The Business Cycle Implications of Banks' Maturity Transformation*

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Abstract

This paper develops a DSGE model in which banks use short term deposits to provide firms with long-term credit. The demand for long-term credit arises because firms must borrow in order to finance their capital stock which they only adjust at infrequent intervals. We show that the presence of maturity transformation in the banking sector has real effects on business cycles. In particular, maturity transformation may reduce or amplify the endogenous propagation of shocks in the economy and generate a *credit maturity attenuator* or a *credit maturity accelerator*.

Keywords: Banks, DSGE model, Financial frictions, Firm heterogeneity, Maturity transformation

JEL: E32, E44, E22, G21.

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1 Introduction

The seminal contributions by Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999) show how financial frictions augment the propagation of shocks in otherwise standard real business cycle (RBC) models.[1] This well-known financial accelerator effect is derived without explicitly modelling the behavior of a banking sector and a growing literature has therefore incorporated this sector into a general equilibrium framework.[2] With a few exceptions, in this recent literature banks are assumed to receive one-period deposits which are instantaneously passed on to firms as one-period credit. Hence, most of the papers in this literature do not address a key activity of the banking sector, namely the maturity transformation of short-term deposits into long-term credit.

The aim of this paper is to examine how banks' maturity transformation affects business cycle dynamics. Our main contribution is to show how maturity transformation in the banking sector can be introduced in an otherwise standard Dynamic Stochastic General Equilibrium (DSGE) model, such as the models by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). We then use a simple RBC model extended with banks that transform short-term deposits into long-term credit in order to study the quantitative implications of maturity transformation for business cycles.

Some implications of maturity transformation have been studied outside a general equilibrium framework. For instance, Flannery and James (1984), Vourougou (1990) and Akella and Greenbaum (1992) document that asset prices of banks with a large maturity mismatch in their balance sheets react more to unanticipated interest rate changes than asset prices of banks with a small maturity mismatch. Additionally, the papers by Gambacorta and Mistrulli (2004) and den Heuvel (2006) argue that banks' maturity transformation also affects the transmission mechanism of a monetary policy shock. In our context, however, a general equilibrium framework is necessary because we are interested not only in explaining how long-term credit affects the economy but also in the important feedback effects from the rest of the economy onto the behavior of banks.

Maturity transformation has to our knowledge not been studied in a general equilibrium setting, although long-term financial contracts have been examined by Gertler (1992) and Smith and Wang (2006). This may partly be explained by the fact that introducing long-term credit and maturity transformation in a general equilibrium framework is quite challenging for at least three reasons. Firstly, one needs to explain why firms demand long-term credit. Secondly, banks' portfolios of outstanding loans at a given point in time are difficult to keep track of in the presence of long-term credit. Finally, and related to the second point, model aggregation is often very difficult or simply infeasible when banks provide long-term credit.

[1] See also Berger and Udell (1992); Peek and Rosengren (2000); Hoggarth, Reis, and Saporta (2002); Dell'Ariccia, Detragiache, and Rajan (2008); Chari, Christiano, and Kehoe (2008); Campello, Graham, and Harvey (2009) for a discussion of the real impact of financial shocks.

[2] See for instance Chen (2001), Aikman and Paustian (2006), Goodfriend and McCallum (2007), Teranishi (2008), Gertler and Karadi (2009), Gertler and Kiyotaki (2009), and Gerali, Neri, Sessa, and Signoretti (2009).

The framework we propose overcomes these three difficulties and remains conveniently tractable. Our novel assumption is to consider the case where, in each period, firms face a constant probability α_k of not being able to adjust their capital stocks. The capital level of firms that cannot adjust is assumed to slowly depreciate over time. This setup generates a demand for long-term credit when we impose the standard assumption that firms need to borrow in order to finance their capital stock. That is, firms require a given amount of credit for potentially many periods because they may be unable to adjust their capital levels for many periods in the future.

Interestingly, our setup with infrequent capital adjustments implies heterogeneity at the firm level. In particular, the dynamics of individual firms' capital in our model is in line with the main stylized fact that the literature on non-convex investment adjustment costs tries to explain, i.e. that firms usually invest in a lumpy fashion (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006). We show, however, that for a wide class of DSGE models without a banking sector the dynamics of *aggregate* variables is unchanged relative to the standard case where firms adjust capital in every period. This result relies on firms' standard Cobb-Doublas production function which implies that the scale of each individual firm is irrelevant for all aggregate quantities and prices. This is a very important result because it shows that the constraint we impose on firms' ability to adjust capital does not affect the aggregate properties of many existing DSGE models. We refer to this result as the '*irrelevance of infrequent capital adjustments*'.

Our next step is to introduce a banking sector into the model. We specify the behavior of banks along the lines suggested by Gertler and Karadi (2009) and Gertler and Kiyotaki (2009). That is, banks receive short-term deposits from the household sector and face an agency problem in the relationship with households. As a result, there is a positive relation between wealth in the banking sector and the amount of credit banks provide. Differently from Gertler and Karadi (2009) and Gertler and Kiyotaki (2009), in our setup banks' assets consist of long-term credit contracts supplied to firms. As we match the life of the credit contracts to the number of periods the firm is not allowed to adjust capital, the average life of banks' assets in the economy as a whole is given by $\mathcal{D} \equiv 1/(1-\alpha_k)$. This implies that, in case $\alpha_k > 0$, banks face a maturity transformation problem because they use short term deposits (and accumulated wealth) to fund the provision of long-term credit. The standard case of no maturity transformation in the banking sector is recovered when $\alpha_k = 0$ (or equivalently when $\mathcal{D} = 1$).

Our simple RBC model shows that introducing maturity transformation in the banking sector has real effects on business cycle dynamics.[3] This is because the degree of maturity transformation in the economy $\mathcal{D}$ is negatively related to the fraction of banks' revenues exposed to interest rate risk. For example, as $\mathcal{D}$ increases the fraction of banks' revenues composed of interest payments negotiated in the past increases, while the fraction linked to newly signed contracts decreases. Since contracts negotiated in the past are unaffected by

[3] This also means that the irrelevance result of infrequent capital adjustments no longer holds.

changes in the current interest rate, banks' revenues become less exposed to interest rate risk as $\mathcal{D}$ increases. This in turn has macroeconomic implications because banks' revenues (and profits) affect the aggregate supply of credit that is used by firms to fund their purchases of capital. Hence, the dynamics of capital and therefore the whole macroeconomic equilibrium is affected by $\mathcal{D}$.

We finally use the derived model to examine the quantitative implications of maturity transformation for business cycles. In particular, we consider two sources of economic fluctuations: the first is a standard shock to technology, whereas the second is a shock to the confidence in the banking sector. Our analysis shows that changes in $\mathcal{D}$ have ambiguous effects on the endogenous amplification and propagation of shocks in the economy. In other words, depending on the type of shock that hits the economy and the level of $\mathcal{D}$, maturity transformation may either increase or decrease amplification and propagation.

For instance, the effects of a technology shock are less pronounced as $\mathcal{D}$ increases, and maturity transformation therefore generates what we call a *credit maturity attenuator* effect. To understand why, consider the case of a negative shock to technology. In this case, a fall in the marginal product of capital induces a reduction in the interest rate on banks' loans. If there is no maturity transformation in the economy ($\mathcal{D} = 1$) all banks' assets are immediately renegotiated at this lower rate and therefore banks' profits fall. In the case with maturity transformation ($\mathcal{D} > 1$), on the other hand, only a fraction of banks' assets is immediately renegotiated and therefore the fall in banks' profits is less pronounced than in case $\mathcal{D} = 1$. As a result, maturity transformation insulates the economy following a technology shock.

In case confidence in the banking sector unexpectedly drops, households reduce the amount deposited in banks which, in turn, suffer a shortage of funds for loans. As a consequence we observe a protracted fall in investment, consumption, and output. Banks' profits are hit not only because the amount of loans decrease, but also because the interest rate on banks' loans falls (due to a fall in labor that makes capital less productive) and the price of capital (Tobin's q) drops. Increasing $\mathcal{D}$ has two implications for the behavior of the economy following this shock. First, as before, a smaller fraction of banks' assets is immediately hit by the fall in the loans rate which in turn tends to attenuate the reduction in banks' profits. At the same time, however, the fall in the price of capital is more pronounced, causing a negative effect on banks' profits. For high enough values of $\mathcal{D}$, the second effect outweighs the first and increasing the degree of maturity transformation causes a more pronounced reduction in banks profits following the shock. In this case maturity transformation amplifies the economic cycle and generates a *credit maturity accelerator* effect.

The remainder of the paper is structured as follows. In section 2 we extend the simple RBC model with infrequent capital adjustments. We then show that this extension has firm-level effects but does not affect the aggregate dynamics in a wide class of DSGE models. We then extend a simple RBC model with infrequent capital adjustments with a banking sector that performs maturity transformation in section 3. Business cycle implications of maturity transformation are examined in section 4, and section 5 concludes.

2 A Standard RBC Model with Infrequent Capital Adjustments

The aim of this section is to describe how a standard real business cycle (RBC) model can be extended to incorporate the idea that firms do not optimally choose capital in every period. An important result is to show that this extension does not affect the dynamics of any aggregate variable in the model. This result holds under very weak assumptions and therefore generalizes to a wide class of DSGE models.

We proceed as follows: sections 2.1 to 2.3 describe how we modify the standard RBC model. Section 2.4 proves and discusses the irrelevance of this modification for the model dynamics at the aggregate level. Detailed derivations of the model can be found in Appendix A.

2.1 Households

Consider a representative household which consumes c_t, provides labor h_t, and accumulates capital k_t^s. The contingency plans for c_t, h_t, and k_t^s are determined by maximizing

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{\left(c_{t+j} - b\, c_{t+j-1}\right)^{1-\phi_0}}{1-\phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1+\phi_1} \right) \tag{1}$$

subject to

$$c_t + i_t = h_t\, w_t + R_t^k\, k_t^s \tag{2}$$

$$k_{t+1}^s = (1-\delta)\, k_t^s + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right] \tag{3}$$

and the usual no-Ponzi game condition. The left-hand side of equation (2) lists expenditures on consumption and investment i_t, and the right-hand side lists the sources of income. We let w_t denote the real wage and R_t^k is the real rental rate of capital. As in Smets and Wouters (2003), household preferences are assumed to display internal habits with intensity parameter b. The capital depreciation rate δ in equation (3) is constant and $S\left(i_t \,/\, i_{t-1}\right) i_t$ captures adjustment costs to investment as in Christiano, Eichenbaum, and Evans (2005) where $S(1) = 0$, $S'(1) = 0$ and $\kappa \equiv S''(1) > 0$.

2.2 Firms

We assume a continuum of firms indexed by $i \in [0,1]$, which are perfectly competitive and maximize the expected discounted value of future profits. Since firms are owned by the household, profits are discounted using the household's stochastic discount factor $\beta^j \lambda_{t+j}/\lambda_t$, where λ_t is the marginal utility of consumption.

Profit in each period is given by the difference between firms' output and costs, where costs are composed of capital rental fees $R_t^k\, k_{i,t}$ and the wage bill $w_t\, h_{i,t}$. Both costs are paid at the end of the period. We assume that output is produced from capital and labor according to a standard Cobb-Douglas production function

$$y_{i,t} = a_t k_{i,t}^{\theta} h_{i,t}^{1-\theta}.$$

The aggregate level of productivity a_t is assumed to evolve according to

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a, \tag{4}$$

where $\varepsilon_t^a \sim \mathcal{NID}(0, \sigma_a)$ and $\rho_a \in (-1, 1)$.

