

# Optimal Research and Development Expenditure: a General Equilibrium Approach\*

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## Abstract

How much should be spent in research and development (R&D)? How should R&D vary over the business cycle? In this paper we answer both questions in the context of a calibrated dynamic general equilibrium model with Schumpeterian endogenous growth. Firstly, we demonstrate that, although the existence of distortions in a decentralized economy produces underinvestment in R&D, a simple proportional subsidy to R&D spending alone cannot restore the first best allocation. The optimal proportional R&D subsidy attains a second best allocation in which R&D spending exceeds its first best level. Secondly, we show how the observed procyclicality of R&D is socially inefficient. However, the welfare loss due to this dynamic inefficiency is much smaller than the loss due to underinvestment in R&D.

*Keywords:* Schumpeterian growth, technology adoption, optimal subsidy.  
JEL E32, O38, O40

## 1 Introduction

*How much should be spent in research and development (R&D)? How should R&D investment vary over the business cycle?* In this paper we try to provide an answer to both questions.

The first question refers to the socially optimum average amount of resources devoted to innovation activities. Romer (1990), Aghion and Howitt (1992) or Grossman and Helpman (1991, ch. 4) underline how in a decentralized economy several distortions may produce an inefficient allocation of resources to R&D. These distortions include the existence of monopolistic profits, the presence of knowledge spillovers, and the redistribution of rents from past innovators to current ones through a process of *creative destruction*. Jones and Williams (2000) analyze these effects in the context of an endogenous growth model calibrated with U.S. data and conclude that there exists underinvestment in R&D.

The second question refers to how R&D spending should respond to different shocks. The notion that macroeconomic shocks might affect R&D activities implies that such shocks can have long-lived consequences, far beyond any particular cyclical episode, as suggested by Comin and Gertler (2006). The traditional Schumpeterian view implies that recessions should promote innovation and restructuring activities, as in Caballero and Hammour (1994) or Aghion and Saint-Paul (1998). This view rests on the idea that the opportunity cost of R&D are lower in recessions, providing incentives to undertake such activities in downturns. Barlevy (2007), however, shows how R&D is procyclical in the United States. He explains this procyclicality as the result of the dynamic externalities that make entrepreneurs concentrate their innovation in booms.<sup>1</sup> He also concludes that optimal R&D would be less procyclical. Notwithstanding, his model is silent about the magnitude of the welfare loss of this ‘dynamic’ inefficiency, compared to the ‘static’ loss due to underinvestment in R&D.

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<sup>1</sup>There are other complementary explanations for the procyclicality of R&D such as credit frictions, as in Aghion et al. (2005) or endogenous labor supply (Fatas, 2000).

In this paper we present a dynamic stochastic general equilibrium (DSGE) model that integrates endogenous growth into an otherwise standard real business cycle (RBC) model. Endogenous growth is based on a Schumpeterian theory of vertical innovations *à la* Aghion and Howitt (1998, ch. 12). This theory provides a robust description of the historical growth process, as discussed in Howitt(2000) or Howitt and Mayer-Foulkes (2005). To analyze how much R&D is socially optimum, we calibrate the steady state solution of the decentralized economy model with U.S. data for the post-war period, and compare it with the counterfactual efficient allocation solution in the case of a social planner, a similar approach to Jones and Williams (2000). We follow a similar approach to analyze how R&D investments should be allocated over the cycle. Firstly we show that our calibrated model is able to replicate the main dynamic features of the data, especially the procyclicality of R&D spending and the high persistence of the responses to the shocks, and then we compare it with the counterfactual efficient allocation.

Our findings can be summarized as follows. The existence of distortions in a decentralized economy produces underinvestment in R&D, which generates lower business turnover and productivity growth than under the efficient allocation. A proportional subsidy to R&D does not allow the economy to reach its Pareto-efficient level. This is due to the fact that the economy is facing several distortions, so mitigating a single distortion may lead to a *second-best* situation. We compute the subsidy that maximizes social welfare under the constraints of a decentralized economy and show how it produces a level of R&D investment *higher* than under a social planner. We also find how the observed procyclicality in R&D expenditure may be explained as the response of rational entrepreneurs facing exogenous shocks. We extend the results of Barlevy (2007) by showing that although this procyclicality is inefficient, the associated welfare loss is more than one order of magnitude smaller than the loss due to the ‘static’ inefficiency. Therefore, even if these findings support the necessity of countercyclical policies, they underline the importance of structural policies that encourage R&D expenditure. Finally, we show how even in the case of an optimal R&D subsidy that partially mitigates steady state inefficiencies, equilibrium R&D is still too procyclical: a result that justifies the necessity of countercyclical policies.

In section 2 we lay out the decentralized model and the counterfactual efficient allocation. In section 3 we calibrate the model and derive results regarding the long-run behavior of the economy. In section 4 we analyze the dynamic properties of the model. Finally, in Section 5 we conclude.

## 2 The Model

We present a model that integrates endogenous growth in an otherwise conventional RBC model. Endogenous growth is based on vertical innovations as in Aghion and Howitt (1998, ch. 12) and Howitt (2000). The sketch of the model is as follows. Final goods producers use labor and a continuum of intermediate goods as inputs. These intermediate goods differ in their relative productivity and each of them is produced by a monopolistic firm using capital. The amount of capital necessary to produce each intermediate good is proportional to its productivity, thus reflecting that more advanced products require increasingly capital-intensive techniques. Each period, there is a probability that the productivity of an intermediate good jumps to the technology frontier due to the innovation activities of entrepreneurs. Entrepreneurs borrow resources and invest them in an attempt to increase their probabilities of making a discovery. If a discovery happens, the successful entrepreneur introduces a new enhanced intermediate product in her sector and becomes the new monopolist until the moment that she is replaced by another entrepreneur. The technology frontier, that is, the productivity level of the most advanced sector, evolves endogenously as the result of positive spillovers from innovation activities.

We introduce the model, characterize its equilibrium conditions and present a counterfactual model where decisions are taken by a benevolent social planner.

