How Do You Make A Time Series Sing Like a Choir? Extracting Embedded Frequencies from Economic and Financial Time Series using Empirical Mode Decomposition

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Abstract

Empirical Mode Decomposition (EMD) was developed late last century, but has still to be introduced to the vast majority of economists. EMD was originally one of the components of Hilbert Huang Transform (HHT) which was a process of extracting the frequency mode features of cycles embedded in any time series using an adaptive data method which can be applied without making any assumption about stationarity or linear data-generating properties of time series. This paper introduces economists to the two constituent parts of the HHT transform, namely EMD and the Hilbert Spectrum, and also a new variant of this methodology, Ensemble EMD (EEMD). Several illustrative applications using the methodology are also included.

Keywords: Business cycles, growth cycles, Hilbert-Huang Transform (HHT), empirical mode decomposition (EMD), ensemble EMD, economic time series, non-stationarity, Hilbert spectral analysis.

JEL Classification: C49, E32

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1 Introduction

Empirical analysis using tools from the frequency domain is a woefully under-researched area in economics. Economics tends to focus on the time dimension of empirical macroeconomics, but developments in frequency domain techniques have extended far beyond the simple spectral analysis originally introduced into the mainstream by Granger through his Granger and Hatanka (1964) and subsequent Granger (1966) contribution. Developments in signal processing and other disciplines have taken place which now give the researcher much more advanced techniques than the very basic spectral analysis that is usually glossed over in a typical graduate econometrics course. Time-frequency analysis has been the area where most value-added is likely to be found for economists, notably in wavelet analysis (see Crowley (2007)), and more recently empirical mode decomposition, the subject matter of this article. The latter, although already over 10 years old, has been, to my knowledge, completely ignored by economists. This article seeks to rectify this deleterious situation, by introducing economists to Empirical Mode Decomposition (EMD) and the Hilbert-Huang transform (HHT), a relatively new and innovative technique together with a novel new variant that was introduced three years ago.

What makes frequency domain analysis important in empirical macroeconomic and financial analysis? Simply put, the time horizon and the interrelationships between macroeconomic and financial variables at different time horizons. In time-series analysis we often search (by using different econometric specifications) for the most appropriate "fit" for the time-series data at hand, and thus attempt to better understand the evolution of the series over time and the drivers behind the series. In time-frequency domain we can take this one step further - we can attempt to understand the evolution of the series over different time horizons and the drivers behind the series at different time horizons. Given the ongoing developments in time-frequency analysis there is a possibility that we might also be able to uncover meaningful sub-series in the data operating at different frequencies.

The only applications of EMD and HHT to economic and financial time series to date can be found in Huang and Shen (2005), Zhang, Lai, and Wang (2007) and Crowley (2008), and of these only one so far appears in journal format. Section 2 explains the technique of empirical mode decomposition, section 3 applies the technique to economic and financial time series, while section 4 concludes.

2 The Hilbert-Huang Transform and Empirical Mode Decomposition

2.1 Background

Identifying the different frequencies at work in economic variables was pioneered by Granger (1966) and has recently been updated by Levy and Dezhbakhsh (2003b) and Levy and Dezhbakhsh (2003a). Both of these latter two articles use traditional spectral analysis techniques, but this approach assumes that time series are stationary and linearly generated, so although the findings are consistent with Granger's this is hardly surprising, given that spectral analysis should not be applied to non-stationary time series[1]. Both wavelet analysis and HHT allow the use of non-stationary data, and although wavelet analysis assumes that variables are linearly generated, HHT, EMD and its variants, do not. Table 1 gives a summary of the different frequency domain methods available to researchers and the implicit assumptions used by each method.

One of the main problems with using traditional spectral analysis is that economic and financial variables and rarely both globally and locally stationary, and although using time-varying spectral analysis is clearly superior to using traditional spectral analysis spurious results may still result from local non-stationarities and from asymmetries in cycles. Wavelet analysis (see Crowley (2007)) is clearly superior to spectral analysis for dealing with most economic and financial variables, but problems still exist, as with discrete wavelet analysis cycles might not always lie within the dyadic frequency ranges imposed by scale separation, and so might lie on the border between these ranges hence with some "bleeding" between the scales might appear in more than one crystal. Also with continuous wavelet analysis there might be problems of frequency resolution and there are also usually only symmetric wavelet functions available "off the shelf", limiting the usefulness for economic and financial series[2].

[1] This is further explored in Crowley (2010) where it is clear that Granger's "typical shape" is not typical at all - it is due to a misuse of spectral analysis to analyze non-stationary variables.

[2] Although see Aguiar-Contraria and Soares (2010) for a primer and software which implements several different wavelet forms for use with the continuous wavelet transform.