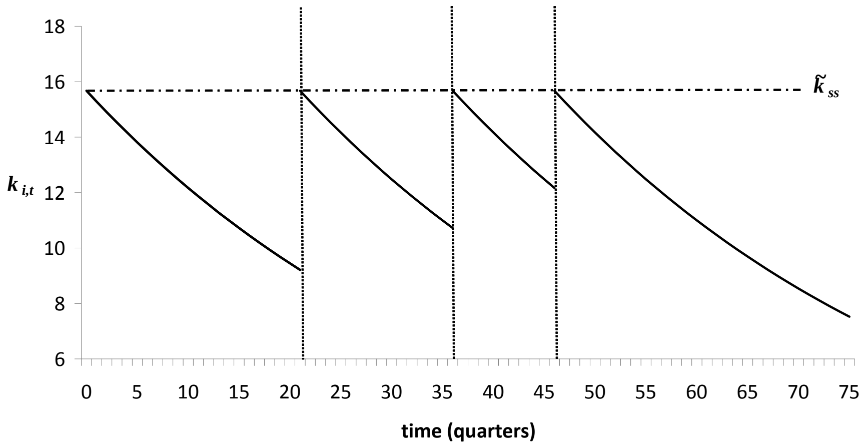
The setup has so far been completely standard. We now depart from the typical RBC setup by assuming that firms can only choose their optimal level of capital with probability $1 - \alpha_k$, where $\alpha_k \in [0, 1[$. This probability is assumed to be the same for all firms and across time. The capital level for firms that cannot reoptimize is assumed to depreciate by the rate δ over time. All firms are, however, allowed to choose labor in every period as in the standard RBC model.

One way of rationalizing the restriction we impose on firms' ability to adjust capital is as follows. The decision of a firm to purchase a new machine or to setup a new plant usually involves large fixed costs. This could be costs related to gathering information, decision making, and training the workforce. We do not attempt to model the exact nature of these costs and how firms choose which period to adjust capital, but our setup still captures the main macroeconomic implications of firms' infrequent changes in capital.

To see how this assumption affects the level of capital for an individual firm i, consider the example depicted in Figure 1. To simplify the analysis this figure represents an economy in steady state. The downward slopping lines denote the levels of capital actually used in production by firm i over time. The dashed horizontal line represents the optimal choice of capital (in steady state) for firms that are able to optimize ($\widetilde{k}_{ss}$), whereas vertical lines mark the periods in which the firm is allowed to reoptimize capital. The firm is not allowed to reoptimize capital from period zero until the first vertical line and simply sees its capital depreciate. Once the vertical line is reached the firm is allowed to adjust capital and chooses the level $\widetilde{k}_{ss}$. In the following periods capital depreciates again until the firm is allowed to adjust capital again. Note that the vertical lines are not equidistant reflecting our assumption of random capital adjustment dates.

It is also important to note that the dynamics of capital at the firm level implied by our assumption is in line with the empirical literature on non-convex investment adjustment costs (Caballero and Engel, 1999; Cooper and Haltiwanger, 2006). This literature uses micro

Figure 1: Infrequent Capital Adjustments - Dynamics at the Firm Level



Notes: Bold lines represent the capital of firm i. Vertical lines mark the periods in which the firm is allowed to reoptimize capital. The dotted horizontal line represents the steady state level of $\widetilde{k}_t$.

data to document that firms usually invest in a lumpy fashion, i.e. there are many periods of investment inaction followed by spikes in the level of investment and capital.

Our assumption on firms' ability to adjust their capital level implies that there in every period are two groups of firms: i) a fraction $1-\alpha_k$ which potentially change their level of capital and ii) the remaining fraction α_k which produce using the depreciated capital chosen in the past. All reoptimizing firms choose the same level of capital due to absence of cross-sectional heterogeneity. We denote this capital level by $\widetilde{k}_t$. By the same token, all firms that produce in period t using the depreciated capital chosen in period $t-m$ also set the same level of labor which we denote by $\widetilde{h}_{t|t-m}$ for $m=\{1,2,...\}$.[4]

To sum up, firms that adjust capital in period t solve

$$\max_{\widetilde{k}} \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[a_{t+j} \left((1-\delta)^j \widetilde{k}_t \right)^{\theta} \widetilde{h}_{t+j|t}^{1-\theta} - R_{t+j}^k (1-\delta)^j \widetilde{k}_t - w_{t+j} \widetilde{h}_{t+j|t} \right]. \tag{5}$$

We see that firms account for the fact that they might not adjust capital for potentially many periods. Note that capital depreciates while the firm does not adjust the capital level, and the amount of capital available in period $t+j$ for a firm that last optimized in period t is $(1-\delta)^j \widetilde{k}_t$.

The first-order condition for the choice of capital $\tilde{k}_t$ is thus given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} \theta (1-\delta)^{j\theta} \tilde{k}_t^{\theta-1} \widetilde{h}_{t+j|t}^{1-\theta} - R_{t+j}^k (1-\delta)^j \right) = 0. \tag{6}$$

If $\alpha_k > 0$, the optimal choice of capital now depends on the discounted value of all future

[4] A similar notation for capital implies $\widetilde{k}_{t|t-m} \equiv \widetilde{k}_{t-m} (1-\delta)^m$.

expected marginal products of capital and rental rates. Note also that the discount factor between periods t and $t+j$ incorporates α_k^j which is the probability that the firm cannot adjust its level of capital after j periods. If $\alpha_k = 0$, equation (6) reduces to the standard case where the firm sets capital until its marginal product equates the rental rate.

The first-order condition for labor is given by

$$h_{i,t} = \left(\frac{w_t}{a_t (1-\theta)} \right)^{-\frac{1}{\theta}} k_{i,t} \quad \text{for } i \in [0,1]. \tag{7}$$

Here, we do not need to distinguish between optimizing and non-optimizing firms because all firms are allowed to optimally set their labor demand each period. It is important to note that the capital-labor ratio only depends on aggregate variables and is therefore identical for all firms in the economy.

2.3 Market Clearing and Aggregation

In equilibrium, the aggregate supply of capital must equal the capital demand of all firms, i.e.

$$k_t^s = \int_0^1 k_{i,t} di.$$

A fraction of $1-\alpha_k$ firms choose $\tilde{k}_t$ in period t. The capital demand among non-reoptimizing firms is equal to the aggregate capital in period $t-1$ rescaled by α_k and adjusted for depreciation. This is because all firms face the same probability of being allowed to adjust capital. Market clearing in the capital rental market is therefore given by

$$k_t^s = (1-\alpha_k) \tilde{k}_t + \alpha_k (1-\delta) k_{t-1}^s. \tag{8}$$

Note that $k_t^s = \tilde{k}_t$ when $\alpha_k = 0$ and all firms are allowed to adjust their capital level in every period.

Market clearing in the labor market implies

$$h_t = \int_0^1 h_{i,t} di. \tag{9}$$

Finally, the good market clears when

$$y_t \equiv \int_0^1 y_{i,t} di = c_t + i_t. \tag{10}$$

2.4 Implications of Infrequent Capital Adjustments

The parameter α_k determines the fraction of firms reoptimizing capital in a given period, or equivalently the average numbers of periods that firms operate without being able to adjust their capital levels. It is therefore natural to expect that different values of α_k imply different business cycle implications for aggregate variables in the model. For instance, large values of α_k imply that adjusting firms are more forward-looking compared to the case where α_k is small, and this could potentially give rise to different dynamics for aggregate variables. This simple intuition turns out *not* to be correct: different values of α_k actually imply exactly the same aggregate dynamics in the RBC model. This important result is stated in the next theorem.

Theorem 1 *For a given initial value of k_t^s, the parameter $\alpha_k \in [0,1[$ does not affect the dynamics of aggregate variables in the RBC model for $t = \{1, 2, ..., \}$.*

Proof. Let $\alpha_k = 0$ and denote the aggregate equilibrium by $\mathbf{x}_t^* \equiv \left(k_t^{s,*}, h_t^*, w_t^*, R^{k,*}, \lambda_t^*, y_t^*, c_t^*, i_t^*, a_t\right)$ and let $\mathbf{x}_t^{\#}$ denote the equilibrium when $\alpha_k > 0$. We thus need to show $\mathbf{x}_t^* = \mathbf{x}_t^{\#}$. The parameter α_k only affects the problem of the firms, and it is therefore sufficient to verify that $\mathbf{x}_t^*$ also fulfills firms' optimality conditions and market clearing conditions when $\alpha_k > 0$.

The initial value of k_t^s is given, so $k_t^{s,*} = k_t^{s,\#}$. Suppose w_t is unaffected by α_k, i.e. $w_t^* = w_t^{\#}$. Aggregate demand for labor is

$$h_t = \left(\frac{w_t}{a_t (1-\theta)}\right)^{-\frac{1}{\theta}} k_t^s,$$

and as a result $h_t^* = h_t^{\#}$. Constant returns to scale in the production function implies that only aggregate levels of labor and capital are important for aggregate output. Hence, we also have $y_t^* = y_t^{\#}$. Equation (6) also holds for $\mathbf{x}_t^*$. To see why, note first that (6) reduces to

$$a_t \theta \left(\frac{w_t^*}{a_t (1-\theta)}\right)^{-\frac{1-\theta}{\theta}} - R_t^{k,*} = 0$$

when $\alpha_k = 0$. This equation must hold for all states and for all periods. Hence,

$$\sum_{l=0}^{+\infty} \left(\frac{\alpha_k \beta}{1-\delta}\right)^j \frac{\lambda_{t+j}^*}{\lambda_t^*} \left(a_{t+j}\theta \left(\frac{w_{t+j}^*}{a_{t+j}(1-\theta)}\right)^{-\frac{1-\theta}{\theta}} - R_{t+j}^{k,*}\right) = 0$$

for all states showing that (6) also holds at $\mathbf{x}_t^*$. That is, $R_{t+j}^{k,*} = R_{t+j}^{k,\#}$.

The problem of the household is unaffected by α_k. Hence, we must have $w_t^* = w_t^{\#}$ as assumed above. As a result, all prices and all other aggregate quantities are also unaffected by α_k in period t. The same argument ensures that all aggregate quantities and all prices in period $t+1, t+2, ...$ are unaffected by α_k. ■

The intuition behind this irrelevance theorem is simple. When the capital supply is predetermined, it does not matter if a fraction of firms cannot change their capital level because the other firms have to demand the remaining amount of capital to ensure equilibrium in the capital market. The fact that the capital-labor ratio is the same across firms further implies that the aggregate demand for labor is identical to the case where all firms can freely adjust their capital level. The aggregate output produced by firms are also unaffected due to the presence of constant returns to scale in the production function. The result in theorem 1 is thus similar to the well-known result from micro-economics for a market in perfect competition and constant returns to scale, where only the aggregate production level can be determined but not the production level of the individual firms.

There are at least two interesting implications of the infrequent capital adjustments at the firm level. Firstly, the distortion on firms' ability to change their capital level does not break the key relation from the standard RBC model that the marginal product of capital is equal to the rental price of capital in all periods. In other words, the induced distortion in the capital market does not lead to any inefficiencies in economy because the remaining part of the economy is sufficiently flexible to compensate for this distortion.

Secondly, the infrequent capital adjustments give rise to firm heterogeneity. There will be firms which have not adjusted their capital levels for a long time and hence have small capital levels due to the effect of depreciation. These firms will therefore produce a small amount of output and will also have a low labor demand due to (7). Similarly, there will also be firms which recently have adjusted their capital levels and therefore produce relatively high quantities and have high labor demands. This firm heterogeneity relates to the literature on firm specific capital as in Woodford (2005).

When proving theorem 1 we only used two assumptions from our RBC model besides a predetermined capital supply. Hence, the irrelevance theorem for α_k holds for all DSGE models with these two properties. We state this important observation in proposition 1.

