their product is not greatly affected. In addition, estimated FH models imply a higher share of intermediate and sticky firms than the standard Calvo model. These firms take longer to reoptimize their prices and therefore are penalized to a larger extent in terms of lost demand. This simply reflects Carvalho (2006) *heterogeneity effect*

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[27] In fact, in the heterogenous economy around 60-70% of firms change their prices every quarter, whereas only 30-40% in the standard Calvo case.

Figure 6: Impulse response functions to a monetary shock: Fully heterogeneous vs Calvo and sectoral Calvo models.
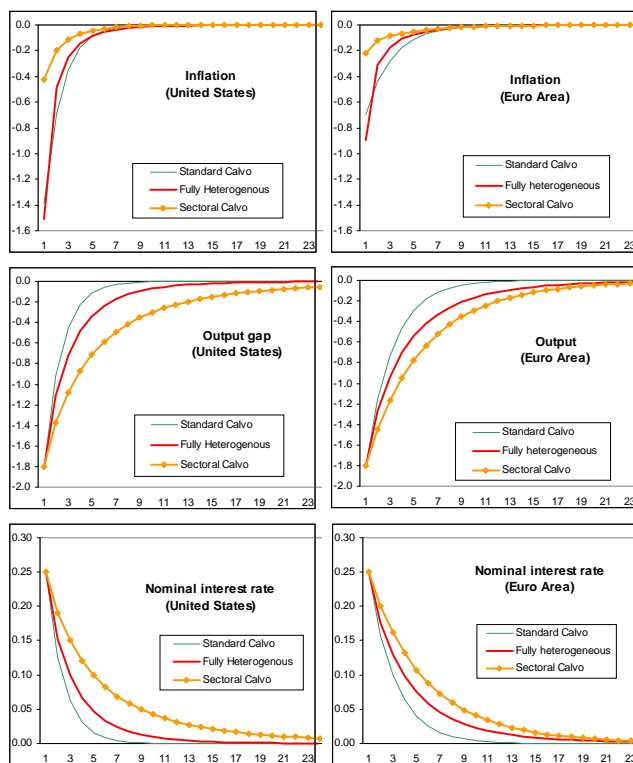

Table 2: Comparison of impulse response functions across price setting models

	Output		**Inflation**		**Sacrifice ratio**	
United States	Initial	Accumulated	Initial	Accumulated	Initial	Accumulated
Standard Calvo [C]	-1.8	-5.0	-0.7	-1.9	2.6	2.6
Fully heterogeneous [FH]	-1.8	-7.5	-0.9	-1.7	2.0	4.3
Sectoral Calvo [SC]	-1.8	-9.8	-0.2	-0.7	8.3	13.7
Euro Area	Initial	Accumulated	Initial	Accumulated	Initial	Accumulated
Standard Calvo [C]	-1.8	-3.7	-1.4	-2.8	1.3	1.3
Fully heterogeneous [FH]	-1.8	-5.4	-1.5	-2.6	1.2	2.1
Sectoral Calvo [SC]	-1.8	-10.3	-0.4	-0.9	4.2	12.1

25 bp interest rate rise

Interestingly, the real impact of monetary shocks is higher in the euro area than in the US (table 4), reflecting stickier prices in the euro area. Prices in the euro area not only remain unchanged for longer periods of time, but also their distribution is more spread out. As seen in figure 2, in the euro area there is a smaller share of firms with very flexible prices than in the US, whereas there is a higher share of firms with rigid prices in the euro area than in the US. The accumulated impact on the output gap is roughly 50% higher in the fully heterogeneous model than in the standard Calvo and the sacrifice ratio is also considerably higher.

We also consider a calibration with price setting according to the sectoral Calvo model of section 3.1 (yellow line with diamonds). In quantitative terms, there are sizable differences between the FH model and the sectoral Calvo model in terms of the response of inflation and the output gap, so quantitative analyses of the impact of monetary policy in the euro area and the US are sounder using the FH model. The sectoral Calvo model substantially overestimates the real impact of monetary shocks. This mostly reflects the fact that the sectoral Calvo model substantially underestimates the share of very flexible price setters. In fact, output changes considerably more, whereas inflation changes less in the sectoral Calvo model than in FH. In particular, in the euro area, the cumulative impact of an interest rate shock on the output gap is roughly twice in the sectoral Calvo model than in the FH model and slightly less so in the US. In terms of the sacrifice ratio, the accumulated impact of a monetary shock in the sectoral Calvo model is about 6 times larger than in the FH model in the euro area and about 3 times in the euro area.

4.3 Core versus non-core inflation

Central banks typically pay substantial attention to underlying or core inflation measures. It is widely recognised that headline inflation is inherently noisy and that some measures are needed to obtain a clearer picture of underlying inflationary pressures[28]. Widely used statistical measures include the CPI ex. food and energy, trimmed means, weighted medians or trend measures. Other methods, like structural VARs, also consider information on additional variables, such as GDP, to

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[28]Mankikar and Paisley (2004) survey this literature.

derive a core measure.

A different approach is suggested by monetary policy considerations in a framework in which there is heterogeneity in pricing behaviour. Aoki (2001) shows that stabilising the headline CPI rate is not the best available policy (see also Woodford(2003) or Benigno (2004)). Stabilising a measure of core inflation in which attention is focussed on the stickier price sector is the best policy given that it leads to the same allocation of resources as in a fully flexible economy in Aoki's (2001) setup.