	Time series	Spectral	Time-varying spectral	DWT	CWT	HHT/EMD
Basis?	A priori	A priori	A priori	A priori	A priori	A posteriori adaptive
Domain?	Time	Frequency	Frequency through time	Time-frequency	Time-frequency	Time
Stationary?	Yes	Yes	Yes within each window	No	No	No
Linearly generated?	Yes	Yes	Yes	Yes	Yes	No
Mathematical underpinning?	Yes	Yes	Yes	Yes	Yes	No, empirical
Asymmetric cycles?	Yes	No	No	Yes	Yes	Yes

Table 1: Summary of frequency domain methods

The EMD method introduced by Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung, and Liu (1998), resolves many of these problems by allowing non-stationary series (both globally and locally), non-linearly generated processes and also asymmetric cycles. The method has subsequently been applied to many areas in physics, mechanics, engineering, astronomy and the environmental sciences, and the US National Aeronautical and Space Administration (NASA) has taken great interest in the new technology, patenting a special application of the method. Unlike both spectral methods and wavelets, the EMD method is entirely empirically based - it has no formal mathematical basis, but rather attempts to break down the series according to how many frequencies are apparent in the data - in other words it allows the data to speak for itself rather than imposing certain *a priori* beliefs about which frequencies are present at any time within a series. As with any statistical method, the advantage of it's "temporal locality" feature has to be weighed against the fact that the methodology relies on unique extrema, and this can lead to problems of what Wu and Huang (2008) call a lack of "physical uniqueness".

Since being introduced a decade ago, a small group of researchers have extended and modified EMD, in a series of publications, notably Huang and Shen (2005) Wu and Huang (2008) and Huang and Wu (2008), and have launched a journal[3] to provide a publication outlet for applications using EMD and to further advance the EMD methodology. This has spurred even more developments (such as multivariate EMD) and recently a conference devoted solely to the HHT and EMD[4].

2.2 Methodology

EMD is actually part of a two-step procedure referred to as the Hilbert-Huang transform (HHT[5]):

1. Do EMD to obtain intrinsic mode functions (IMFs); and

2. use the Hilbert spectrum to assess instantaneous frequency for each IMF.

The Hilbert transform is not new, but EMD is. The main advantage of using EMD over other frequency domain techniques is that it not only identifies separate processes at work

[3] Journal of Adaptive Data Analysis

[4] See http://ldaa.fio.org.cn/index.html

[5] Named after Norden Huang who invented the EMD part of the process and David Hilbert who is the mathematician who originated the notion of a Hilbert spectrum. The methodology was designated HHT by NASA.

in a series, but it also separates each of these out and resolves them in time-frequency space. The IMFs should satisfy the following properties: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. An exhaustive introduction and comparison with continuous wavelet analysis is provided in Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung, and Liu (1998).

These processes or IMFs could be meaningful in that they represent the separate processes operating at different frequencies embedded within the data. To quote Huang and Wu (2008) (p19):

> HHT offers a potentially viable method for nonlinear and nonstationary data analysis, especially for time-frequency-energy representations. It has been tested widely in various applications other than geophysical research but only empirically. In all the cases studied, HHT gives results much sharper than most of the traditional analysis methods. And in most cases, it reveals true physical meanings.

The essence of the method is to identify the intrinsic oscillatory modes by their characteristic time scales in the data empirically, and then decompose the data accordingly. EMD does not impose any *a priori* conditions on the data (such as stationarity or linearity), but rather allows the data to speak for itself. EMD sifts the data by identifying maxima and minima in the data so as to identify cycles within the data at different frequencies, using a spline algorithm as follows:

i) identify maxima and minima of $x(t)$

ii) generate upper and lower envelopes with cubic spline interpolation $e_{\min}(t)$ and $e_{\max}(t)$.

iii) calculate mean of upper and lower envelopes:

$$m(t) = (e_{\max}(t) + e_{\min}(t))/2 \tag{1}$$

- this process is shown in figure 1.

iv) the mean is then subtracted from the series to yield a difference variable, $d(t)$:

$$d(t) = x(t) - m(t) \tag{2}$$

v) if the stopping criterion (SC):

$$\sum_{t=1}^{T} \frac{[d_j(t) - d_{j+1}(t)]^2}{d_j^2(t)} < SC \tag{3}$$

is met, where $d_j(t)$ is the result from the jth iteration, then denote $d(t)$ as the ith IMF and replace $x(t)$ with the residual

$$r(t) = x(t) - d(t) \tag{4}$$

vi) if the stopping criterion it is not an IMF, replace $x(t)$ with $d(t)$.

vii) repeat steps i) to v) until residual $r_n(t)$ has at most only one local extremum or becomes a monotonic function from which no more IMFs can be extracted.

The EMD process can also be illustrated by a diagrammatic flow chart, as in figure 2. The resultant decomposition of the series can be written as:

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{5}$$

where $c_j(t)$ represents the jth IMF.

Once the IMFs have been obtained, given the fact that (unlike traditional spectral analysis which typically uses Fourier analysis with constant frequency) variable frequency cycles can occur, it is more appropriate to use measures of instantaneous frequency and amplitude. This follows on from the observation that cycles can either change within a single period (known as "intrawave" frequency modulation) or between cycles (known as "interwave" frequency modulation), or with a combination of both types of modulation. Spectral analysis can detect the latter, particularly when using time-varying spectral analysis, but it cannot detect the former, and yet the former is likely, particularly with the non-linear types of processes that characterize variables found in economics and finance.