### 2.1 Final Goods Output

In the model, a country economy produces a final good under perfect competition by using labor and a continuum of intermediate products. Final goods firms maximize their profits

$$\max_{m_{j,t}, l_t} \left( Y_t - W_t l_t - \int_0^1 p_{j,t} m_{j,t} dj \right),$$

subject to

$$Y_t = z_t l_t^{1-\alpha} \left( \int_0^1 A_{j,t} m_{j,t}^\alpha dj \right), \quad (1)$$

where  $m_{j,t}$  is the flow output of intermediate product  $j \in [0, 1]$ ,  $l_t$  is labor supply<sup>2</sup>, and  $A_{j,t}$  is a productivity parameter attached to the latest version of intermediate product  $j$ .  $z_t$  is an aggregate productivity shock that follows a stationary AR(1) process with persistence  $\rho_z$  and variance of the innovation  $\sigma_z^2$ .<sup>3</sup> The model displays decreasing marginal products in each of the intermediate products and in labor. The first-order conditions are

$$p_{j,t} = \alpha A_{j,t} z_t l_t^{1-\alpha} m_{j,t}^{\alpha-1}, \quad (2)$$

and the wages

$$W_t = (1 - \alpha) \frac{Y_t}{l_t}. \quad (3)$$

## 2.2 Intermediate Goods Firms

Final output can be used interchangeably as a consumption or capital good, or as an input to innovation. Each intermediate product is produced by an incumbent monopolist using capital, according to the production function:

$$m_{j,t} = K_{j,t-1} / A_{j,t}, \quad (4)$$

where  $K_{j,t-1}$  is the capital in sector  $j$  at time  $t$ , installed in period  $t - 1$ . Division by  $A_{j,t}$  indicates that successive vintages of the intermediate product are produced by increasingly capital-intensive techniques. The incumbent monopolist of each sector solves the problem

$$\max_{m_{j,t}} (p_{j,t} m_{j,t} - q_t K_{j,t-1}),$$

subject to (2) and (4), where  $q_t$  is the rental cost of capital. Marginal costs and marginal revenues are proportional to  $A_{j,t}$ . Therefore all intermediate producers choose to supply the same amount of intermediate product  $m_t = \left( \frac{q_t}{\alpha^2 z_t l_t^{\alpha-1}} \right)^{1/(\alpha-1)}$ . The aggregate capital in the economy is  $K_{t-1} = \int_0^1 K_{j,t-1} dj = m_t A_t$ , where  $A_t = \int_0^1 A_{j,t} dj$  is the average productivity across all sectors in final-goods production. As a result, the aggregate production function of the economy (1) can be reduced to the standard constant returns to scale one  $Y_t = z_t K_{t-1}^\alpha (A_t l_t)^{1-\alpha}$ .

The cost of capital can be expressed as a function of the aggregate level of capital

$$q_t = \alpha^2 \frac{Y_t}{K_{t-1}}, \quad (5)$$

and the flow of profits that each incumbent earns is

$$\Pi_{j,t}(A_{j,t}) = \alpha (1 - \alpha) Y_t \frac{A_{j,t}}{A_t}, \quad (6)$$

so that a share  $(1 - \alpha)$  of final output is allocated to wages,  $\alpha^2$  to capital costs and  $\alpha(1 - \alpha)$  to profits.

## 2.3 Productivity

Innovations result from entrepreneurship that uses technological knowledge. At any date there is a “technology frontier” that represents the most advanced technology across all the sectors:

$$A_t^{\max} \equiv \max\{A_{j,t} \mid j \in [0, 1]\}. \quad (7)$$

<sup>2</sup>We abstract from population growth by assuming that variables are scaled by the working age population. This may be seen as a special case of Howitt (2000) where the number of sectors is not constant, but grows asymptotically at the same rate as the population so that the model does not exhibit the sort of scale effect that Jones (1995) argues is contradicted by postwar trends in R&D spending and productivity.

<sup>3</sup>Throughout the paper we denote with capital letters  $S_t$  the non stationary variables whereas we reserve lowercase letters  $s_t$  for stationary variables. Variables in steady state are denoted without time subscript  $s$ .

Each period, productivity in sector  $j$  evolves according to

$$A_{j,t+1} = \left\{ \begin{array}{ll} A_t^{\max}, & \text{with probability } n_{j,t} \\ A_{j,t}, & \text{with probability } 1 - n_{j,t} \end{array} \right\}. \quad (8)$$

Once an innovation happens, it creates an improved version of the existing product by raising its productivity  $A_{j,t+1}$  to the technology frontier  $A_t^{\max}$ . The entrepreneur then enters into Bertrand competition with the previous incumbent in that sector, who by definition produces a good of inferior quality. Rather than facing a price war with a superior rival, the incumbent exits. Having exited, the former incumbent cannot threaten to reenter. Therefore, in  $t + 1$  the former entrepreneur has become the new incumbent.

The probability  $n_{j,t}$  is a function of the quantity of final output devoted to R&D in this sector  $X_{j,t}$ :

$$n_{j,t} = \left( \frac{X_{j,t}}{\lambda A_t^{\max}} \right)^{\frac{1}{(\eta+1)}}, \quad \eta > 0. \quad (9)$$

Equation (9) displays decreasing returns to scale in innovation<sup>4</sup>. The parameter  $\lambda$  accounts for the productivity of resources devoted to R&D. The amount of resources is adjusted by the technology frontier variable  $A_t^{\max}$  to represent the increasing complexity of progress: as technology advances, the resource cost of further advances increases proportionally.

## 2.4 Entrepreneurs

The value of being the incumbent in period  $t$  in a sector with productivity  $\bar{A}$ ,  $V_{j,t}(\bar{A})$ , is the discounted flow of profits that the incumbent may obtain by taking into account the probability of obsolescence due to the arrival of a new innovation in this sector, so

$$V_{j,t}(\bar{A}) = \Pi_{j,t}(\bar{A}) + \frac{(1 - n_{j,t})}{r_t} E_t [V_{j,t+1}(\bar{A})], \quad (10)$$

where  $r_t$  is the risk-free interest rate. The first term reflects the flow of profits of the monopolist whereas the second term is the discounted value of still being the incumbent at  $t + 1$ .

We consider that each period there is a single entrepreneur in each sector. Her problem can be expressed as

$$\max_{X_{j,t}} \frac{n_{j,t}}{r_t} E_t [V_{j,t+1}(A_t^{\max})] - (1 - \tau) X_{j,t}, \quad (11)$$

subject to (9), where  $\tau X_{j,t}$  is the amount of government-subsidized R&D. It means that the entrepreneur maximizes the discounted value of becoming the incumbent the next period, weighted by the probability of doing so, which is a function of the amount of R&D spending. The first order condition is that the marginal costs of an extra unit of goods allocated to research  $(1 - \tau)$  equal the discounted marginal expected benefit  $\frac{E_t[V_{j,t+1}(A_t^{\max})]mc_t}{A_t^{\max}r_t}$ , where  $mc_t \equiv A_t^{\max} \frac{dn_{j,t}}{dX_{j,t}} = \frac{1}{\lambda n_{j,t}^{\eta}(\eta+1)}$ . Since the value of becoming the incumbent in the next period  $V_{j,t+1}(A_t^{\max})$  is the same for all sectors (as all of them jump to the technology frontier if an innovation happens), the input invested in R&D in each intermediate sector is the same:  $X_{j,t} = X_t$  and  $n_{j,t} = n_t$ .