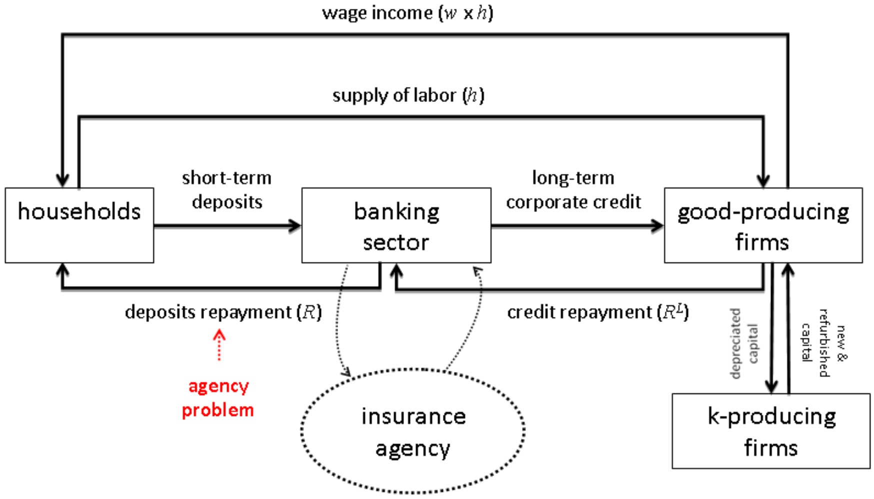
Proposition 1 *Theorem 1 holds for any DSGE model with the following two properties:*

1. *The capital labor ratio is identical for all firms*
2. *The parameter α_k does not affect the remaining part of the economy*

Examples of DSGE models with these properties are models with sticky prices, sticky wages, monopolistic competition, habits, a variable capital utilization rate to name just a few. The three most obvious ways to break the irrelevance of the infrequent capital adjustments can be inferred from (7). That is, if firms i) do not have a Cobb-Douglass production function, ii) face firm-specific productivity shocks, or iii) face different wage levels due to imperfections in the labor market.

Another way to break the irrelevance of infrequent capital adjustments is to make α_k affect the remaining part of the economy. We will in the next section show how this can be accomplished by introducing a banking sector into the model. With long-term financial contracts, the irrelevance theorem does not hold because α_k directly affects banks' balance sheets and the amount of credit provided by banks.

Figure 2: RBC Model With Banks and Maturity Transformation



3 An RBC Model With Banks and Maturity Transformation

This section incorporates a banking sector into the RBC model developed above. Here, we impose the standard assumption that firms need to borrow to finance their capital stock. This requirement combined with the infrequent capital adjustments generate a demand for long-term credit at the firm level. Banks use one period deposits from households and accumulated wealth, i.e. net worth, to meet this demand. As a result, banks face a maturity transformation problem because they use short term deposits to provide long-term credit.

Having outlined the novel feature of our model, we now turn to the details. The economy is assumed to have four agents: i) households, ii) banks, iii) good-producing firms, and iv) capital-producing firms. The latter type of firms are standard in the literature and only introduced to facilitate the aggregation (see for instance Bernanke, Gertler, and Gilchrist (1999)). The interactions between the four types of agents are displayed in Figure 2.[5]

Households supply labor to the good-producing firms and make short-term deposits in banks. Banks then use these deposits together with their own wealth to provide long-term credit to good-producing firms. The good-producing firms hire labor and use the obtained credit to buy capital from the capital-producers. The latter firms simply repair the depreciated capital and build new capital which they sell to good-producing firms in a competitive market.

We proceed as follows. Sections 3.1 and 3.2 revisit the problem of the household and good-producing firms when banks are present. Sections 3.3 and 3.4 are devoted to the

[5] For simplicity, Figure 2 does not show the profit flows going from firms and banks to households.

behavior of banks and the capital-producing firms, respectively, and section 3.5 closes the model. Detailed derivations are available in Appendix B.

3.1 Households

Each household is inhabited by workers and bankers. Workers provide labor h_t to good-producing firms and in exchange receive labor income $w_t h_t$. Each banker manages a bank and accumulates wealth that is eventually transferred to his respective household. It is assumed that a banker becomes a worker (and vice-versa) with probability α_b in each period, and only in this event is the wealth of the banker transferred to the household. Each household postpones consumption from periods t to $t+1$ by holding short-term deposits in banks.[6] Deposits b_t made in period t are repaid in the beginning of period $t+1$ at the gross deposit rate R_t.

The preferences of the households are as in section 2.1, and these preferences are maximized with respect to c_t, b_t, and h_t subject to

$$c_t + b_t = h_t\, w_t + R_{t-1} b_{t-1} + \pi_t.$$

Here, π_t denotes the net transfers of profits from firms and banks. Note that the household is not allowed to accumulate capital as in the previous model but is forced to postpone consumption through deposits in banks.

3.2 Good-Producing Firms

We impose the requirement on good-producing firms that they need credit to finance their capital stock. With infrequent capital adjustments these firms therefore face a demand for long-term credit which we assume is provided by banks.

It is convenient in this setup to match the number of periods a firm cannot adjust capital to the duration of its financial contract with the bank. That is, the financial contract lasts for all periods where the firm cannot adjust its capital level, and a new contract is signed whenever the firm is allowed to adjust capital. Since the latter event happens with probability $1-\alpha_k$ in each period, the exact maturity of a contract is not known when the contract is signed. The average maturity of contracts, however, is known and given by $\mathcal{D} = 1/\left(1-\alpha_k\right)$.

The specific obligations in the financial contract are as follows. When entering a contract, the bank offers the firm credit at the rate r_t^L throughout the contract. It is further assumed that the bank does not want to carry any risk due to changes in the price of capital within a

[6] As in Gertler and Karadi (2009), it is assumed that a household is only allowed to deposit savings in banks owned by bankers from a different household. Additionally, it assumed that within a household there is perfect consumption insurance.

given contract.[7] This implies that the firm is required to clear its account in the bank each period due to gains and losses associated with changes in the price of capital. The assumption further implies that the firm must borrow in each period based on the current market value of the capital stock and not on the market value of capital when signing the contract. Finally, the depreciation of capital during the contract is incorporated in the required amount of credit. Note that we do not introduce informational asymmetries between banks and firms. This implies that firms do not deviate from the signed contract or attempt to renegotiate the contract [do a reference to Hart mand Moore 1989].

As in the standard RBC model, good-producing firms also use labor in the production. We continue to assume that the wage bill is paid after production takes place, which implies that demand for credit is uniquely associated with firms' capital level.

The assumptions above are summarized in the expression for $\pi_{t+j|t}$, i.e. the profit in $t+j$ for a firm that entered a financial contract in period t:

$$\pi_{t+j|t} = \underbrace{a_{t+j}\left[(1-\delta)^{l}\widetilde{k}_t\right]^{\theta} h_{t+j}^{1-\theta}}_{\text{production revenue}} + \underbrace{\left(q_{t+j}\left(1-\delta\right)-q_{t+j-1}\right)\left(1-\delta\right)^{j}\widetilde{k}_t}_{\text{clearing price gains and losses}} - \underbrace{w_{t+j}h_{t+j}}_{\text{wage bill}} - \underbrace{r_t^L q_{t+j-1}(1-\delta)^{j}\widetilde{k}_t}_{\text{pay interest}}$$

where r_t^L is the net interest rate on loans and $R_t^L \equiv 1+r_t^L$ is the corresponding gross interest rate.

Profits are discounted by the stochastic discount factor of the households because households own all shares of the good-producing firms. The optimal level of capital chosen by firms which adjust their capital level in period t is therefore given by

$$\max_{\widetilde{k}_t} \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \pi_{t+j|t}. \tag{11}$$

This gives the first-order condition

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \left(\beta\alpha_k\right)^j \frac{\lambda_{t+j}}{\lambda_t}\left[\theta\left(1-\delta\right)^{j\theta} a_{t+j}\tilde{k}_t^{\theta-1}\tilde{h}_{t+j|t}^{1-\theta} + q_{t+j}\left(1-\delta\right)^{j+1} - q_{t+j-1}\left(1-\delta\right)^{j} R_t^L\right] = 0. \tag{12}$$

The gross lending rate R_t^L is seen to be a weighted average of the expected marginal product of capital and the price of capital. When $\alpha_k = 0$ and the duration of the financial contract is one period, we get a first-order condition for capital which is very similar to the one in Gertler and Karadi (2009). The only practical difficulty rests in finding a recursive version of this first-order condition. This is shown in Appendix B.

The first-order condition for the optimal choice of labor is exactly as in the standard RBC model, i.e. as in equation (7).

[7] The firm's capital stock is the bank's collateral for the provided credit, and our assumption ensures that the bank only faces minimal default risk if the firm were allowed to default. The focus in this paper is solely on banks maturity transformation problem and this motivates the assumption.

3.3 The banking sector

We incorporate banks as suggested by Gertler and Kiyotaki (2009) and Gertler and Karadi (2009). Their specification has two key elements. The first is an agency problem that characterizes the interactions between households and banks and limits households' deposits into banks. This in turn limits the amount of credit provided by banks to the good-producing firms.

The agency problem only constraints banks' supply of credit as long as banks cannot accumulate sufficient wealth to be independent of deposits from households. The second key element is therefore to assume that bankers retire with probability α_b in each period, and when doing so, transfer wealth back to their respective households. The retired bankers are assumed to be replaced by new bankers with a sufficiently low initial wealth to make the aggregate wealth of the banking section bounded.[8]

Although our model is very similar to the model in Gertler and Karadi (2009), the existence of long-term financial contracts complicates the aggregation. This is because new bankers must inherit the outstanding long-term contracts from the retired bankers, but the new bankers may not be able to do so with a low initial wealth. We want to maintain the assumption of bankers having to retire with probability α_b because this justifies the transfers of wealth from the banking sector to the households and in turn to consumption. Our solution is to introduce an insurance agency financed by a proportional tax on banks' profit. When a banker retires, the role of this agency is to create a new bank with an identical asset and liability structure and effectively guarantee the outstanding contracts of the old bank. This agency therefore ensures the existence of a representative bank and that the wealth of banks are bounded if the tax rate is appropriately calibrated.

The remainder of this section is organized as follows: section 3.3.1 describes the balance sheet of the representative bank and section 3.3.2 presents the agency problem.

3.3.1 Banks' Balance Sheets

As mentioned earlier, the representative bank uses accumulated wealth n_t and short-term deposits from households b_t to provide credit to good-producing firms. This implies the following identity for the bank's balance sheet

$$q_t s_t \equiv n_t + b_t, \tag{13}$$

where s_t represents the volume of provided credit by the representative bank.

The net wealth accumulated by the bank in period t is given by

$$n_{t+1} = (1 - \tau)\left[rev_t - R_t b_t\right], \tag{14}$$

[8] Their second assumption thus generates heterogeneity in the banking section and a representative bank does not exist.

where τ is the proportional tax rate and rev_t denotes revenue from lending to good-producing firms. The term $R_t b_t$ constitutes the value of deposits repaid to consumers. Combining the last two equations give the following law of motion for the bank's net wealth

$$n_{t+1} = (1-\tau)\left[rev_t - R_t q_t s_t + R_t n_t\right].$$

The imposed structure for firms' inability to adjust capital imply simple recursions for s_t and rev_t. First, let $\widetilde{s}_{t|t-m}$ denote the volume of credit provided in period t to firms which last adjusted their capital level in period $t-m$. An argument similar to the one used in section 2.3 implies that

$$s_t = (1-\alpha_k)\,\widetilde{s}_{t|t} + \alpha_k\,(1-\delta)\,s_{t-1}.$$

The amount of credit is thus equal to the amount of credit provided to adjusting firms $(1-\alpha_k)\,\widetilde{s}_{t|t}$ plus the amount of credit to non-adjusting firms with a correction for capital depreciation.

The revenue for the representative bank is given by

$$rev_t = (1-\alpha_k)\sum_{j=0}^{\infty}\alpha_k^j R_{t-j}^L q_t \widetilde{s}_{t|t-j}. \tag{15}$$

The intuition for this equation is as follows. A fraction $(1-\alpha_k)$ of the bank's revenue in period t relates to credit provided to adjusting firms in the same period. Likewise, a fraction $(1-\alpha_k)\,\alpha_k$ of revenue relates to credit provided to firms that last adjusted capital in period $t-1$, and so on. For all contracts, the loans made j periods in the past are repaid at the rate R_{t-j}^L. Thus, a large values of α_k makes the bank's balance sheet less exposed to changes in R_t^L compared to small values of α_k. The most important thing to notice, however, is that α_k affects the bank's revenue and its balance sheet, and this implies that the irrelevance theorem of infrequent capital adjustments in section 2.4 does not hold in this model.