The Hilbert spectrum lends itself directly to the task of estimating instantaneous frequency, thus allowing the researcher to account for all types of frequency modulation. In mathematical terms, for any function $x(t)$ of L^p class, its Hilbert transform $y(t)$ is:

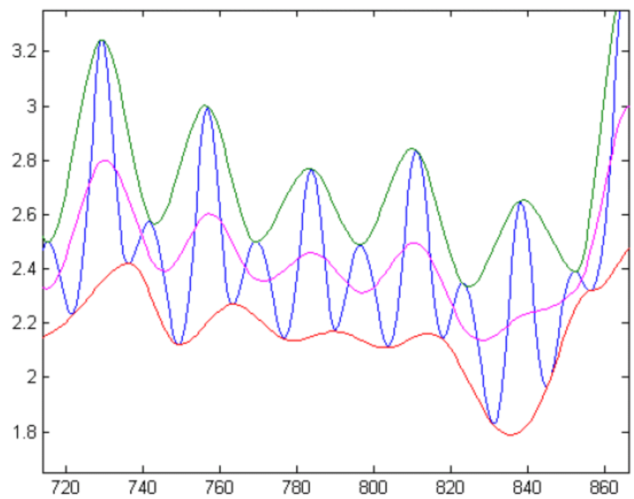

Figure 1: The spline-envelope process under EMD for a hypothetical series

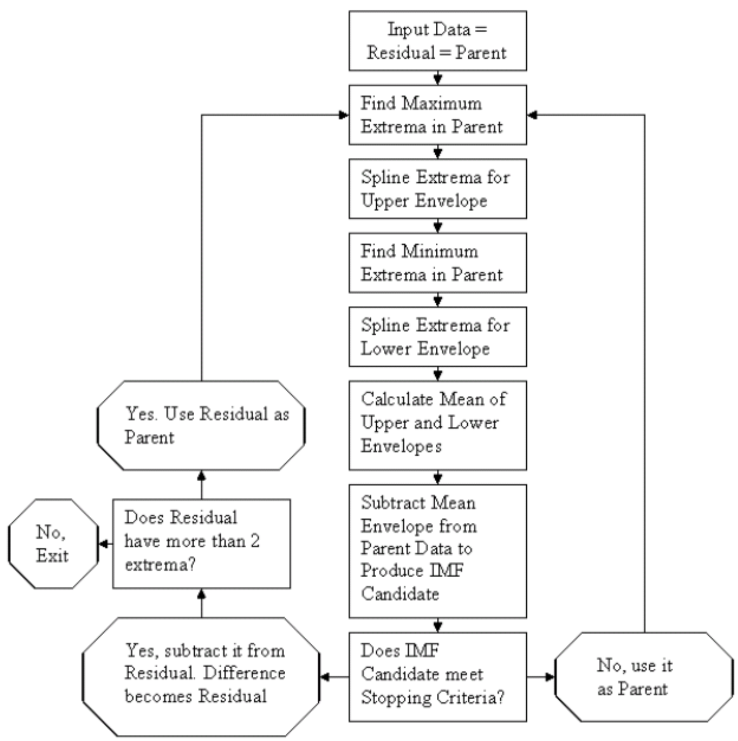

Figure 2: Flow chart of EMD sifting process

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau \tag{6}$$

where P is the Cauchy principal value of the singular integral. The Hilbert transform $y(t)$ of any real-valued function $x(t)$ will yield the analytic function:

$$z(t) = x(t) + iy(t) = a(t) \exp\left[i\phi(t)\right] \tag{7}$$

where $i = \sqrt{-1}$, $a(t)$ represents the amplitude and $\phi(t)$ the phase ($\phi(t) = \arg(x(t))$). $a(t)$ is then given by

$$a(t) = \left(x^2 + y^2\right)^{1/2} \tag{8}$$

and:

$$\phi(t) = \tan^{-1}\left[\frac{y}{x}\right] \tag{9}$$

Instantaneous frequency, ω, then is given by:

$$\omega = \frac{d\phi}{dt} \tag{10}$$

The instantaneous frequency introduced here is physical and depends on the differentiation of the phase function, which is fully capable of describing not only interwave frequency changes due to nonstationarity but also the intrawave frequency modulation due to nonlinearity. The Hilbert transform as applied to each IMF can now be expressed as:

$$z(t) = \sum_{j=1}^{t} a_j(t) \exp\left[i \int \omega_j(t) dt\right] \tag{11}$$

so that the instantaneous amplitude $a_j(t)$ can be separately extracted from the instantaneous phase $\omega_j(t)$ for each IMF and combined into a Hilbert (amplitude) spectrum, $H(\omega, t)$ (see Huang and Shen (2005)). The power (or energy) spectrum is given by $[H(\omega, t)]^2$ so accordingly the marginal average power spectrum is:

$$h(\omega) = \frac{1}{T} \int_0^T H^2(\omega, t) dt \tag{12}$$

2.3 Other issues

The first concern, as with spectral analysis and wavelet analysis, relates to end effects. At the beginning and at the end of the time series the cubic spline is not defined, so where the cubic spline fitting can have large swings. Left by themselves, the end swings can eventually propagate inward and corrupt the whole data span especially in the low-frequency components. A numerical method of adding extra waves to eliminate the end effects has been implemented.

The second concern is with "mode mixing", which is defined as a single IMF either consisting of signals of widely disparate scales or a signal of a similar scale residing in different IMF components. Usually mode mixing is identified by the frequencies of different (usually adjacent) IMFs intersecting each other. To overcome this problem a new noise-assisted data analysis method was proposed, the ensemble EMD (EEMD), which defines the true IMF components as the mean of an ensemble of trials, each consisting of the signal plus a white noise of finite amplitude. More details can be obtained from Wu and Huang (2008). In this study EEMD is utilized, with parameters specified for each example.