Figure 7: Impulse response for different definitions of core inflation.
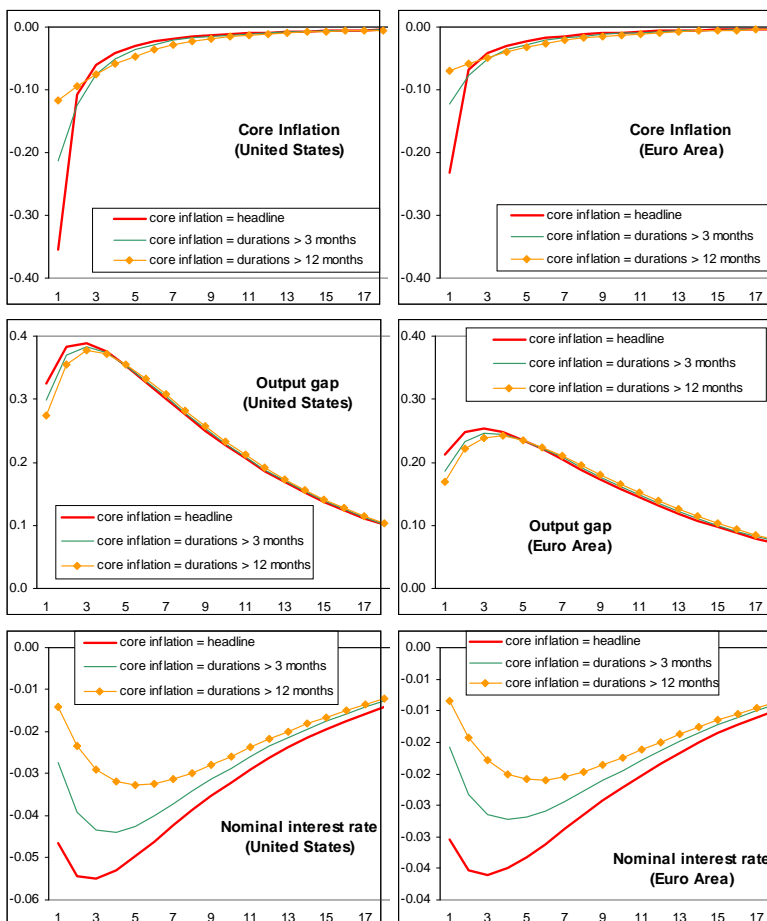
We model a transitory relative price shock by considering a temporary mark-up rise of the most flexible firms in the economy. Figure 7 shows the impulse responses to this shock under different

assumptions on central bank behaviour. It is considered that the central bank targets headline inflation (pink line) and 2 core inflation measures corresponding to price developments of firms that on average reset prices more than every 3 months (light blue line) or more than every 12 months (dark blue line). As expected, the central bank that targets headline inflation needs to increase interest rates to a considerably larger extent, with the corresponding dampening effect on output, than the central bank that focusses on core inflation. Under the assumption that the central bank mechanically follows a Taylor rule, targeting headline inflation is expected to generate higher volatility in interest rates and the output gap. than core inflation targeting.

5   Concluding remarks

Most pricing models used in the literature make the convenient assumption that economies are peopled by homogeneous firms. Recent research has shown that heterogeneity in price adjustment distorts the real impact of monetary shocks, stressing the need to properly account for differences in firm pricing behaviour. Furthermore, it has been widely documented that the frequency of price changes greatly varies both across and within sectors, reflecting a number of different explanatory factors.

Introducing the marked heterogeneity present in real data into tractable macro models poses some challenges. The approach followed in this paper is to carefully model observed differences in the frequency of price adjustment by assuming that there is a continuum of firms, which only differ in how often they adjust prices. We find that standard approaches of using calibrations based on sectoral frequencies of adjustement substantially overestimate the real impact of monetary shocks and distort the distribution of price durations. In contrast, our fully heterogeneous model accurately matches the distribution of price durations found in euro area and US data and account for a declining population hazard rate. Our model also captures interesting sectoral reallocation of resources following shocks. An alternative approach followed in some recent work within the framework of state dependent models is to only partially account for heterogeneity in pricing

behaviour and use calibrated instead of estimated models. We feel that there are important gains to be made by merging these two approaches. In our view, a fruitful area of future research is to build and estimate state dependent models which carefully deal with the marked heterogeneity present in micro data both in terms of the frequency of price adjustment and the size of price changes and are able to account for flat hazard rates at the individual level.[29]

Incorporating micro-based estimated models for the US and the euro area into a DSGE model, we find that monetary policy shocks in the euro area have a stronger real impact. Euro area monetary policy has a greater leverage on the real economy in the short run than US monetary policy reflecting the fact that prices are stickier in the euro area, but also that the share of low frequency price adjusters is much higher than in the US. There is more heterogeneity in the frequency of price adjustment across euro area countries and regions than across US States. This is likely to be related to a number of factors, including differences in consumption patterns, the extent of competition or the regulatory regime. Cultural differences across euro area countries and regions are unlikely to diminish in the near future, but the growing importance of common European legislation and harmonized measures to increase competition in some sectors are likely to reduce the gap between the effectiveness of monetary policy in the euro area and the US.

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[29]Extending Danziger(1999) to account for heterogeneity is on our research agenda.

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