It is straightforward to show that the recursive representation for rev_t is given by

$$rev_t = (1-\alpha_k)\,q_t R_t^L \widetilde{s}_{t|t} + \alpha_k\,(1-\delta)\,\frac{q_t}{q_{t-1}} rev_{t-1}.$$

Finally, our timing assumption for the good-producing firms is that capital used in period t is financed at the end of the previous period. Hence,

$$\widetilde{s}_{t|t} = \widetilde{k}_{t+1}.$$

3.3.2 The Agency Problem

As in Gertler and Karadi (2009), we assume that bankers can divert a fraction Λ of its deposits and wealth at the beginning of the period, and transfer this amount of money back to their corresponding households. The cost for bankers of diverting is that the depositors can force them into bankruptcy and recover the remaining fraction $1-\Lambda$ of assets. Bankers therefore choose to divert whenever the benefit from diverting, i.e. $\Lambda q_t s_t$, is greater than the value associated with staying in business as a banker, i.e. V_t. This gives the following incentive constraint

$$\underbrace{V_t}_{\substack{\text{banker's loss} \\ \text{from diverting}}} \geq \underbrace{\Lambda q_t s_t}_{\substack{\text{banker's gain} \\ \text{from diverting}}} .$$

for households to have deposits in banks.

The continuation value V_t of a bank is given by

$$V_t = \mathbf{E}_t \sum_{j=0}^{+\infty} (1-\alpha_b)\, \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} n_{t+j+1}. \tag{16}$$

This expression reflects the idea that bankers attempt to maximize their expected wealth at the point of retirement where they transfer n_t to their respective household. Note that the discount factor in (16) is adjusted by $(1-\alpha_b)\, \alpha_b^j$ to reflect the fact that retirement itself is stochastic and therefore could happen with positive probability in any period.

We assume that lending is profitable for banks, which implies that banks lend as much as possible to the good-producing firms as allowed by the incentive constraint. This constraint therefore holds with equality. Consequently, the amount of credit provided by the representative bank is limited by its accumulated wealth through the relation

$$q_t s_t = (lev_t)\, n_t$$

where

$$lev_t \equiv \frac{x_{2,t}}{\frac{\Lambda}{1-\tau} - x_{1,t}}$$

is the bank's leverage ratio. The two variables $x_{1,t}$ and $x_{2,t}$ have the recursive structure (see Appendix B)

$$\begin{aligned} x_{1,t} &= (1-\alpha_b)\, \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{rev_t}{q_t s_t} - R_t \right) \right] + \mathbf{E}_t \left[x_{1,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1} s_{t+1}}{q_t s_t} \right] \\ x_{2,t} &= (1-\alpha_b) + \mathbf{E}_t \left[x_{2,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{n_{t+1}}{n_t} \right]. \end{aligned}$$

3.4 Capital-Producing Firms

Perfectly competitive capital-producer are assumed to control the supply of capital. These firms purchase the depreciated capital $k_t(1-\delta)$ and invest in new capital before they resell the 'refurbished' capital stock k_{t+1}. Accordingly, their problem is to choose contingency plans for investment i_t to maximize the expected discounted value of profits

$$\max_{i_t} \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_j} \left[q_{t+j} k_{t+j+1} - q_{t+j} k_{t+j} (1-\delta) - i_{t+j} \right],$$

subject to

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right].$$

Profits are discounted by the household's stochastic discount because the households also own the capital-producing firms.

3.5 Market Clearing

Market clearing conditions in the capital, labor, and good markets are similar to those derived in section 2.3, and technology evolves according to the AR(1) process in (4).

4 Business Cycle Implications of Maturity Transformation

This section examines the quantitative implications of maturity transformation in the RBC model from section 3. We start by presenting our calibration in section 4.1. The effects of a technological shocks and a shock to the confidence in the banking sector are then examined in section 4.1 and 4.2, respectively.

4.1 Calibration

Table 1 shows our parameter choice under the baseline calibration. We chose standard values for the discount factor $\beta = 0.995$, the capital share $\theta = 0.36$, the coefficient of relative risk aversion $\phi_0 = 1$, and the rate of depreciation $\delta = 0.025$. In line with the estimated values in Christiano, Eichenbaum, and Evans (2005), we set the intensity of habits to $b = 0.70$ and investment adjustment costs to $\kappa = 2.5$. The inverse Frisch elasticity of the labor supply ϕ_1 is set to 1/3. This is slightly below the value estimated in Smets and Wouters (2007) but preferred to account for the fact that there are no wage rigidities in our model. The parameters affecting the evolution of total factor productivity are calibrated as in King and Rebelo (1999), implying $\rho_a = 0.98$ and $\sigma_a = 0.07$.

There are three parameters that directly affect the behavior of banks in our model: i) the fraction of banks' assets that can be diverted Λ, ii) the probability that a banker retires α_b, and iii) the tax rate on banks' wealth τ. We calibrate these parameters to generate an external financing premium of 100 basis points and a leverage ratio in the banking sector of 4 in steady state as in Gertler and Karadi (2009).[9] The value of α_k determines the average

[9] Note that the steady state level of the external financing premium implied by our model does not depend on α_k.

Table 1: Baseline Calibration

β	0.995	Λ	0.2
b	0.7	α_b	0.972
ϕ_0	1	τ	0.015
ϕ_1	1/3	κ	2.5
θ	0.36	δ	0.025
α_k	$free$	ρ_a	0.98
		σ_a	0.07

duration of financial contracts and is left as a free parameter to explore the macroeconomic implications of maturity transformation.

The complete list of equations in the model is shown in Appendix C. We approximate the model solution using the standard log-linear approximation method.[10]

4.2 A Negative Technology Shock

To fully explain the key mechanisms from long-term financial contracts, we first consider the simplified case in section 4.2.1 without investment adjustment costs, i.e. $\kappa = 0$, and a constant price of capital. Section 4.2.2 then examines the full version of our model with $\kappa > 0$, where movements in the price of capital provide an additional channel through which technological shocks transmit to the economy.

4.2.1 The Case With a Constant Price of Capital

Figure 3 displays the impulse responses for a negative shock to technology of one standard deviation in case $\kappa = 0$. In this case the linearity of the capital accumulation equation effectively implies that the supply of capital is perfectly elastic and therefore the price of capital does not move. In each graph, the continuous line without markers represents the model with banks and no maturity transformation, i.e. $\mathcal{D} = 1$. The continuous lines with markers represent the model with increasing degree of maturity transformation corresponding to an average duration of loans of one, five and ten years. Finally, the dashed lines represent impulse responses from the model without banks described in section 2.

Starting with one-period financial contracts, i.e. $\mathcal{D} = 1$, note that the shock generates a reduction in labour supply, consumption, output, investments and capital that are qualitatively in line with the results of a standard RBC model without a banking sector. The reduction in consumption lowers the deposit rate R_t to encourage the households to transfer future consumption into the present. According to equation (12) note that a fall in productivity lowers marginal product of capital and the loans rate R^L. This fall is smaller than the

[10] All versions of the model are implemented in Dynare. Codes are available on request.

fall in R_t and implies an increase in banks' wealth. This increase, however, is insufficient to compensate for the fall in deposits and we therefore see a fall in the aggregate levels of banks' loans and capital.

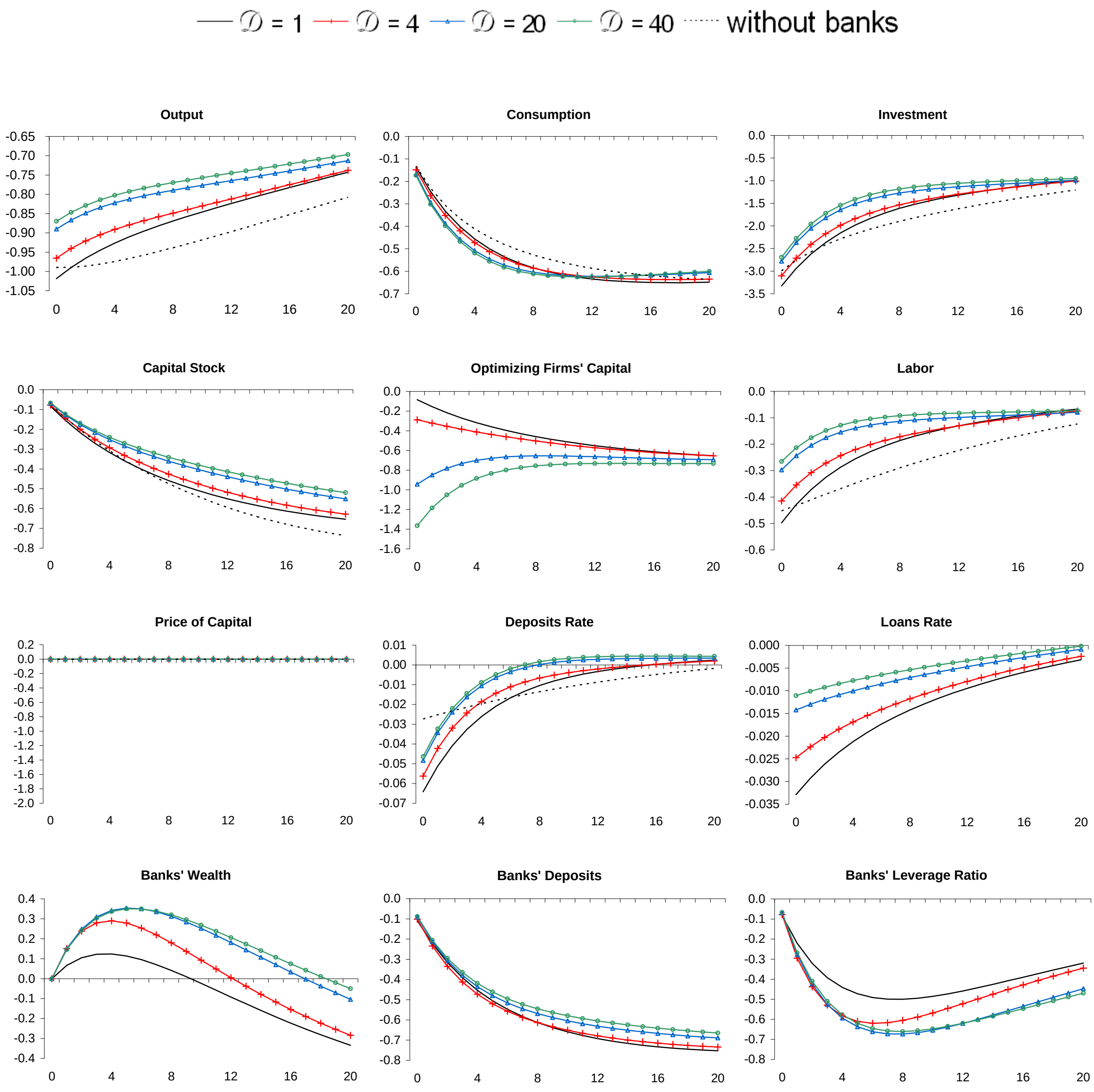
Comparing the model with banks and $\mathcal{D} = 1$ to the one without banks, note that the inclusion of a banking sector does not enhance the internal propagation to the technological shock. In other words, when $\kappa = 0$, there is no financial accelerator effect in the sense of Bernanke, Gertler, and Gilchrist (1999). This is because the output contraction generated by the negative technological shock is attenuated by the increase in banks' wealth; as we will see in the next section, this effect is reversed once we allow for changes in the price of capital.

We now study the effects of maturity transformation by gradually increasing the average duration of financial contracts, $\mathcal{D}$. There are two important differences relative to the case without maturity transformation. First, with $\mathcal{D} > 1$ only a fraction of banks' loans are renegotiated at the new and lower loan rate R_t^L after the shock hits. A second difference is that the loans rate falls by less as we increase the average duration of contracts. Banks benefit in these two dimensions, causing an increase in banks' wealth by even more than in the case where $\mathcal{D} = 1$. Because of the agency problem and its associated incentive constraint, the increase in banks' wealth implies that more credit is supplied to good-producing firms. This in turn implies a smaller reduction in the level of capital, labour supply, investments and output than with one-period financial contracts. In other words, long-term lending attenuates the effects of a technology shock compared to the case with one-period financial contracts.