The third concern relates to that of the Bedrosian theorem[6]. To summarize from Huang and Wu (2008), there are problems in defining the instantaneous frequency with the Hilbert spectrum unless the amplitude of the fluctuations are relatively constant - something that is rarely satisfied in empirical data. Therefore a method was developed in **?**) which normalizes the IMFs and then the phase function of each IMF is measured using the arc-cosine of the normalized data. This method is called the “direct quadrature” (DQ) method and it is used in the calculation of the frequencies for most of the examples given below, although implementation using the original Hilbert spectrum is also shown for selected variables.

3 Illustrative Applications

In this section HHT, EMD and EEMD are applied to a selection of financial and economic time series. Data sources are listed in an appendix.

3.1 Financial time series

3.1.1 The Dow Jones Industrial Average

Monthly data for the Dow Jones Industrial Average: 1896-2011 are first used. The data is transformed by taking natural logs, and is displayed in figure 3. The recent fall in the index is clearly significant, but nevertheless the stockmarket crash of 1929 clearly dominates the

[6] The Bedrosian theorem states that the Hilbert transform for product functions can only be expressed in terms of the product of the low-frequency function with the Hilbert transform of the high-frequency one if the spectra of the two functions are disjointed.

series. No further transformation of the series is done, and the IMFs obtained as well as the residual are shown in figure 4. Nine IMFs are obtained (as numbered down the left hand side of the figure), with the residual shown superimposed onto the original data at the top of the panel in red. EEMD was used with white noise added with 30% of the volatility of the series as a whole, and 800 iterations were done on to provide the ensemble for averaging. The residual obviously indicates the trend of the series, with more marked increases in the series shown by two accelerating waves which begin in the 1950s and repeat in the 1990s. Clearly, unlike what one would obtain with traditional spectral analysis, non-regular cycles are extracted as is particularly noticeable with IMFs 3, 4 and 5, while IMFs 1 and 2 appear to contain mostly noise. Interestingly IMFs 7 and 8 appear to have relatively regular cycles and IMF 9 seems to consist of one small amplitude undulation from the early 1940s to 1960s so is not really considered cyclical activity. The longest cycle detected in the data, then, is in IMF8, and it is roughly a 40 year cycle with persistent and roughly constant amplitude.

The quality of the decomposition is dependent on the frequency resolution of the IMFs as noted above, and this should also be apparent from the frequency resolution which is plotted in figure 5. Clearly nearly all the IMFs are well resolved, but there is some mode mixing between IMF7 and 8 and IMF3 and 4 - nevertheless, apart from this there is little mode mixing among other IMFs. IMF9 hardly contains any identifiable cyclical activity, so the longest cycle in the data is only picked up at around a 40 year frequency.

Figure 6 shows a significance test for the cyclicality of the IMFs versus white noise which is generated at a variety of different periodicities. As the mean period is extended the energy levels drop off, and confidence intervals can be constructed around a decomposition of white noise. These are displayed in the figure by a widening green cone and these green lines represent the 1% and 99% significance points according to what would be expected with white noise. The red stars then indicate the energy of IMFs 2-9 and in this instance they are all significantly different from what would be expected with white noise, but once again IMF8 is shown to be a particularly strong cycle at the 40 year frequency.

In figure **??** the equivalent results using EMD are presented - here the Hilbert spectrum (- with the colour bar on the right hand side of the figure showing that low energy levels are in blue, middling energy levels in red and high energy frequencies in yellow) is shown and it is clear that the lower frequency IMFs tend to generally contain more energy than the higher frequency IMFs, although there are circumstances (for example around 1915 and between 1920 and 1930) when shorter cycles appeared to have more energy. IMF5 appears

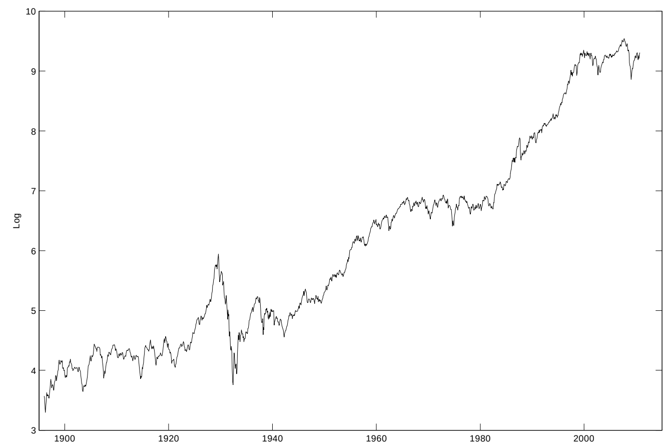

Figure 3: The Dow-Jones Industrial Average (DJIA) from 1896 to 2011

to correspond most closely to the business cycle, particularly as it has troughs in around 1993 and 2001. Lastly figure 8 shows the marginal Hilbert power spectrum which indicates that the IMF with most consistent energy lies at roughly a 40 year cycle. Beyond this, there is nothing evident in terms of the IMFs from the original data.

3.1.2 The US dollar - British pound exchange rate

Figure 9 gives the US dollar-British pound exchange rate from March 1973 to June 2011 using monthly data, and including the precipitous fall in the pound during the first half of 2009. The data obviously displays some irregular wave-like features, with both irregular amplitude and frequency. Figure 10 shows that the EEMD method (with 0.3 of the standard deviation and 500 runs), extracts 7 IMFs, with the possibility of another cycle remaining in the residual (- note that unless a full cycle is observed, any incomplete cycles remain in the residual). IMF5 clearly extracts the general cycle directly observed in the data, particularly from 1973 to 1988. Interestingly though 6 other IMFs are apparent in the data, of various frequency and amplitude. What is also noticeable is that during the short-lived period when sterling entered the ERM of the EMS[7] the volatility observed in IMFs 1 to 3 clearly increased in a synchronized manner, but the lower frequency IMFs were not affected. Also it is interesting to note that the current fall in the value of the pound has been contained mostly in IMF4, with a smaller amount appearing in IMF3.