**Proposition 1** *In a decentralized economy, the aggregate business turnover  $n_t$  is given by*

$$n_t^{\eta} = E_t \left[ \frac{\alpha(1 - \alpha)Y_{t+1}}{(1 - \tau)\lambda(\eta + 1)A_{t+1}r_t} + \frac{(1 - n_{t+1})n_{t+1}^{\eta}}{r_t} \right]. \quad (12)$$

**Proof.** See Appendix A.1. ■

Growth in the leading-edge parameter  $A_t^{\max}$  occurs as a result of the knowledge spillovers produced by innovations, as in Aghion and Howitt (1992). At any moment in time, the technology frontier is available to any successful innovator, and this publicly available knowledge grows at a rate proportional to the aggregate rate of innovations. Therefore we have

$$g_t \equiv \frac{A_t^{\max}}{A_{t-1}^{\max}} = 1 + \sigma n_{t-1}, \quad (13)$$

where  $\sigma$  is the spillover coefficient.

<sup>4</sup>Previous studies have found decreasing returns in R&D expenditure, such as Kortum (1993).

## 2.5 Households

The representative household solves

$$\max_{C_t, I_t, B_t, K_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{l_t^{1+\psi}}{1+\psi} \right], \quad (14)$$

with  $0 < \beta < 1$ , subject to

$$C_t + I_t + \frac{B_t}{r_t} + T_t = W_t l_t + q_t K_{t-1} + D_t + B_{t-1}, \quad (15)$$

$$K_t = I_t + (1 - \delta) K_{t-1}, \quad (16)$$

where  $C_t$  is consumption,  $I_t$  is investment,  $B_t$  is the amount of state-contingent bonds,  $T_t$  is a government tax and  $D_t = \int_0^1 (\Pi_{j,t} - (1 - \tau) X_{j,t}) dj$  are the dividends from an investment fund that finances entrepreneurs' investments and collects the profits from the ownership of the monopolist firms

The solution of the households' problem yields the standard Euler equations for the risk-free interest rate and the cost of capital and the relationship of wages with the marginal rate of substitution between consumption and labor:

$$1 = E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) r_t \right], \quad (17)$$

$$1 = E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) (q_{t+1} + (1 - \delta)) \right], \quad (18)$$

$$W_t = l_t^\psi C_t. \quad (19)$$

## 2.6 Equilibrium

A competitive equilibrium for this economy is a set of prices and allocations so that given prices households, final and intermediate firms and entrepreneurs solve their maximization problems and markets clear. The capital rental market clears when the demand for capital by intermediate good producers equals the supply by households. The labor market clears when firms' demand for labor equals labor supply by households. The government always runs a balanced budget so that taxes are equal to government subsidies  $T_t = \tau X_t$ . Finally, the final goods market clears if production equals demand for consumption, capital accumulation and entrepreneurship

$$Y_t = C_t + I_t + X_t. \quad (20)$$

In equilibrium, the evolution of the average productivity of the economy is given by the number of sectors that experience an innovation:

$$A_t = \int_0^1 [n_{j,t-1} A_{t-1}^{\max} + (1 - n_{j,t-1}) A_{j,t-1}] dj = n_{t-1} (A_{t-1}^{\max} - A_{t-1}) + A_{t-1}, \quad (21)$$

which describes how the productivity increases due to the distance to the technology frontier  $A_{t-1}^{\max} - A_{t-1}$  multiplied by the entry rate of new firms  $n_{t-1}$  (the number of sector where a new incumbent appears).

## 2.7 The Efficient Allocation

In the model presented above the competitive equilibrium may not be socially optimal. This is due to the existence of monopolistic competition in the intermediate goods sector and to the spillovers associated with the decentralized innovation process, which entrepreneurs do not internalize when making their R&D decisions. The existence of these distortions may produce an inefficient allocation of resources so that the equilibrium is not Pareto optimal, as discussed in Aghion and Howitt (1992).

To see how the economy behaves in the Pareto optimal case we assume that the economy is managed by a benevolent social planner who maximizes

$$\max_{C_t, I_t, K_t, l_t, X_t, A_t, A_t^{\max}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{l_t^{1+\psi}}{1+\psi} \right]$$

subject to the aggregate budget constraint (20), the capital accumulation law (16), the production function of entrepreneurs (9), the evolution of aggregate productivity (21) and the spillover effect of innovation into technology growth (13).

The first order conditions are

$$\begin{aligned} E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) (\gamma_{1,t+1} n_t + \gamma_{2,t+1} g_{t+1}) \right] &= X_t + \gamma_{2,t}, \\ E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) (\gamma_{1,t+1} (1 - n_t) + \gamma_{2,t+1} g_{t+1}) \right] &= -\frac{(1 - \alpha) Y_t}{A_t} + \gamma_{1,t}, \\ E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) (\gamma_{1,t+1} (1 - a_t) + \gamma_{2,t+1} \sigma) \right] &= (1 + \eta) \frac{X_t}{n_t}, \\ E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) \left( \alpha \frac{Y_{t+1}}{K_t} + (1 - \delta) \right) \right] &= 1 \end{aligned}$$

and equation (19), where  $\gamma_{1,t}$  and  $\gamma_{2,t}$  are the Lagrange multipliers of (21), and (13), respectively. In this case, there are no government subsidies ( $\tau = 0$ ).

### 3 How much R&D Expenditure?

#### 3.1 Calibration

In this section we use a calibrated version of the model to study the long-run implications of R&D expenditure. The model has a deterministic steady state that displays a balanced-growth path, where variables  $Y_t$ ,  $C_t$ ,  $I_t$ ,  $X_t$ ,  $A_t$ ,  $A_t^{\max}$ ,  $W_t$ ,  $\Pi_t$ , and  $K_t$  grow at rate  $g = 1 + \sigma n$ , whereas  $n_t$ ,  $q_t$ ,  $l_t$ , and  $g_t$  are stationary. In order to solve the model, we divide  $Y_t$ ,  $C_t$ ,  $I_t$ ,  $X_t$ ,  $A_t$ ,  $W_t$ ,  $\Pi_t$ , and  $K_t$  and by  $A_t^{\max}$  to make them stationary.