The fact that banks' balance sheets are less exposed to R_t^L as we set $\mathcal{D} > 1$ is a somewhat mechanical implication of introducing maturity transformation. The fact that R_t^L falls by less as we increase $\mathcal{D}$, however, is a less obvious result and highlights the benefits of studying maturity transformation in a general equilibrium framework. Going back to equation (12), the loans rate R_t^L is, roughly speaking, associated with the current and future expected marginal products of capital of optimizing firms. On the one hand, lower technology levels are associated with lower capital productivity and therefore requires lower loan rates. On the other hand, Figure 3 shows that, as $\mathcal{D}$ increases, optimizing firms choose lower levels of capital, which tends to increase the marginal product of capital for optimizing firms[11]. The interplay between these two effects implies that R_t^L falls by less as $\mathcal{D}$ increases and produces an even higher rise in banks' wealth.

[11] When the technology shock hits, optimizing firms are the only ones that are able to change the level of capital demanded. They are therefore the ones that need to adjust in order to guarantee that the aggregate demand and supply of capital equalize. As the value of $\mathcal{D}$ increases, the impulse responses of optimizing firms' capital ($\widetilde{k}_t$) to a technological shock get more pronounced because a smaller number of firms are responsible for clearing the market for capital.

Figure 3: Impulse Responses to a Negative Technological Shock - $\kappa = 0$



Notes: Impulse response to a one standard deviation negative shock to technology. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

4.2.2 The Case With Changes in The Price of Capital

Figure 4 shows the model impulse responses when κ is set equal to 2.5. In this case the supply of capital is not perfectly elastic anymore and therefore firms are exposed to changes in the price of capital, q_t. This generates an extra channel for shock propagation which, in the case of a negative shock to technology, starts off with a fall in q_t caused by the lower productivity of capital.

Starting from the model without maturity transformation, note that the aggregate value of banks' credit ($q_t s_t$) falls with the fall in the price of capital. Additionally, the deposit rate (which was reduced after the shock when $\kappa = 0$) now increases[12], whereas the loans rate R_t^L decreases significantly more than when q_t was constant. As a consequence, the aggregate wealth in the banking sector, which in the previous section increased following the shock, is now severely hit by the shock. Accordingly, banks are obliged to restrict credit in order to respect their incentive compatibility constraint and, as a result, there are large negative effects on investment and output. Note also that even though the aggregate supply of credit in the economy decreases, the leverage ratio in the banking sector increases reflecting the fall in banks' wealth.

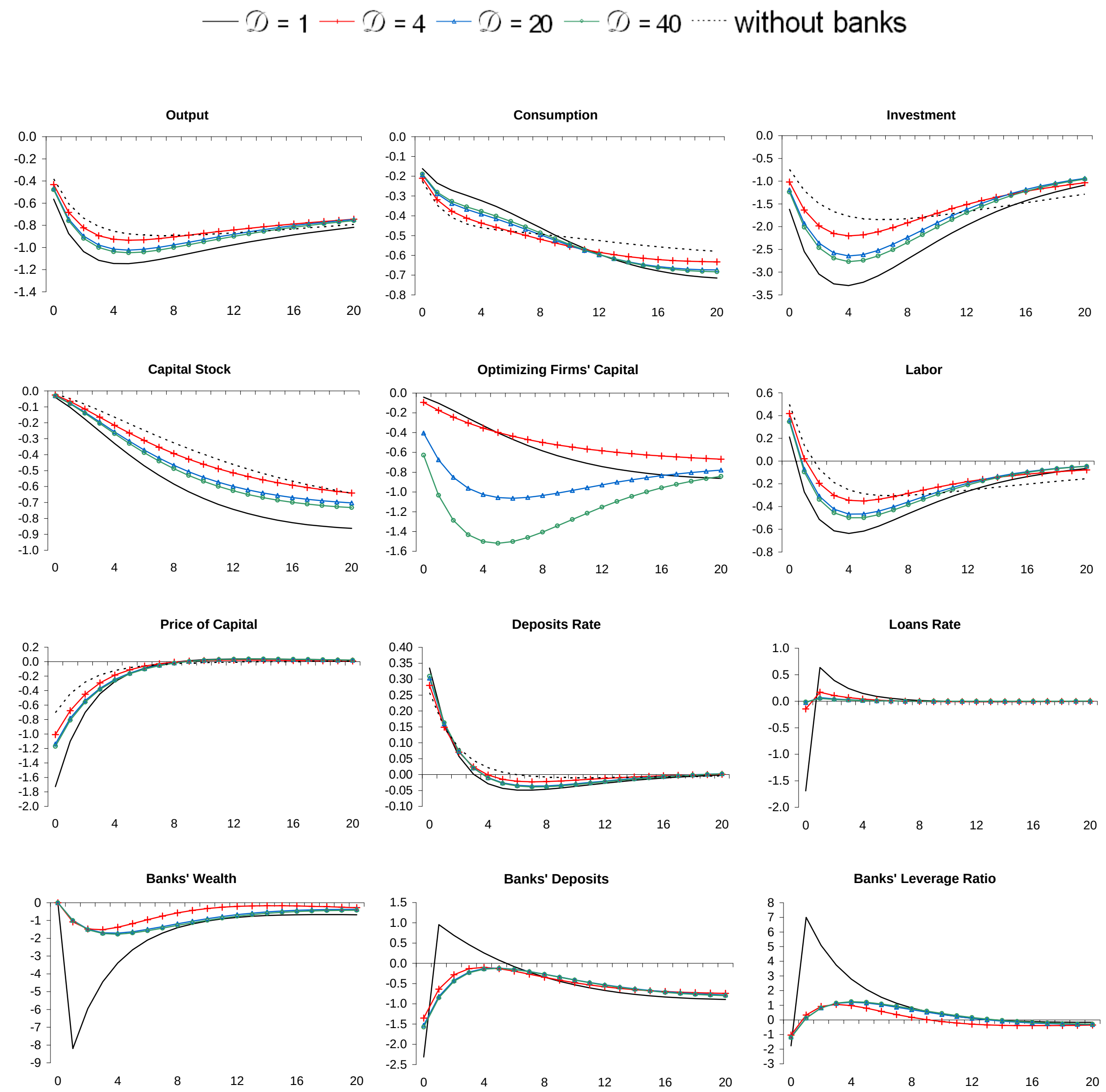
Note that, after we allow for movements in the price of capital, the introduction of banks significantly enhances the internal amplification and propagation of the model with respect to technology shocks. This happens because, differently from the model with $\kappa = 0$, now the response of banks' wealth is procyclical implying that banks restrict the supply of credit exactly when output is low, deepening the recession.

When we increase the average duration of financial contracts to $\mathcal{D} = 4$, we obtain the same result as in the previous section: long-term financial contracts reduce the effects of the technology shock. As before, when long-term credit is provided, only a fraction of banks' loans are renegotiated at the new and lower loans rate R_t^L. In addition, both the loans rate and the price of capital falls by a smaller amount compared to the model without maturity transformation. These effects combined imply a lower reduction in banks' wealth and therefore a smaller reduction in the aggregate amount of credit provided.

Note that, when increasing the average duration of financial contracts from $\mathcal{D} = 4$ to $\mathcal{D} = 20$ or 40, we obtain a marginal increase in amplification of the output, investment and capital impulse responses. Since this non-linearity in the impulse responses with respect to changes in $\mathcal{D}$ was not present in the previous section (see Figure 3), it must be related to changes in the response of the price of capital. In fact, the price of capital falls by a larger amount when we move from $\mathcal{D} = 4$ to $\mathcal{D} = 20$ or 40, causing bankers' wealth to also fall by a larger amount. Under our calibration, however, high degrees of maturity transformation such as $\mathcal{D} = 20$ or 40 still attenuate cycles if compared to the case without maturity transformation.

[12] The fact that the response of R_t to a technology shock changes direction when we introduced investment adjustment costs is not a feature of the model with banks. In fact, the same happens if we consider the impulse responses obtained from a simple RBC model without banks, in which case R_t represents the gross return of a riskless asset.

Figure 4: Impulse Responses to a Negative Technological Shock - $\kappa = 2.5$



Notes: Impulse response to a one standard deviation negative shock to technology. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

4.3 A Negative Financial Shock

We model a financial shock as a reduction in the confidence of households on the banking sector. This can be done by replacing Λ, the fraction of deposits that banks may divert, with $\Lambda\epsilon_t$, where ϵ_t follows

$$\begin{aligned} \ln \epsilon_t &= \rho_\epsilon \ln \epsilon_{t-1} + \xi_t \\ \xi_t &\sim \mathcal{NID}(0, \sigma_\xi) . \end{aligned}$$

We let $\sigma_\xi = 0.05$ and set $\rho_\epsilon = 0.98$ to generate a persistent increase in Λ from 0.2 to 0.25. This increase makes it more attractive for bankers to divert and is therefore equivalent to a reduction in the confidence in the banking sector.

The impulse response functions are shown in Figure 5 for the case where $\kappa = 2.5$. We first observe that the loss of confidence in the banking sector implies a large reduction in deposits which is translated into a short-lived consumption boom. The reduction in deposits leaves banks with a shortage of funds for loans and we therefore observe a protracted fall in the capital stock. This in turn lowers output, investment, and eventually also consumption.

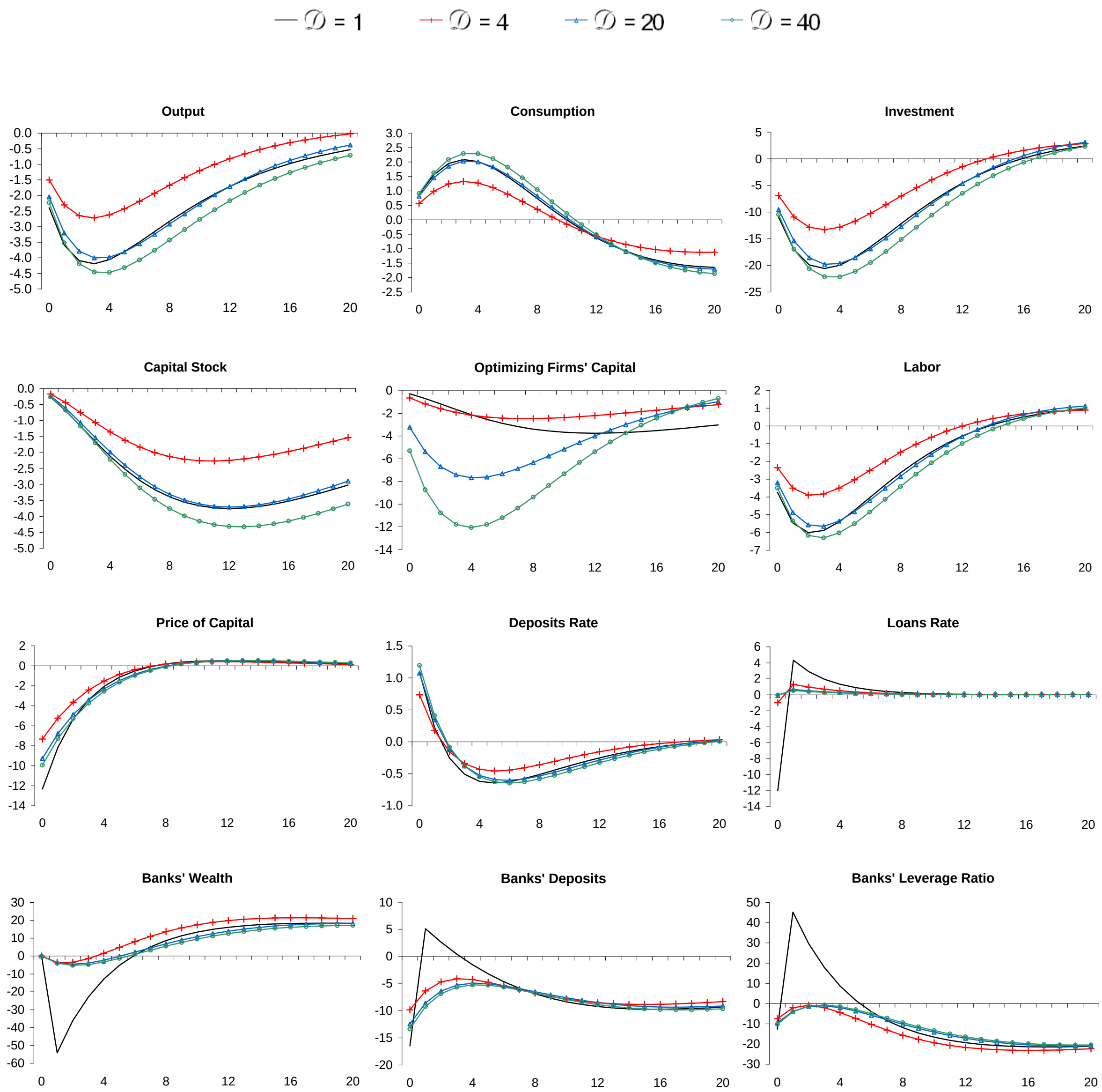
Banks' profits are affected not only by the fall in the amount of credit provided, but also by the movements in prices. First, the fall in labor reduces the marginal product of capital which, in turn, causes the loans rate to fall. At the same time, it takes a much lower price of capital to convince firms to hold the aggregate stock of capital. Finally, banks increase R_t in order to prevent deposits from falling by even a larger amount.