[7] The exchange rate mechanism of the European monetary system.

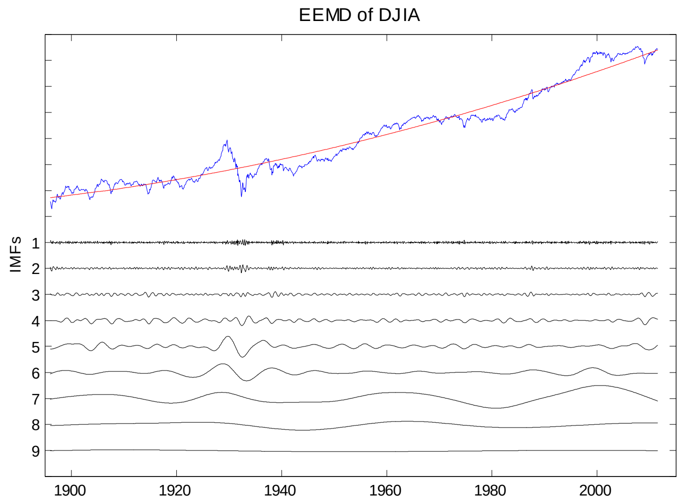

Figure 4: IMFs for DJIA

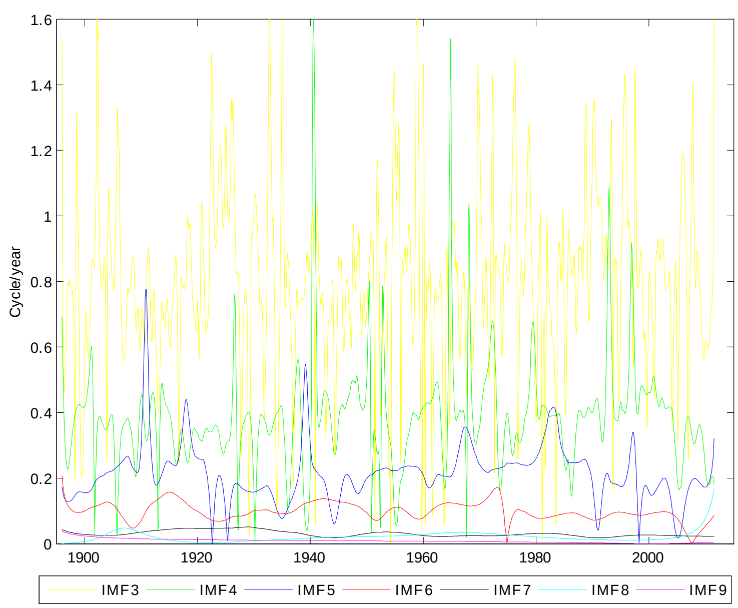

Figure 5: Instantaneous frequencies for DJIA IMFs

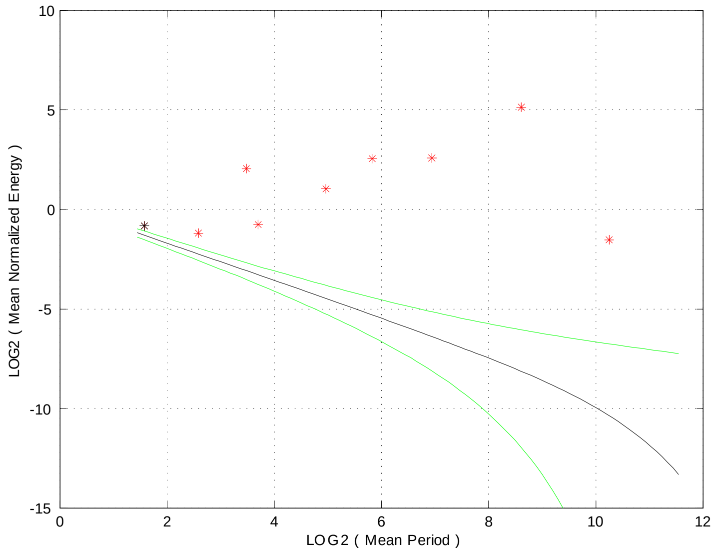

Figure 6: Significance test of DJIA IMFs against white noise

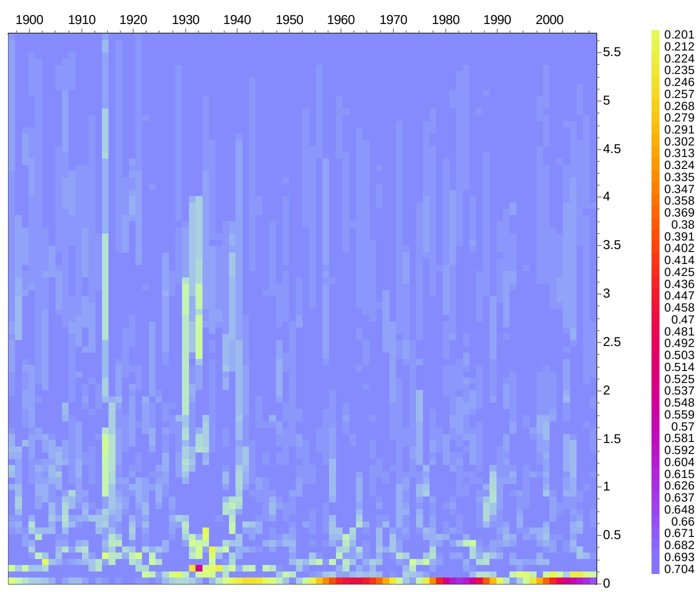

Figure 7: Hilbert spectrum for DJIA IMFs

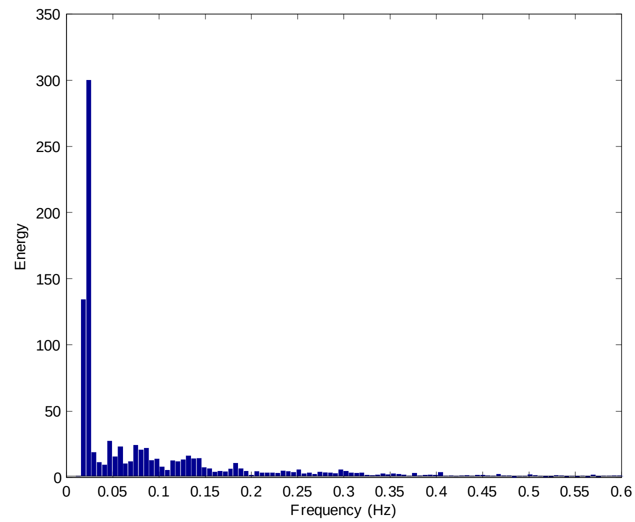

Figure 8: Marginal Hilbert power spectrum for DJIA IMFs

Figure 11 shows the instantaneous frequency for each IMF and it appears as though once again the measure of instantaneous frequency is well measured for the lower frequency IMFs, but the 1980s saw some mode mixing occurring. In figure 12 the test of significance of the IMFs compared to white noise is implemented, and all are found to be significant. The Hilbert spectra are shown in an appendix.

3.2 Economic time series

3.2.1 US industrial production

In figure 13 (natural) log of monthly US industrial production is shown from 1919 to March of 2011. The series is clearly highly volatile before around 1947 but then appears to exhibit much less volatility in the post-war era. It is well known that recessions in the US usually tend to adversely affect the manufacturing sector much more than other sectors, so the post-war recessions can be very clearly seen in the data. Figure 14 shows the extracted IMFs for the series[8], 8 in all, with the business cycle not apparent in any one particular IMF but apparently a result of fluctuations in several different IMFs[9]. There does appear to be the possibility of a weak long cycle in the data, with roughly an 80 years cycle, but apart from that only a cycle operating at roughly a 20-25yrs cycle length. This is an important

[8]Here EEMD is used with 800 replications using 0.8 of the standard deviation of the series.

[9]This is a result that mirrors the result shown for US growth in Crowley (2010).