The parameters of the model are calibrated to match key empirical evidence in the United States for the post-war period. Information about data sources is provided in Appendix B. The value of  $\alpha$  is set to 0.35 so that the share of output devoted to labor compensation is 65%. The average growth of GDP per working-age population is 1.9% so that assuming a real interest rate of 4%, we obtain a value of  $\beta = \frac{g}{r}$  of 0.9804. Capital depreciation  $\delta$  is set to 10% per year and Frisch labor supply elasticity  $1/\psi$  to 1, all standard values in the literature.

We match the values of business turnover and public and private R&D spending with the parameters  $\lambda$ ,  $\eta$  and  $\tau$ . The average business turnover of U.S. firms  $n$  (the rate of creation/destruction of firms in the economy) in the last two decades has been 10%. As shown in Figure 1, this value is also consistent with the empirical evidence about average survival rates for the 1963 and 1976 cohorts of U.S. manufacturing firms. The total share of R&D spending  $X$  has been roughly stable at about 2.6 percent for most of the post-war period. However, the shares of private and public R&D have significantly changed during this period. We concentrate on the last three decades so that the average share of public R&D is around 1% and therefore we set  $\tau$  to 0.4,  $\log(\lambda)$  to 25.73 and  $\eta$  to 12, respectively. Finally, given a business turnover of 10%, to replicate the value of GDP growth of 1.9% we set the spillover coefficient  $\sigma$  to 0.19. Table 1 displays the comparison between the benchmark calibration and the data.

#### 3.2 Pareto-efficient R&D Investment

To analyze efficient R&D expenditures, we solve the social planner's problem for the set of calibrated parameters presented above. Table 1 presents the results. In this case, total R&D spending (public plus

private) represents 4.8 percent of the GDP, which results in an increase in long-term growth (2.1%) and in business turnover (11.2%). The ratio between observed R&D investment and its efficient is close to 2. This is in line with the calibration exercise of Jones and Williams (2000), who find ratios between 1 and 3 for plausible parametrizations.<sup>5</sup>

To compute how this translates to social welfare, we compute the steady state utility as

**Proposition 2** *Normalizing  $A_0^{\max} = 1$ , the steady state value of the representative household's utility is*

$$U = \frac{\log(c)}{(1-\beta)} - \frac{l^{1+\psi}}{(1-\beta)(1+\psi)} + \frac{\beta \log(g)}{(1-\beta)^2}. \quad (22)$$

**Proof.** See Appendix A.2. ■

Welfare depends on steady state effective consumption ( $c_t \equiv \frac{C_t}{A_t^{\max}}$ ), leisure *and growth*. Naturally, the impact of growth is quite significant as it is weighted by  $\frac{\beta}{(1-\beta)^2}$  which is higher than  $\frac{1}{(1-\beta)}$  for the calibrated value of  $\beta$ . This is so as a higher growth rate allows households to consume more in the future. As presented in Table 1, the social welfare in the Pareto optimal case is more than 9 times higher than in a decentralized equilibrium.

### 3.3 R&D Subsidies

To mitigate the welfare loss due to the suboptimal allocation of resources in the decentralized equilibrium, governments may subsidize R&D expenditures so that innovators internalize some of the spillovers derived from their activities. The change in steady state utility  $U$  as a function of the value of  $\tau$  is shown in Figure 2. The scale is normalized so that  $U(\tau = 0.4)$  is set to 1 (the benchmark case). Total utility grows as subsidies increase until a maximum is reached (at  $\tau = 0.78$ ). We define this value as the “optimal subsidy”, that is, the subsidy that maximizes welfare under the constraints of a decentralized economy.

Table 1 displays the results for the case of no subsidies and for the optimal subsidy. When the optimal subsidy is applied, total R&D expenditures rise to 6.9% of the GDP, a higher value than in the Pareto-efficient allocation. Most of this increase is due to public R&D (5.3%) whereas private R&D increases slightly (only an additional 0.1%). The increase in R&D spending raises growth and business turnover. The net impact on welfare is a two-fold increase with respect to the benchmark. Notwithstanding, social welfare with the optimal subsidy is still below the level with a benevolent social planner.

Why is total R&D investment different with the optimal subsidy than under a social planner? This is due to the fact that the economy is facing several distortions, so mitigating one distortion may lead to a *second-best* situation. Figure 3 shows how public subsidies affect the different components of welfare. An increase in public R&D redirects resources from consumption and investment to innovation, thus reducing effective output and consumption. This fall in output forces agents to work more hours, with the consequent loss in leisure utility. At the same time, the increase in public R&D increases growth and stimulates private R&D. The growth effect is more significant for subsidies below 0.78 whereas the consumption and leisure effects prevail above this subsidy level.

Finally, and for the matter of comparison, we include in Table 1 the case of no subsidies. In this situation growth and welfare fall in comparison to the benchmark, as entrepreneurs decide to invest slightly less (the difference is in the second decimal) and there is no public R&D.

### 3.4 R&D Subsidies versus Corporate Subsidies

R&D subsidies allow entrepreneurs to internalize some of the spillovers of innovation by reducing their R&D sunk costs. An alternative approach would be to increase their prospective profits so that the value of becoming the incumbent in a sector  $V_{j,t}$  increases. It can be done by subsidizing profits (or cutting corporate taxes, if they are present).

**Proposition 3** *In a decentralized economy, a subsidy to corporate profits  $\phi$  financed by lump-sum taxes is equivalent to a proportional R&D subsidy  $\tau = \frac{\phi}{1+\phi}$ .*

<sup>5</sup>Estimating the social return to R&D, Jones and Williams (1998) conclude that a “conservative” lower bound to the ratio between optimal R&D investment and actual investment would be around 4.

**Proof.** See Appendix A.3. ■

This is so in the context of the model due to the fact that incumbents do no research. Thus, corporate profits affect exclusively the prospective value of becoming the next monopolist (Aghion and Howitt, 1998). We leave for further research the study of how this result would change in the case of R&D activities by incumbents.

### 3.5 Robustness

The main results presented in this section are robust to reasonable changes in parameter values. Notwithstanding, it is interesting to explore what happens in an extreme case where current observed public R&D investment cannot be considered as R&D subsidies and therefore it has no effect on innovation ( $\tau = 0$ ). We then recalibrate parameters  $\lambda$ ,  $\eta$  and to explain the stylized facts. The values of  $\log(\lambda)$  and  $\eta$  are 25.32 and 12 (as in the standard case), respectively.

Table 2 suggests how the results change very little. R&D expenditure under a social planner is now 5.0% and it produces a 2.2% growth rate. Optimal R&D subsidies are roughly the same as with the standard calibration, now producing a higher effect on business turnover and growth. In conclusion, the main results of this section (the suboptimality of R&D spending and the possibility of partially mitigating it by increasing public subsidies) remain unaltered.