As we increase $\mathcal{D}$ the impulse responses of output, investment, and capital change in a non-linear fashion. Initially, when we move from $\mathcal{D} = 1$ to $\mathcal{D} = 4$ maturity transformation attenuates the cycle. As in the previous section, this effect is explained by the combination of two factors: (i) the fact that with $\mathcal{D} = 4$ only a fraction of banks' loans are renegotiated at the new and lower loans rate R_t^L; (ii) both the loan rate and the price of capital falls by a smaller amount compared to the model without maturity transformation. As we move to higher degrees of maturity transformation, the attenuator effect is again reduced. As before, this happens because the fall in the price of capital is larger in case $\mathcal{D} = 20$ or 40 than when $\mathcal{D} = 4$, causing banks' wealth to drop more in the former cases. Interestingly, by increasing $\mathcal{D}$ we eventually reach a point where maturity transformation actually amplifies the cycle relative to the case where only one period loans are provided.

5 Conclusions

This paper shows how to introduce a banking sector with maturity transformation into an otherwise standard DSGE model. To facilitate aggregation, and in line with micro-data evidence, we introduce infrequent firm-level capital adjustments and subsequently characterize the class of DSGE models in which these have no aggregate implications. We then additionally introduce banks and impose the requirement that firms require credit to finance the

Figure 5: Impulse Responses to a Negative Financial Shock



Notes: Impulse response to a one standard deviation negative shock to technology. In each graph the vertical axis measures percentage deviation from the deterministic steady state of the respective variable, whereas the horizontal axis measures quarters after the shock hits.

capital stock used in production. Two simulation studies illustate that maturity transformation may reduce or amplify the endogenous propagation of shocks in the economy and generate a credit maturity attenuator or a credit maturity accelerator.

Our way of incorporating maturity transformation is only a first step in analyzing this topic in a dynamic stochastic general equilibrium setup. Interesting extensions could introduce extra financing options, possibly by breaking the match between the duration of firms' exposures and their financial contract. This would also have the potential to create a time-varying maturity transformation problem. Adding a nominal side to the model is straightforward and would enable us to study monetary policy effectiveness under long-term financial contracts. Finally, studying higher order effects and the impact of risk on banks' behavior would also make for an interesting extension.

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A A Standard RBC Model with Infrequent Capital Adjustments: Detailed Derivations

A.1 Households

The lagrangian for problem of the representative household is

$$\begin{aligned}
\mathcal{L} \; =& \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{\left(c_{t+j} - b\, c_{t+j-1}\right)^{1-\phi_0}}{1-\phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1+\phi_1} \right) + \\
& \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \lambda_{t+j} [h_{t+j}\, w_{t+j} + R_{t+j}^k\, k_{t+j} - c_{t+j} - i_{t+j}] + \\
& \mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j q_{t+j} \lambda_{t+j} \left[(1-\delta)\, k_{t+j}^s + i_{t+j} \left[1 - S\left(\frac{i_{j+j}}{i_{t-1+j}} \right) \right] - k_{t+1+j}^s \right],
\end{aligned}$$

where λ_t is the Lagrange multiplier associated with the budget constraint. The first order conditions are:

i **Consumption,** c_t:

$$\lambda_t = \mathbf{E}_t \left[\frac{1}{\left(c_t - bc_{t-1}\right)^{\phi_0}} - \frac{\beta b}{\left(c_{t+1} - bc_t\right)^{\phi_0}} \right]$$

ii **Labor,** h_t:

$$\phi_2 h_t^{\phi_1} = \lambda_t w_t$$

iii **Physical capital stock,** k_{t+1}^s:

$$1 = \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{R_{t+1}^k + q_{t+1}\left(1-\delta\right)}{q_t} \right) \right]$$

iv **Investments,** i_t :

$$q_t = \frac{1 - \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left(\frac{i_{t+1}}{i_t} \right)^2 S' \left(\frac{i_{t+1}}{i_t} \right) \right]}{\left[1 - S\left(\frac{i_t}{i_{t-1}} \right) - \frac{i_t}{i_{t-1}} S' \left(\frac{i_t}{i_{t-1}} \right) \right]}$$

A.2 Firms

The profit of firm i in period $t+j$ is

$$a_t k_{i,t+j}^{\theta} h_{i,t+j}^{1-\theta} - R_{t+j}^k k_{i,t+j} - w_{t+j} h_{i,t+j},$$

and the firm seeks to maximize its expected discounted value of profits given by

$$\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} k_{i,t+j}^{\theta} h_{i,t+j}^{1-\theta} - R_{t+j}^k k_{i,t+j} - w_{t+j} h_{i,t+j} \right).$$

This problem is divided in two steps. We first derive the i^{th} firm's demand of labor, which takes the standard form since labor is optimally chosen in every period. In the second step, we derive the optimal value of capital $\widetilde{k}_{i,t}$ for the firms which can adjust their capital stock. Note that a firm adjusting capital in period t faces a probability α_k^j of not being able to reoptimize after j periods in the future and hence have $(1-\delta)^j \widetilde{k}_{i,t}$ in period $t+j$.

i **Labor,** h_t :

In every period $t+j$, for $j = 0, 1, 2, ...$, all firms are allowed to adjust their labour demand. Hence, we can ignore the dynamic dimension of the firm's problem which implies

$$h_{i,t+j} = \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1}{\theta}} k_{i,t+j}.$$

The period $t+j$ demand for labor for a firm that last reoptimized in period t is given by

$$\tilde{h}_{i,t+j|t} = \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1}{\theta}} (1-\delta)^j \widetilde{k}_{i,t}$$

ii **Capital,** $\widetilde{k}_t$:

A firm adjusting capital in period t chooses $\widetilde{k}_{i,t}$ to maximize the present discounted value of profits. This firm therefore solves

$$\max_{\widetilde{k}_{i,t}} \mathbf{E}_t \sum_{j=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j} \left((1-\delta)^j \widetilde{k}_{i,t} \right)^{\theta} \widetilde{h}_{i,t+j|t}^{1-\theta} - R_{t+j}^k (1-\delta)^j \widetilde{k}_{i,t} - w_{t+j} h_{i,t+j|t} \right)$$

$$\Downarrow$$

$$\mathbf{E}_t \sum_{l=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j}\theta (1-\delta)^{\theta j} \tilde{k}_{i,t}^{\theta-1} \widetilde{h}_{i,t+j|t}^{1-\theta} - R_{t+j}^k (1-\delta)^j \right) = 0.$$

An equivalent expression of this condition is

$$\mathbf{E}_t \sum_{l=0}^{+\infty} \alpha_k^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{1}{(1-\delta)^j} \left(a_{t+j}\theta (1-\delta)^{(\theta-1)j} \tilde{k}_{i,t}^{\theta-1} \widetilde{h}_{i,t+j|t}^{1-\theta} - R_{t+j}^k \right) = 0$$

$$\Updownarrow$$

$$\mathbf{E}_t \sum_{l=0}^{+\infty} \left(\frac{\alpha_k \beta}{1-\delta} \right)^j \frac{\lambda_{t+j}}{\lambda_t} \left(a_{t+j}\theta \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1-\theta}{\theta}} - R_{t+j}^k \right) = 0$$

A.3 Market Clearing

All reoptimizing firms face the same problem when setting the optimal capital stock (because they have no other state variables), and these firms therefore choose the same value of capital and labor, i.e. $\widetilde{k}_{i,t} = \widetilde{k}_t$ and $\tilde{h}_{i,t+l|t} = \tilde{h}_{t+l|t}$. Note also that the first-order condition for $\widetilde{k}_{i,t}$ can easily be rewritten in recursive form, and standard solution method can therefore be applied.

The aggregate demand for capital is given by equation (8) in the text. We use similar arguments to derive the market clearing condition for the labor market:

$$
\begin{aligned}
h_t &= \int_0^1 h_{i,t} di \\
&= (1-\alpha_k)\,\tilde{h}_{t|t} + \int_{Z_t} h_{i,t} di \\
&= (1-\alpha_k)\,\tilde{h}_{t|t} + \left(\frac{w_t}{a_t\,(1-\theta)}\right)^{-\frac{1}{\theta}} \int_{Z_t} k_{i,t} di \\
&= (1-\alpha_k)\,\tilde{h}_{t|t} + \left(\frac{w_t}{a_t\,(1-\theta)}\right)^{-\frac{1}{\theta}} \alpha_k\,(1-\delta)\,k_{t-1}
\end{aligned}
$$

where Z_t represents the set of firms that do not reoptimize capital in period t. Here, we use the fact that $h_{i,t} = \left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1}{\theta}} k_{i,t}$. Note also that $\tilde{h}_t = \left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1}{\theta}} \widetilde{k}_t$, and we therefore have

$$
\begin{aligned}
h_t &= \left(\frac{w_t}{a_t\,(1-\theta)}\right)^{-\frac{1}{\theta}} \left[(1-\alpha_k)\,\widetilde{k}_t + \alpha_k\,(1-\delta)\,k_{t-1}\right] \\
&= \left(\frac{w_t}{a_t\,(1-\theta)}\right)^{-\frac{1}{\theta}} k_t
\end{aligned}
$$

Finally, equilibrium in the aggregated goods market is given by equation (10) in the text.