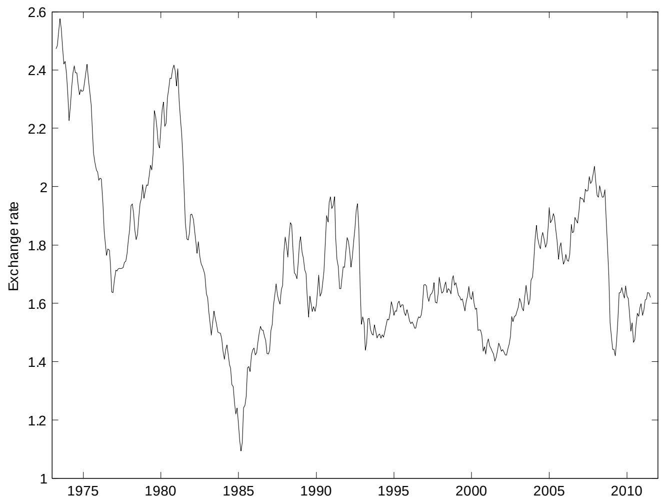

Figure 9: US dollar-British pound exchange rate

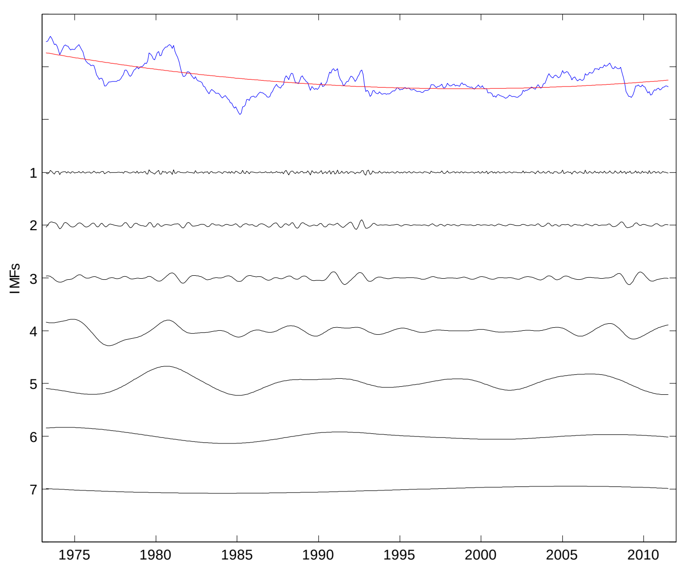

Figure 10: IMFs for the US dollar-British pound exchange rate

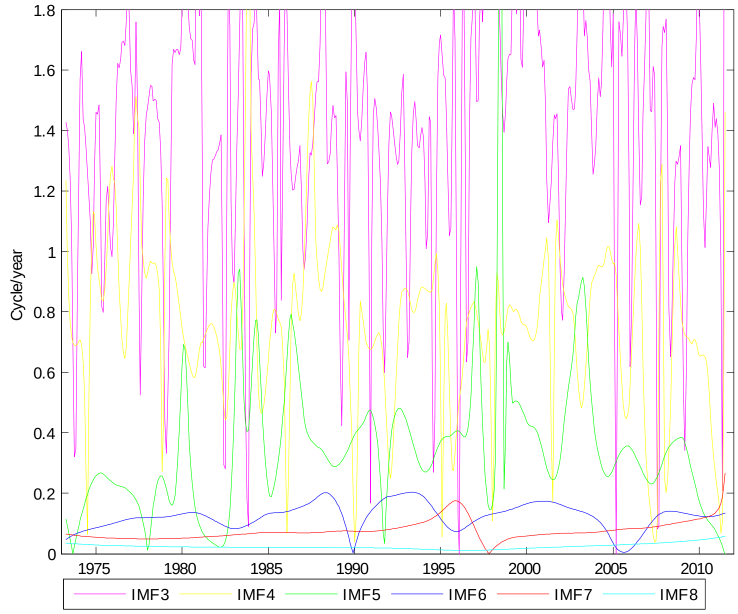

Figure 11: Instantaneous frequency for the IMFs from British pound - US dollar exchange rate

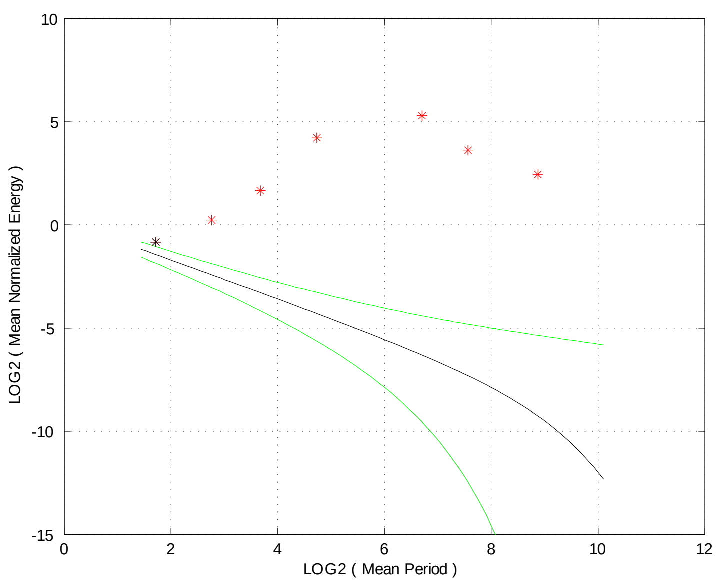

Figure 12: Significance test of British pound exchange rate IMFs against white noise

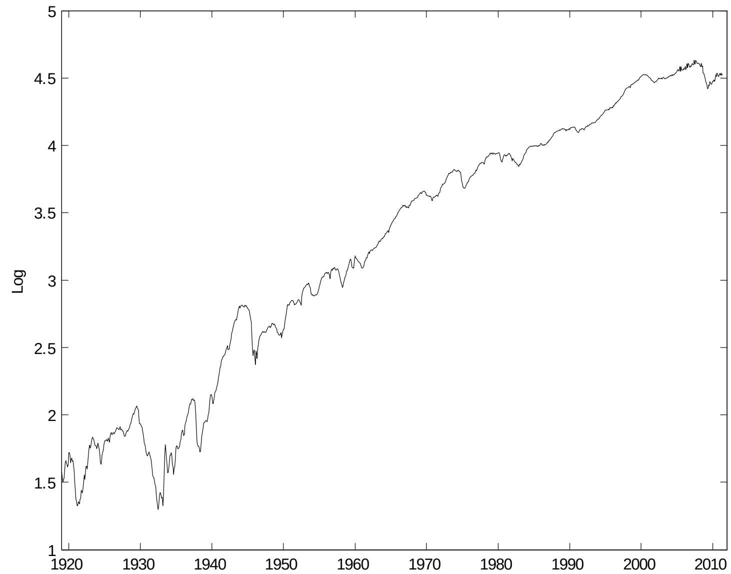

Figure 13: US industrial production (natural log)

result, as it implies that higher frequency cycles, along with the business cycle account for most of the fluctuations in industrial production data for the US[10]. With the higher frequency IMFs, IMF1 shows no significant post-war change, and has very little energy, but IMFs 2 and 3 exhibit reduced volatility from the early 1980s with IMF3 not altering in the 1980s but virtually disappearing from around 1990 onwards. This tends to suggest that the "great moderation" only affects two cycles in growth in industrial production, but longer term business cycle fluctuations appear not to have been affected. This result also mirrors the results of Crowley and Hughes Hallett (2011).