## 4 How Should it Vary over the Cycle?

### 4.1 Procyclical R&D

Barlevy (2007) finds a positive correlation between the growth rate of R&D expenditure and GDP growth of 0.39. As our model is in per capita terms,<sup>6</sup> we recompute this correlation in per capita terms for the growth rates of R&D  $dx_t$  and GDP  $dy_t$  to find it to be 0.26, which confirms the procyclicality of R&D. To check whether this procyclicality may be reproduced in the context of our model, we calibrate the parameters  $\rho_z$  and  $\sigma_z$  of the temporary aggregate productivity shock  $z_t$  so that the model replicates the volatility and first autocorrelation coefficient of the GDP growth ( $\rho_z = 0.96$ ,  $\sigma_z = 0.02$ ).<sup>7</sup>

Results in table 3 show that in this case the benchmark model produces a correlation between GDP and R&D of 0.997, higher than the one observed in the data. Additionally, the model generates a volatility of R&D growth of 1.9%, roughly half of the one observed in the data. Compared to Barlevy (2007), we do not need to introduce any fixed cost to generate procyclicality due to the endogeneity of labor supply and the use of final output in R&D activities. Figure 4 shows the impulse responses to a  $z_t$  shock of GDP, the R&D share, business turnover and employment. GDP is expressed as log-deviations from the steady state linear growth trend. When a positive shock increases output, the size of the potential market for entrepreneurs increases, thus encouraging innovation. The rise in the prospects of higher profits also induce entrepreneurs to expect higher business turnover in the future, which discourages innovation. This trade-off between higher profits and shorter monopolies produces a temporary increase in R&D expenditure accompanied by a temporary increase in the growth rate of productivity, which generates a *permanent* increase in the deviation of the GDP from its trend. The increase in R&D is slightly less procyclical than the GDP, so that the R&D share decreases by almost 0.01 percentage points during the first years, to remain constant thereafter.

We extend the analysis to other types of shocks. Firstly, we set a labor shock  $\mu_t$  so that  $W_t = \mu_t l_t^\psi C_t$ . This is the shock considered in Comin and Gertler (2006) and can be considered as a shock to the labor disutility or a wage markup shock. We consider the shock to follow an AR(1) and calibrate it as in the previous case. Results in table 3 show how this shock produces quantitatively similar results to the aggregate productivity shock.<sup>8</sup> This confirms the results by Comin and Gertler (2006), who show how this non-technological shock may drive business cycles at the high frequencies and generate the strong medium frequency movements in productivity observed in the data. The main difference is that Comin

<sup>6</sup> More precisely, in terms *per working age person* terms, as we do not model any kind of population dynamics.

<sup>7</sup> Computations are performed in dynare. Data frequency is annual. We always refer to the GDP and R&D expenditure per person aged 15-64. Additional information about data sources may be found in Appendix B.

<sup>8</sup> We omit a figure as the impulse response is similar to that of a productivity shock.



and Gertler consider endogenous technological change à la Romer (1990), so that the product survival rate is constant throughout the cycle, whereas in our model it is also endogenous via creative destruction.

Both the productivity shock and the labor shock generate correlations between GDP and R&D higher than the observed in the data. This result does not apply to all the shocks. For example in table 3 we also consider an i.i.d. shock to the spillover parameter  $\sigma_t$  that we calibrate to replicate the variance of the GDP growth. In this case the model is not able to replicate the output autocorrelation,<sup>9</sup> but it generates a volatility of R&D higher than the empirical one and a negative (and small) correlation between GDP and R&D growth. Figure 5 displays the impulse responses to this shock. When a shock increases  $\sigma_t$  at year 1, it immediately raises the growth rate of the technology frontier, given (13). The rise in  $A_t^{\max}$  increases the expected value of becoming the incumbent in a sector and the amount of resources necessary to achieve an innovation in equation (9). Therefore, the initial effects of the shocks are a fall in the business turnover and an surge in R&D expenditure, which reduces consumption, investment and labor supply. In the coming years, these contractionary effects are compensated by an increase in the productivity of the economy as the increased R&D investments begin to pay results. The final effect is that an i.i.d. shock generates a permanent increase in the GDP level that reaches its plateau after at least a couple of decades. This type of response resemble the models of “general purpose technologies” such as Helpman and Trajtenberg (1994), where the arrival of a new technology that raises output and productivity in the long run can also cause cyclical fluctuations while the economy adjusts to it, a fact documented in cases such as the steam engine or the dynamo by Lipsey, Carlaw and Bekar (2005).

The level of procyclicality of R&D observed in the data could be explained as the result of the interaction of shocks that produce strong procyclicality, such as shocks to labor supply or productivity, and countercyclical shocks, such as the spillover shock.

## 4.2 Pareto-efficient Dynamic R&D Investment

According to Barlevy (2007), socially optimal R&D spending can be countercyclical under very restrictive assumptions, such as constant labor supply. When these constraints are relaxed, socially optimal R&D seems to be procyclical, although less than decentralized one. To analyze whether this conclusion still holds in the context of our model we compare the moments and impulse responses between the benchmark and the efficient allocation under the same shocks.

Results in table 3 show how the correlation between GDP and R&D growth for productivity and labor shocks is 0.97. This value extend Barlevy’s conclusion that socially optimal R&D investment is less procyclical than the decentralized one. The difference is larger in the case of the spillover shock, where the socially optimal response is quite countercyclical, with a correlation coefficient of -0.28. Figures 4 and 5 display a comparison between the benchmark and the response with a social planner. In both the cases the socially optimal response induces a more pronounced change in the R&D share in the first years after the shock. In addition, the autocorrelation of output is smaller under a social planner than in a decentralized economy. The conclusion is that the presence of economic distortions inefficiently prolong the effects of exogenous shocks.

How important is this dynamic effect compared to the ‘static’ one, that is, to the welfare loss in steady state?. To analyze this, we compute numerically the unconditional expected utility of the representative agent (14) by MonteCarlo methods.<sup>10</sup> Results in table 4 show how the divergence between this simulated approach and the theoretical steady state utility equation (22) is less than 1%. We employ this approach to assess the welfare loss in the case of productivity shocks  $z_t$  with the calibration presented above. Table 4 shows how the presence of shocks generates a welfare loss of 15 percent with respect to the benchmark steady state utility. In the optimal case, this welfare loss would be of 10 percent of the benchmark utility. Therefore, the differential impact of the shocks in both cases is around a 5 percent of the benchmark steady state utility. This amount is small compared to the potential gains of mitigating the steady state

<sup>9</sup>This output autocorrelation could have been replicated by considering this shock to be AR(1) instead of i.i.d.. We leave this for further research.