B An RBC Model With Banks and Maturity Transformation: Detailed Derivations

B.1 Households

The lagrangian for problem of the representative household is

$$
\begin{aligned}
\mathcal{L} \; = &\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \left(\frac{(c_{t+j} - b\,c_{t+j-1})^{1-\phi_0}}{1-\phi_0} - \phi_2 \frac{h_{t+j}^{1+\phi_1}}{1+\phi_1} \right) + \\
&\mathbf{E}_t \sum_{j=0}^{+\infty} \beta^j \lambda_{t+j} [h_{t+j}\,w_{t+j} + R^k_{t+j-1} b_{t+j-1} + \pi_{t+j} - c_{t+j} - b_{t+j}],
\end{aligned}
$$

where λ_t is the Lagrange multiplier associated with the budget constraint. The first order conditions are:

i **Consumption,** c_t :

$$
\lambda_t = \mathbf{E}_t \left[\frac{1}{(c_t - bc_{t-1})^{\phi_0}} - \frac{\beta b}{(c_{t+1} - bc_t)^{\phi_0}} \right]
$$

ii **Deposits,** b_t :

$$\mathbf{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}R_t\right]=1$$

iii **Labor,** h_t :

$$\phi_2 h_t^{\phi_1}=\lambda_t w_t$$

B.2 Good-Producing Firms

The i^{th} firm's profit in period $t+j$ is

$$\pi_{i,t+j}=a_{t+j}k_{i,t+j}^{\theta}h_{i,t+j}^{1-\theta}+q_{t+j}\left(1-\delta\right)k_{i,t+j}-q_{t+j-1}k_{i,t+j}R_{i,t+j}^{L}-w_{t+j}h_{i,t+j}.$$

Note that the gross interest rate on loans depends on i because in a given period each firm potentially faces a difference long-term financial contract. The firm chooses capital and labor to maximize the expected discounted value of profits.

i **Labor,** $h_{i,t}$:

As in the simple RBC model, the firm is always allowed to adjust its labor demand, and therefore

$$h_{i,t+j}=\left(\frac{w_{t+j}}{a_{t+j}\left(1-\theta\right)}\right)^{-\frac{1}{\theta}}k_{i,t+j}.$$

ii **Capital,** $k_{i,t}$:

The first-order condition for the optimal choice of capital is given

$$\mathbf{E}_t\sum_{j=0}^{+\infty}\left(\beta\alpha_k\right)^j\frac{\lambda_{t+j}}{\lambda_t}\left[\theta\left(1-\delta\right)^{j\theta}a_{t+j}\tilde{k}_t^{\theta-1}\tilde{h}_{t+j|t}^{1-\theta}+q_{t+j}\left(1-\delta\right)^{j+1}-q_{t+j-1}\left(1-\delta\right)^{j}R_t^L\right]=0$$

To derive the recursive form of this equation, let

$$\begin{aligned}
z_{1,t} &\equiv \mathbf{E}_t\sum_{j=0}^{+\infty}\left(\beta\alpha_k\right)^j\frac{\lambda_{t+j}}{\lambda_t}\theta\left(1-\delta\right)^{j\theta}a_{t+j}\tilde{k}_t^{\theta-1}\tilde{h}_{t+j|t}^{1-\theta}\\
z_{2,t} &\equiv \mathbf{E}_t\sum_{j=0}^{+\infty}\left(\beta\alpha_k\right)^j\frac{\lambda_{t+j}}{\lambda_t}\left[q_{t+j}\left(1-\delta\right)^{j+1}\right]\\
z_{3,t} &\equiv \mathbf{E}_t\sum_{j=0}^{+\infty}\left(\beta\alpha_k\right)^j\frac{\lambda_{t+j}}{\lambda_t}q_{t+j-1}\left(1-\delta\right)^{j}R_t^L.
\end{aligned}$$

This implies that we can express the first order condition as

$$z_{1,t}+z_{2,t}=z_{3,t}$$

The recursive forms for $z_{1,t}$, $z_{2,t}$, and $z_{3,t}$ are as follows:

- **The recursive law of motion for** $z_{1,t}$:

$$\begin{aligned} z_{1,t} &= \theta a_t \tilde{k}_t^{\theta-1} \tilde{h}_{t|t}^{1-\theta} + \mathbf{E}_t \sum_{j=1}^{+\infty} (\beta\alpha_k)^j \frac{\lambda_{t+j}}{\lambda_t} \theta (1-\delta)^{j\theta} a_{t+j} \tilde{k}_t^{\theta-1} \tilde{h}_{t+j|t}^{1-\theta} \\ &= a_t \theta \tilde{k}_t^{\theta-1} \tilde{h}_{t|t}^{1-\theta} + \mathbf{E}_t \left[z_{1,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \right] \end{aligned}$$

where the latter follows from the fact that

$$\begin{aligned} z_{1,t+1} &= \mathbf{E}_{t+1} \sum_{j=0}^{+\infty} (\beta\alpha_k)^j \frac{\lambda_{t+1+j}}{\lambda_{t+1}} \theta (1-\delta)^{j\theta} a_{t+1+j} \tilde{k}_{t+1}^{\theta-1} \tilde{h}_{t+1+j|t+1}^{1-\theta} \\ &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^{m-1} \frac{\lambda_{t+m}}{\lambda_{t+1}} \theta (1-\delta)^{(m-1)\theta} a_{t+m} \tilde{k}_{t+1}^{\theta-1} \tilde{h}_{t+m|t+1}^{1-\theta}. \end{aligned}$$

The last equality implies $m = j+1 \Leftrightarrow j = m-1$. Note that:

$$\begin{aligned} \tilde{h}_{t+j|t} &= \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1}{\theta}} (1-\delta)^j \widetilde{k}_t \\ &= \left(\frac{w_{t+j}}{a_{t+j}(1-\theta)} \right)^{-\frac{1}{\theta}} (1-\delta)^{j-1} \widetilde{k}_{t+1} \left(\frac{(1-\delta)^j \widetilde{k}_t}{(1-\delta)^{j-1} \widetilde{k}_{t+1}} \right) \\ &= \tilde{h}_{t+j|t+1} \left[(1-\delta) \frac{\widetilde{k}_t}{\widetilde{k}_{t+1}} \right]. \end{aligned}$$

Therefore:

$$\begin{aligned} z_{1,t+1} \beta\alpha_k \frac{\lambda_{t+1}}{\lambda_t} (1-\delta) &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^m \frac{\lambda_{t+m}}{\lambda_t} \theta (1-\delta)^{m\theta} a_{t+m} \tilde{k}_{t+1}^{\theta-1} \tilde{h}_{t+m|t}^{1-\theta} \\ &\Downarrow \\ \mathbf{E}_t \left[z_{1,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \right] &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^m \frac{\lambda_{t+m}}{\lambda_t} \theta (1-\delta)^{m\theta} a_{t+m} \tilde{k}_{t+1}^{\theta-1} \tilde{h}_{t+m|t}^{1-\theta} \end{aligned}$$

due to the law of iterated expectations.

- **The recursive law of motion for** $z_{2,t}$:

$$\begin{aligned} z_{2,t} &= q_t (1-\delta) + \mathbf{E}_t \sum_{j=1}^{+\infty} (\beta\alpha_k)^j \frac{\lambda_{t+j}}{\lambda_t} \left[q_{t+j-1} (1-\delta)^{j+1} \right] \\ &= q_t (1-\delta) + \mathbf{E}_t \left[z_{2,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \right] \end{aligned}$$

where the latter follows from the fact that:

$$\begin{aligned} z_{2,t+1} &= \mathbf{E}_{t+1} \sum_{j=0}^{+\infty} (\beta\alpha_k)^j \frac{\lambda_{t+1+j}}{\lambda_{t+1}} \left[q_{t+j} (1-\delta)^{j+1} \right] \\ &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^{m-1} \frac{\lambda_{t+m}}{\lambda_{t+1}} \left[q_{t+m-1} (1-\delta)^m \right]. \end{aligned}$$

The last equality implies $m = j+1 \Leftrightarrow j = m-1$. Then:

$$\begin{aligned} z_{2,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^m \frac{\lambda_{t+m}}{\lambda_t} \left[q_{t+m-1} (1-\delta)^{m+1} \right] \\ &\Downarrow \\ \mathbf{E}_t \left[z_{2,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \right] &= \mathbf{E}_t \sum_{m=1}^{+\infty} (\beta\alpha_k)^m \frac{\lambda_{t+m}}{\lambda_t} \left[q_{t+m-1} (1-\delta)^{m+1} \right] \end{aligned}$$

due to the law of iterated expectations.

- **The recursive law of motion for** $z_{3,t}$:

$$\begin{aligned} z_{3,t} &= q_{t-1} R_t^L + \mathbf{E}_t \sum_{j=1}^{+\infty} (\beta\alpha_k)^j \frac{\lambda_{t+j}}{\lambda_t} q_{t+j-1} (1-\delta)^j R_t^L \\ &= q_{t-1} R_t^L + \mathbf{E}_t \left[z_{3,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^L}{R_{t+1}^L} \right] \end{aligned}$$

where the latter follows from the fact that:

$$\begin{aligned} z_{3,t+1} &= \mathbf{E}_{t+1} \sum_{j=0}^{+\infty} (\beta\alpha_k)^j \frac{\lambda_{t+1+j}}{\lambda_{t+1}} q_{t+j} (1-\delta)^j R_{t+1}^L \\ &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^{m-1} \frac{\lambda_{t+m}}{\lambda_{t+1}} q_{t+m-1} (1-\delta)^{m-1} R_{t+1}^L. \end{aligned}$$

The last equality implies $m = j+1 \Leftrightarrow j = m-1$. Then:

$$\begin{aligned} z_{3,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^L}{R_{t+1}^L} &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (\beta\alpha_k)^m \frac{\lambda_{t+m}}{\lambda_t} q_{t+m-1} (1-\delta)^m R_t^L \\ &\Downarrow \\ \mathbf{E}_t \left[z_{3,t+1} \beta\alpha_k (1-\delta) \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^L}{R_{t+1}^L} \right] &= \mathbf{E}_t \sum_{m=1}^{+\infty} (\beta\alpha_k)^m \frac{\lambda_{t+m}}{\lambda_t} q_{t+m-1} (1-\delta)^m R_t^L \end{aligned}$$

due to the law of iterated expectations.

B.3 The banking sector

B.3.1 The balance sheets of banks

The wealth of bank f evolves according to

$$n_{f,t+1} = (1-\tau)\left[rev_{f,t} - R_t b_{f,t}\right].$$

Let $q_t s_{f,t(i)}$ be the value of lending provided at the end of period t from bank f to firm i. Then revenue for bank f is

$$rev_{f,t} = (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m R_{t-m}^L q_t \widetilde{s}_{f,t|t-m}.$$

The aggregate revenue in the banking sector is therefore given by

$$\begin{aligned} rev_t &\equiv \int_0^1 rev_{f,t} df = (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m R_{t-m}^L q_t \int_0^1 \widetilde{s}_{f,t|t-m} df \\ &\Downarrow \\ rev_t &= (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m R_{t-m}^L q_t \widetilde{s}_{t|t-m} \end{aligned}$$

where $\widetilde{s}_{t|t-m} \equiv \int_0^1 \widetilde{s}_{f,t|t-m} df$.

Let $s_{f,t}$ denote the total volume of outstanding lending by bank f. The bank faces a balance sheet constraint given by

$$q_t s_{f,t} \equiv q_t \int_0^1 s_{f,t(i)} di = n_{f,t} + b_{f,t}.$$

Note that $s_{f,t}$ can also be written as

$$s_{f,t} = (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m \widetilde{s}_{f,t|t-m}.$$

Combining the results from above, we the have

$$n_{f,t+1} = (1-\tau)\left[rev_{f,t} - R_t q_t s_{f,t} + R_t n_{f,t}\right].$$

When aggregating across all banks, we get

$$\begin{aligned} \int_0^1 s_{f,t} df &= (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m \int_0^1 \widetilde{s}_{f,t|t-m} df \\ &\Downarrow \\ s_t &= (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m \widetilde{s}_{t|t-m}. \end{aligned}$$

The aggregate wealth accumulated by the banking sector is therefore

$$\begin{aligned}\int_0^1 n_{f,t+1}dj &= (1-\tau)\left[\int_0^1 rev_{f,t}df - R_t q_t \int_0^1 s_{f,t}df + R_t \int_0^1 n_{f,t}df\right] \\ &\Downarrow \\ n_{t+1} &= (1-\tau)\left[rev_t - R_t q_t s_t + R_t n_t\right]\end{aligned}$$

which is possible due to the existence of the insurance company.

- **Finding the recursive law of motion for** rev_t :

$$\begin{aligned}rev_t &= (1-\alpha_k) R_t^L q_t \widetilde{s}_{t|t} + (1-\alpha_k)\sum_{m=1}^{+\infty} \alpha_k^m R_{t-m}^L q_t \widetilde{s}_{t|t-m} \\ &= (1-\alpha_k) q_t R_t^L \widetilde{s}_{t|t} + \alpha_k (1-\delta)\frac{q_t}{q_{t-1}} rev_{t-1}.\end{aligned}$$

- where the latter follows from the fact that[13]:

$$\begin{aligned}rev_{t-1} &= (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m R_{t-1-m}^L q_{t-1} \widetilde{s}_{t-1|t-m-1} \\ &= (1-\alpha_k)\sum_{j=1}^{+\infty} \alpha_k^{j-1} R_{t-j}^L q_{t-1} \widetilde{s}_{t-1|t-j}.\end{aligned}$$

  The last equality implies $j = m+1 \Leftrightarrow j = m-1$. Then:

$$rev_{t-1}\alpha_k (1-\delta)\frac{q_t}{q_{t-1}} = (1-\alpha_k)\sum_{j=1}^{+\infty} \alpha_k^j R_{t-j}^L q_t \widetilde{s}_{t|t-j}.$$

- **Finding the recursive law of motion for** s_t :

$$\begin{aligned}s_t &= (1-\alpha_k)\widetilde{s}_{t|t} + (1-\alpha_k)\sum_{m=1}^{+\infty} \alpha_k^m \widetilde{s}_{t|t-m} \\ &= (1-\alpha_k)\widetilde{s}_{t|t} + \alpha_k (1-\delta) s_{t-1}\end{aligned}$$

  where the latter follows from the fact that:

$$\begin{aligned}s_{t-1} &= (1-\alpha_k)\sum_{m=0}^{+\infty} \alpha_k^m \widetilde{s}_{t-1|t-m-1} \\ &= (1-\alpha_k)\sum_{l=1}^{+\infty} \alpha_k^{l-1} \widetilde{s}_{t-1|t-l}\end{aligned}$$

[13] In these derivations we use the following: $\widetilde{s}_{t|t-i} = (1-\delta)^i \tilde{k}_{t-i+1} \Rightarrow \widetilde{s}_{t-1|t-i} = (1-\delta)^{i-1} \tilde{k}_{t-i+1} \Rightarrow \widetilde{s}_{t|t-i} = (1-\delta)\widetilde{s}_{t-1|t-i}$.