<sup>10</sup>In each simulation, we generate a vector of  $T = 500$  years of shocks  $\omega_i = [\omega_1^i, \dots, \omega_T^i]'$ , simulate the corresponding endogenous variables  $C_t(\omega_i)$  and  $l_t(\omega_i)$ , and compute the cumulative discounted utility  $U(\omega_i) = \sum_{t=0}^T \beta^t \left[ \log(C_t(\omega_i)) - \frac{l_t(\omega_i)^{1+\psi}}{1+\psi} \right]$ . We then perform the average over  $N = 1,000$  simulations to compute the expected utility  $\sum_{i=0}^N U(\omega_i)$ .

welfare differences. The conclusion is that, even if a countercyclical policy is welfare-enhancing, structural policies aimed at increasing R&D expenditures have a higher pay-off in terms of utility.

### 4.3 The Case with Optimal Constant R&D Subsidies

In section 2 we have shown how the ‘static’ welfare loss is partially mitigated by introducing a proportional subsidy to R&D expenditure or to corporate profits. Does this subsidy also help to improve the dynamic response of R&D? To check it, we simulate the decentralized model under the shocks assuming a subsidy rate of  $\tau = 0.78$ . Results are shown in table 3 and figure 6. In general there is not a considerable change in second order moments. Notwithstanding, the simulated welfare analysis of figure 4 shows how the dynamic welfare loss in this case (13 percent of the benchmark) represents a midpoint between the losses in the benchmark and optimal cases.

The conclusion is that the subsidy helps to mitigate not only the static losses, but the dynamic ones. However, as commented above, the static welfare differences are one order of magnitude larger than the dynamic ones.

## 5 Conclusions

This paper examines which is the optimal R&D expenditure from a social welfare point of view. The main results are two. Firstly, the use of a proportional R&D subsidy is not able to completely mitigate the distortions introduced by monopolistic competition, creative destruction and technology spillover. The optimal R&D subsidy induces a level of R&D higher than in the efficient allocation, which for the U.S. seems to be around 7% of the GDP. Secondly, although optimal R&D investment is procyclical, it is less procyclical than in the case of a decentralized economy. Notwithstanding, from a welfare perspective, there are more gains in structural policy interventions, such as an increase in the R&D subsidy, than in countercyclical policies.

Given the potential gains from increasing R&D expenditure and the suboptimality of a proportional subsidy, an interesting question is how a tax system should be designed in order to incentivize innovation. In particular, it is not clear whether this optimal tax system would allow the economy to reach the first best optimum. We leave this question for future research.

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## Appendix A: Proofs

### Proposition 1

**Proof.** We can write equation (10) as

$$V_t(\bar{A}) = E_t \left[ \sum_{i=t}^{\infty} \left( \prod_{l=t}^{i-1} \frac{(1-n_l)}{r_l} \right) \frac{Y_i}{A_i} \right] \alpha (1-\alpha) \bar{A},$$

so the first order condition of the entrepreneur results in

$$\frac{(1-\tau)r_t}{mc_t} = \frac{E_t [V_{t+1}(A_t^{\max})]}{A_t^{\max}} = E_t \left[ \sum_{i=t+1}^{\infty} \left( \prod_{l=t+1}^{i-1} \frac{(1-n_l)}{r_l} \right) \frac{Y_i}{A_i} \right] \alpha (1-\alpha).$$

We can use this condition to express  $V_t(A_{t-1}^{\max})$  as

$$\begin{aligned} V_t(A_{t-1}^{\max}) &= \frac{\alpha(1-\alpha)Y_t A_{t-1}^{\max}}{A_t} + \frac{(1-n_t)}{r_t} E_t \left[ \sum_{i=t+1}^{\infty} \left( \prod_{l=t+1}^{i-1} \frac{(1-n_l)}{r_l} \right) \frac{Y_i}{A_i} \right] \alpha(1-\alpha) A_{t-1}^{\max} \\ &= \frac{\alpha(1-\alpha)Y_t A_{t-1}^{\max}}{A_t} + \frac{(1-n_t)(1-\tau)}{mc_t} A_{t-1}^{\max}, \end{aligned}$$

and taking expectations of the next period

$$E_t [V_{t+1}(A_t^{\max})] = E_t \left[ \frac{\alpha(1-\alpha)Y_{t+1}}{A_{t+1}} + \frac{(1-n_{t+1})(1-\tau)}{mc_{t+1}} \right] A_t^{\max},$$

that can be re-introduced in the first order condition of the entrepreneur to get

$$E_t \left[ \frac{\alpha(1-\alpha)Y_{t+1}}{A_{t+1}} + \frac{(1-n_{t+1})(1-\tau)}{mc_{t+1}} \right] = \frac{(1-\tau)r_t}{mc_t}.$$

■

## Proposition 2

**Proof.** Given equation (14), a household's expected utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{l_t^{1+\psi}}{1+\psi} \right] = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) + \log(A_{t-1}^{\max}) + \log(g_t) - \frac{l_t^{1+\psi}}{1+\psi} \right].$$

In steady state and with  $A_0^{\max} = 1$ , this expression can be simplified to

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \left[ \log(c) + \log(A_0^{\max}) + \log(g^t) - \frac{l^{1+\psi}}{1+\psi} \right] &= \left[ \log(c) - \frac{l^{1+\psi}}{1+\psi} \right] \left( \sum_{t=0}^{\infty} \beta^t \right) + \log(g) \left( \sum_{t=0}^{\infty} t \beta^t \right) \\ &= \frac{\left[ \log(c) - \frac{l^{1+\psi}}{1+\psi} \right]}{1-\beta} + \frac{\log(g)\beta}{(1-\beta)^2}. \end{aligned}$$

■

## Proposition 3

**Proof.** A subsidy  $\phi$  to corporate profits modifies equation (10) as

$$V_{j,t}(\bar{A}) = (1 + \phi) \Pi_{j,t}(\bar{A}) + \frac{(1 - n_{j,t})}{r_t} E_t [V_{j,t+1}(\bar{A})],$$

so that proceeding like in proposition 1, the first order condition for entrepreneurs results in

$$E_t \left[ (1 + \phi) \frac{\alpha(1 - \alpha) Y_{t+1}}{A_{t+1}} + \frac{(1 - n_{t+1})}{mc_{t+1}} \right] = \frac{r_t}{mc_t},$$

and therefore if  $(1 + \phi) = \frac{1}{(1-\tau)}$  then  $\tau = \frac{\phi}{1+\phi}$ . ■

## Appendix B: Data Sources

GDP data for the period 1950-2007 comes from the *Bureau of Economic Analysis*. Data about civilian noninstitutional population aged 16 and older 1950-2007 comes from the *Bureau of Labor Statistics*. Information about business turnover is provided for the period 1990-2003 by the *U.S. Small Business Administration*. Evidence about survival rates for the 1963 and 1976 cohorts is obtained from Dunne, Roberts and Samuelson (1988) and Audretsch (1991), respectively. Data about R&D expenditures 1953-2007 comes from the *National Science Foundation*.

## Appendix C: Tables and figures

Table 1. Comparison between data, benchmark, social optimum and optimal subsidy

%	$\tau$	GDP growth	Welfare	Turnover	R&D Subsidies	Private R&D	Total R&D
Data	-	1.9	-	10.0	1.1	1.5	2.6
Benchmark	40	1.9	100	10.0	1.0	1.5	2.5
Social Planner	-	2.1	927	11.2	-	-	4.8
No subsidy	0	1.8	50	9.7	0	1.5	1.5
Optimal subsidy	78	2.0	154	10.9	5.3	1.6	6.9

Welfare refers to the steady state utility compared to that of the benchmark

GDP growth refers to the growth of GDP per working age population (civilian noninstitutional population 16 years and older)

Table 2. Robustness check when there are no public subsidies in the benchmark

%	$\tau$	GDP growth	Welfare	Turnover	R&D Subsidies	Private R&D	Total R&D
Benchmark	0	1.9	100	10.0	0	1.5	1.5
Social Planner	-	2.2	939	11.6	-	-	5.0
Optimal subsidy	79	2.1	205	11.3	5.3	1.6	7.2

The benchmark refers to the new calibration

Table 3. Second moments of GDP and R&D growth

%		$\sigma(dy_t)$	$\rho(dy_t, dy_{t-1})$	$\sigma(dx_t)$	$\rho(dy_t, dx_t)$
Data	-	2.2	8.8	4.4	26.6
Benchmark	$z_t$ shock	2.2	8.2	1.9	99.7
Benchmark	$\mu_t$ shock	2.2	8.2	1.9	99.7
Benchmark	$\sigma_t$ shock	2.2	91.7	5.8	-1.9
Social Planner	$z_t$ shock	2.4	4.1	1.5	97.2
Social Planner	$\mu_t$ shock	2.4	4.1	1.5	97.2
Social Planner	$\sigma_t$ shock	2.7	81.4	11.6	-27.9
Optimal Tax	$z_t$ shock	2.2	8.2	1.9	99.7
Optimal Tax	$\mu_t$ shock	2.2	8.2	1.9	99.7
Optimal Tax	$\sigma_t$ shock	2.4	92.7	6.3	1.1

GDP and R&D growth per working age person

$\sigma(\cdot)$  denotes volatility and  $\rho(\cdot)$  is the correlation coefficient

Table 4. Dynamic welfare analysis with productivity shocks

%	Benchmark	Social Planner	Optimal subsidy
Steady state welfare	100	927	154
No shocks welfare	99	926	153
Dynamic welfare	84	916	140
Dynamic welfare loss	15	10	13