The last equality implies $l = m + 1 \Leftrightarrow l = m - 1$. Then:

$$\alpha_k (1-\delta) s_{t-1} = (1-\alpha_k) \sum_{l=1}^{+\infty} \alpha_k^l \widetilde{s}_{t|t-l}$$

B.3.2 The continuation value of banks

As in Gertler and Karadi (2009), it is convenient to consider another way of representing V_t. For this purpose, consider

$$V_t = (1-\tau) [q_t s_t x_{1,t} + n_t x_{2,t}]$$

where we have defined

$$\begin{aligned} x_{1,t} &\equiv \mathbf{E}_t \sum_{j=0}^{+\infty} (1-\alpha_b) \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} \left[\frac{rev_{t+j}}{q_t s_t} - R_{t+j} \frac{q_{t+j} s_{t+j}}{q_t s_t} \right] \\ x_{2,t} &\equiv \mathbf{E}_t \sum_{j=0}^{+\infty} (1-\alpha_b) \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} \frac{R_{t+j} n_{t+j}}{n_t} \end{aligned}$$

We will now derive the recursions for $x_{1,t}$ and $x_{2,t}$

- **Finding the recursive law of motion for** $x_{1,t}$:

$$\begin{aligned} x_{1,t} &= (1-\alpha_b) \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{rev_t}{q_t s_t} - R_t \right) \right] + \mathbf{E}_t \sum_{j=1}^{+\infty} (1-\alpha_b) \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+1}}{\lambda_t} \left[\frac{rev_{t+j}}{q_t s_t} - R_{t+j} \frac{q_{t+j} s_{t+j}}{q_t s_t} \right] \\ &= (1-\alpha_b) \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{rev_t}{q_t s_t} - R_t \right) \right] + \mathbf{E}_t \left[x_{1,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1} s_{t+1}}{q_t s_t} \right] \end{aligned}$$

where the latter follows from the fact that:

$$\begin{aligned} x_{1,t+1} &= \mathbf{E}_{t+1} \sum_{j=0}^{+\infty} (1-\alpha_b) \alpha_b^j \beta^{j+1} \frac{\lambda_{t+j+2}}{\lambda_{t+1}} \left[\frac{rev_{t+j+1}}{q_{t+1} s_{t+1}} - R_{t+j+1} \frac{q_{t+j+1} s_{t+j+1}}{q_{t+1} s_{t+1}} \right] \\ &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (1-\alpha_b) \alpha_b^{m-1} \beta^m \frac{\lambda_{t+m+1}}{\lambda_{t+1}} \left[\frac{rev_{t+m}}{q_{t+1} s_{t+1}} - R_{t+m} \frac{q_{t+m} s_{t+m}}{q_{t+1} s_{t+1}} \right]. \end{aligned}$$

The last equality implies $m = j + 1 \Leftrightarrow j = m - 1$. Then:

$$\begin{aligned} x_{1,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1} s_{t+1}}{q_t s_t} &= \mathbf{E}_{t+1} \sum_{m=1}^{+\infty} (1-\alpha_b) \alpha_b^m \beta^{m+1} \frac{\lambda_{t+m+1}}{\lambda_t} \left[\frac{rev_{t+m}}{q_t s_t} - R_{t+m} \frac{q_{t+m} s_{t+m}}{q_t s_t} \right] \\ &\Downarrow \\ \mathbf{E}_t \left[x_{1,t+1} \alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1} s_{t+1}}{q_t s_t} \right] &= \mathbf{E}_t \sum_{m=1}^{+\infty} (1-\alpha_b) \alpha_b^m \beta^{m+1} \frac{\lambda_{t+m+1}}{\lambda_t} \left[\frac{rev_{t+m}}{q_t s_t} - R_{t+m} \frac{q_{t+m} s_{t+m}}{q_t s_t} \right] \end{aligned}$$

due to the law of iterated expectations.

- **Finding the recursive law of motion for $x_{2,t}$:**

$$
\begin{aligned}
x_{2,t} &= (1-\alpha_b)\,\mathbf{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\right]R_t + \mathbf{E}_t\sum_{j=1}^{+\infty}(1-\alpha_b)\,\alpha_b^j\beta^{j+1}\frac{\lambda_{t+j+1}}{\lambda_t}\frac{R_{t+j}n_{t+j}}{n_t} \\
&= (1-\alpha_b) + \mathbf{E}_t\left[x_{2,t+1}\alpha_b\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{n_{t+1}}{n_t}\right]
\end{aligned}
$$

where the last equality follows from the fact that:

$$
\begin{aligned}
x_{2,t+1} &= \mathbf{E}_{t+1}\sum_{j=0}^{+\infty}(1-\alpha_b)\,\alpha_b^j\beta^{j+1}\frac{\lambda_{t+j+2}}{\lambda_{t+1}}\frac{R_{t+j+1}n_{t+j+1}}{n_{t+1}} \\
&= \mathbf{E}_{t+1}\sum_{m=1}^{+\infty}(1-\alpha_b)\,\alpha_b^{m-1}\beta^{m}\frac{\lambda_{t+m+1}}{\lambda_{t+1}}\frac{R_{t+m}n_{t+m}}{n_{t+1}}
\end{aligned}
$$

The last equality implies $m = j+1 \Leftrightarrow j = m-1$. Then:

$$
\begin{aligned}
x_{2,t+1}\alpha_b\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{n_{t+1}}{n_t} &= \mathbf{E}_{t+1}\sum_{m=1}^{+\infty}(1-\alpha_b)\,\alpha_b^m\beta^{m+1}\frac{\lambda_{t+m+1}}{\lambda_t}\frac{R_{t+m}n_{t+m}}{n_t} \\
&\Downarrow \\
\mathbf{E}_t\left[x_{2,t+1}\alpha_b\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{n_{t+1}}{n_t}\right] &= \mathbf{E}_t\sum_{m=1}^{+\infty}(1-\alpha_b)\,\alpha_b^m\beta^{m}\frac{\lambda_{t+1+m}}{\lambda_t}\frac{R_{t+m}n_{t+m}}{n_t}
\end{aligned}
$$

due to the law of iterated expectations.

B.3.3 The agency problem

Each bank f is subject to the following incentive constraint

$$V_{f,t} \geq \Lambda q_t s_{f,t}$$

We will assume that this constraint holds with equality in aggregation, therefore:

$$
\begin{aligned}
\int_0^1 V_{f,t}dj &= \Lambda q_t\int_0^1 s_{f,t}dj \\
&\Updownarrow \\
V_t &= \Lambda q_t s_t
\end{aligned}
$$

Following Gertler and Karadi (2009) we can now substitute the expression for V_t into the equation above to get:

$$
\begin{gathered}
(1-\tau)\left[q_t s_t x_{1,t} + n_t x_{2,t}\right] = \Lambda q_t s_t \\
\Updownarrow \\
\frac{q_t s_t}{n_t} = lev_t
\end{gathered}
$$

where we have defined $lev_t \equiv \frac{x_{2,t}}{\frac{\Lambda}{1-\tau}-x_{1,t}}$ as the leverage ratio.

B.4 Capital Producing Firms

The problem for the representative capital producing firm is then given by

$$\max_{\{i_{t+j}\}_{j=0}^{+\infty}} \mathbf{E}_t \sum_{i=0}^{+\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left[q_{t+i} \left(k_{t+1+i} - k_{t+i} \right) + q_t \delta k_{t+i} - i_{t+i} \right]$$

subject to

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right]$$

The first order condition associated to the optimal choice of i_t implies that:

$$q_t = \frac{1 - \mathbf{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left(\frac{i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right]}{\left[1 - S\left(\frac{i_t}{i_{t-1}} \right) - S'\left(\frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right]}$$

C Complete List of Equations for the Model with Banks

Household:

1) $\lambda_t = E_t\left[(c_t - bc_{t-1})_t^{-\phi_0} - \beta b\,(c_{t+1} - bc_t)^{-\phi_0}\right]$

2) $1 = E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} R_t\right]$

3) $\phi_2 h_t^{\phi_1} = \lambda_t w_t$

Good-Producing Firms:

4) $y_t = a_t k_t^{\theta} h_t^{1-\theta}$

5) $\tilde{h}_{t|t} = \left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1}{\theta}} \widetilde{k}_t$

6) $z_{1,t} + z_{2,t} = z_{3,t}$

7) $z_{1,t} = a_t \theta \tilde{k}_t^{\theta-1} \tilde{h}_{t|t}^{1-\theta} + \mathbf{E}_t\left[z_{1,t+1}\beta\alpha_k\,(1-\delta)\,\frac{\lambda_{t+1}}{\lambda_t}\right]$

8) $z_{2,t} = q_t\,(1-\delta) + \mathbf{E}_t\left[z_{2,t+1}\beta\alpha_k\,(1-\delta)\,\frac{\lambda_{t+1}}{\lambda_t}\right]$

9) $z_{3,t} = q_{t-1} R_t^L + \mathbf{E}_t\left[z_{3,t+1}\beta\alpha_k\,(1-\delta)\,\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t^L}{R_{t+1}^L}\right]$

The banking sector:

10) $n_{t+1} = (1-\tau)\left[rev_t - R_t q_t s_t + R_t n_t\right]$

11) $s_t = (1-\alpha_k)\,\widetilde{s}_{t|t} + \alpha_k\,(1-\delta)\,s_{t-1}$

12) $\widetilde{s}_{t-1|t-1} = \tilde{k}_t$

13) $rev_t = (1-\alpha_k)\,q_t R_t^L \widetilde{s}_{t|t} + \alpha_k\,(1-\delta)\,\frac{q_t}{q_{t-1}} rev_{t-1}$

14) $q_t s_t = (lev_t)\,n_t$

15) $lev_t = \frac{x_{2,t}}{\frac{\Lambda u_t}{1-\tau} - x_{1,t}}$

16) $x_{1,t} = (1-\alpha_b) E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{rev_t}{q_t s_t} - R_t\right)\right] + E_t\left[x_{1,t+1}\alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t}\frac{q_{t+1}s_{t+1}}{q_t s_t}\right]$

17) $x_{2,t} = (1-\alpha_b) + E_t\left[x_{2,t+1}\alpha_b \beta \frac{\lambda_{t+1}}{\lambda_t}\frac{n_{t+1}}{n_t}\right]$

Capital-Producing Firms:

18) $k_{t+1} = (1-\delta)\,k_t + i_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right]$

19) $q_t = \frac{1 - \mathbf{E}_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]}{\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right]}$

Market Clearing Conditions:

20) $k_t = (1-\alpha_k)\,\tilde{k}_t + \alpha_k\,(1-\delta)\,k_{t-1}$

21) $h_t = \left(\frac{w_t}{a_t(1-\theta)}\right)^{-\frac{1}{\theta}} k_t$

22) $y_t = c_t + i_t$

Exogenous Processes:

23) $\log a_t = \rho_a \log a_{t-1} + \varepsilon_t^a$