Welfare refers to utility compared to that of the steady state benchmark

Mean values after 1.000 Monte Carlo simulations of length 500 years

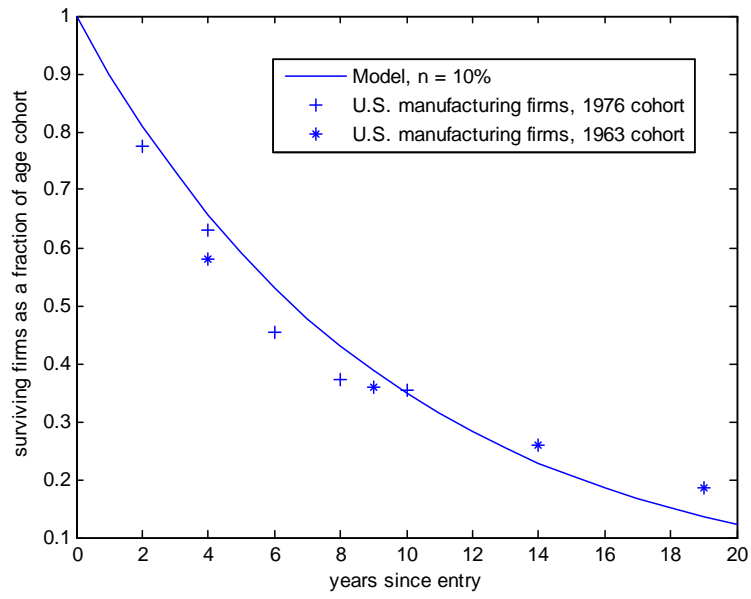


Figure 1: Survival rates of firms in the economy

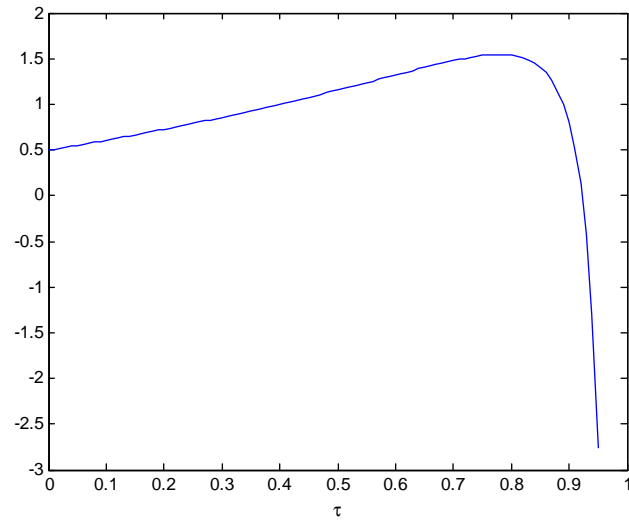


Figure 2: Impact of a general R&D subsidy on welfare

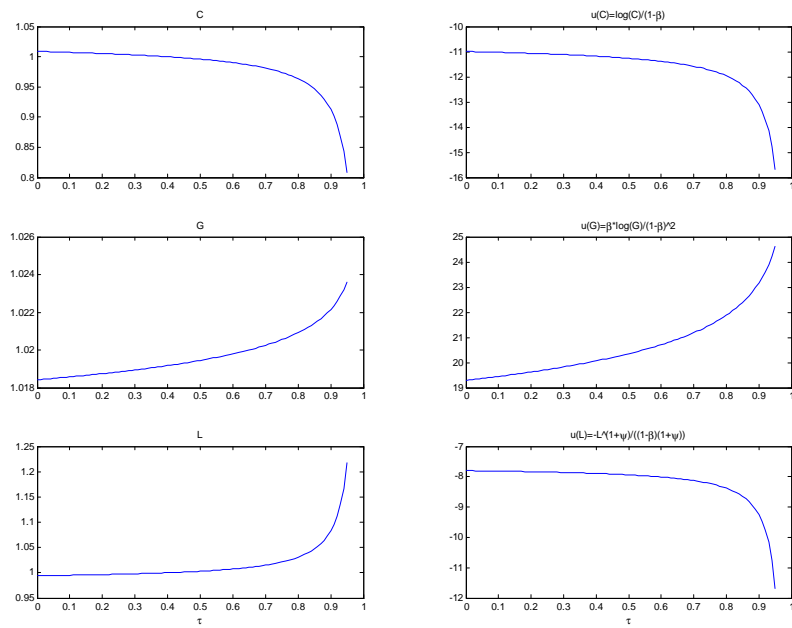


Figure 3: Impact of a R&D subsidy on consumption, growth and labor

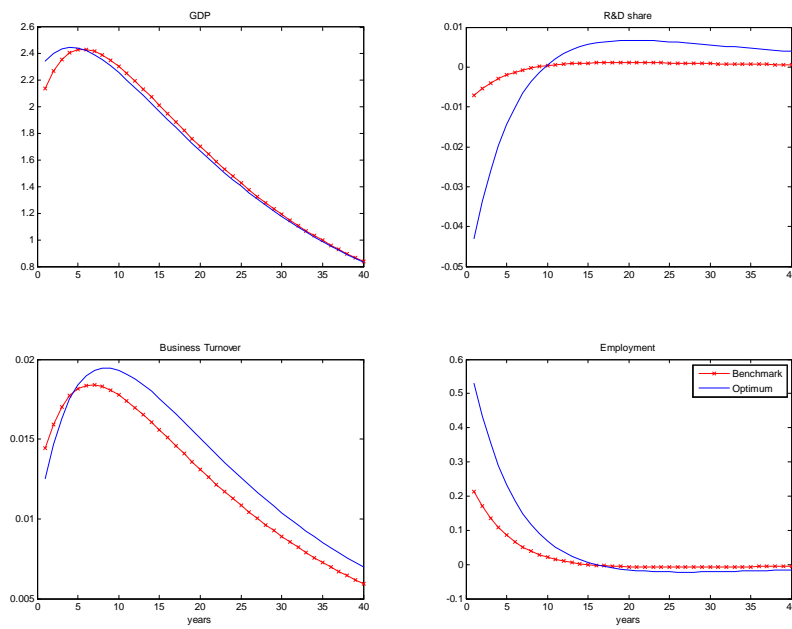


Figure 4: Impulse responses to a productivity shock  $z_t$ . Decentralized economy ('Benchmark') versus social planner ('Optimum').

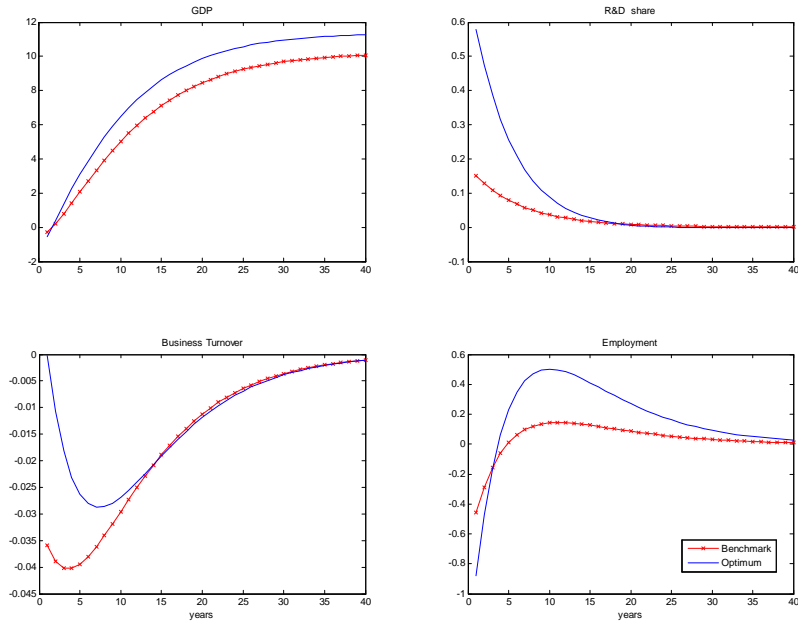


Figure 5: Impulse responses to a spillover shock  $\sigma_t$ . Decentralized economy ('Benchmark') versus social planner ('Optimum').

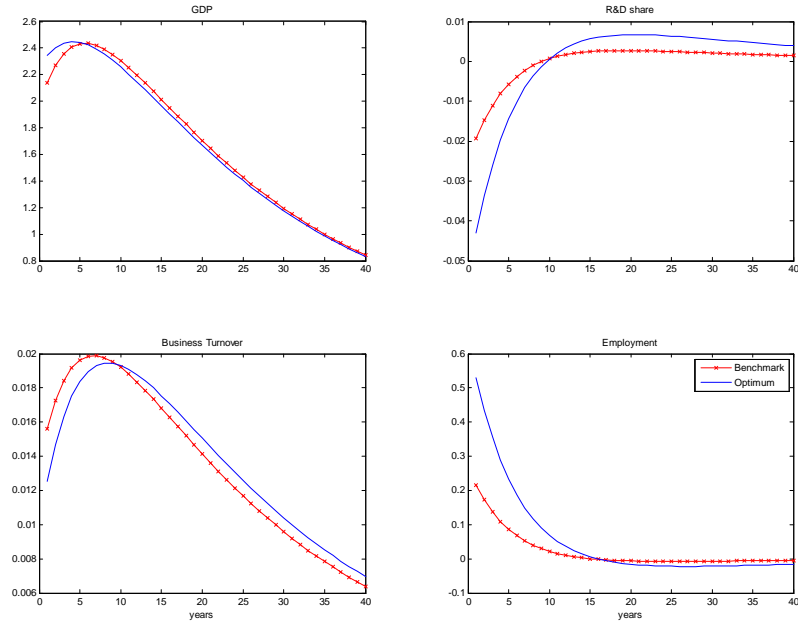


Figure 6: Impulse responses to a productivity shock  $z_t$  in the case of optimal constant tax ( $\tau = 0.78$ ). Decentralized economy ('Benchmark') versus social planner ('Optimum').