As figure 15 illustrates, the frequencies of the IMFs are fairly well separated, but there are some instances when "mode mixing" occurs. What is of interest here is if there are changes in cycle frequencies that occur before and after WWII. From figure 15 it appears there was very little change in average frequency cycles for nearly all IMFs pre- and post-WWII with perhaps the exception of IMF3. Figure 16 shows the significance test for the IMFs, and all appear to be significant when tested against white noise. Hilbert spectra are for this variable are relegated to the appendix.

[10]This is important, as it directly contradicts the results found in Granger (1966).

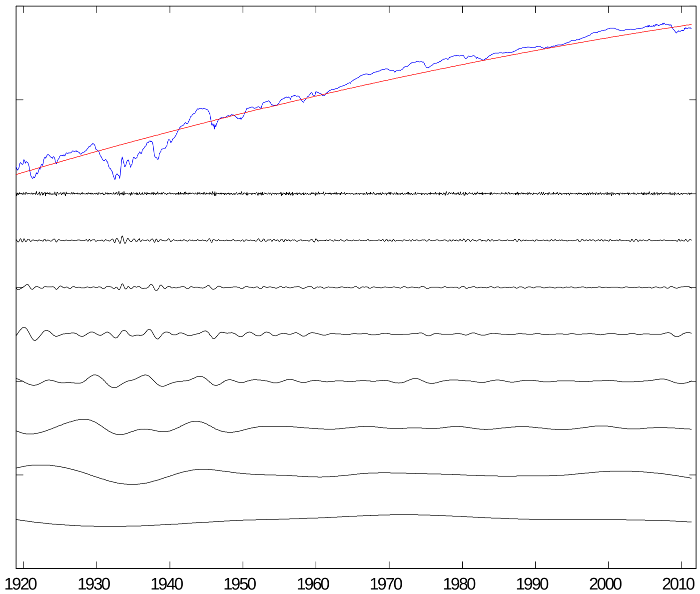

Figure 14: IMFs for US industrial production

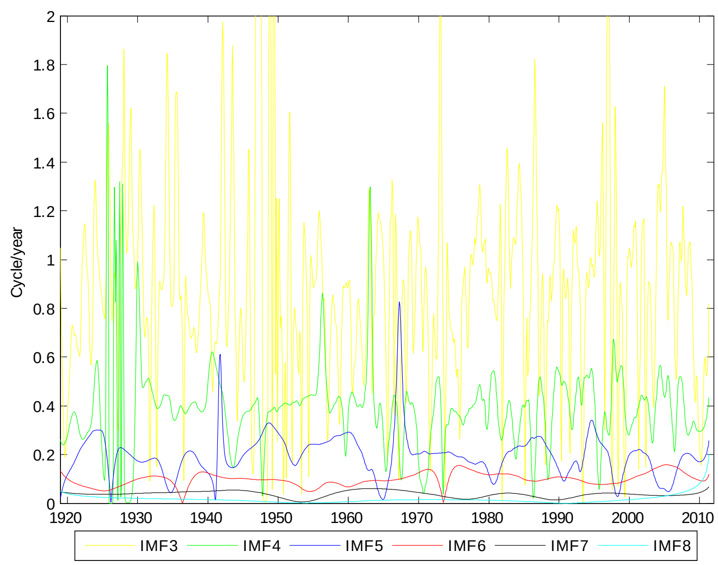

Figure 15: Instantaneous frequencies for IMFs from US industrial production

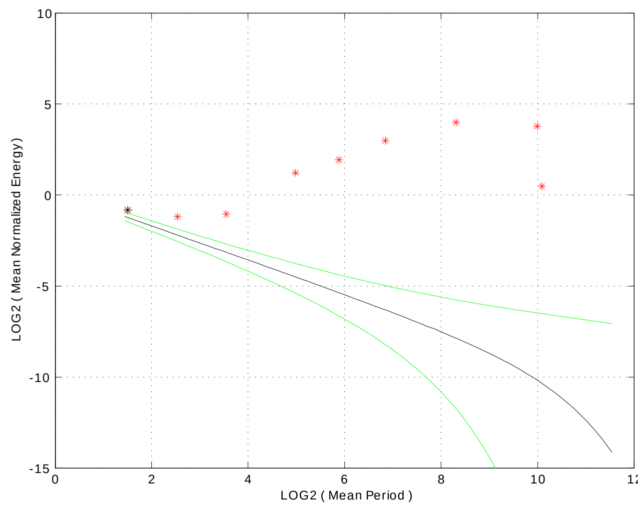

Figure 16: Significance test of US industrial production IMFs against white noise

3.2.2 UK Retail Price Index

The log change in the UK retail price index (RPI) appears to exhibit considerable cyclical fluctuations, as shown in figure 17, but these have tended be less volatile since the early 1990s. In terms of stationarity, inflation is usually considered a non-stationary variable because of inflation persistence, and this can clearly be seen in the clear upward move in the inflation rate during the 1970s.

Figure 18 shows that EEMD extracts 8 IMFs and a inverted U-shaped residual[11], showing that inflation has dropped to consistently low long-term levels (- despite the fact that in the short term inflation appears to have increased in the last 2 years). IMFs 1 and 2 appear to just show high frequency noise, but IMFs 3 and 4 appear to include bursts of cyclical volatility[12], some with large amplitude. IMFs 4 and 5 contain most of the momentum in the series, with IMF5 containing a 3 year cycle, with IMF4 exhibiting a similar cycle but at a higher frequency. IMF6 appears to contain a very irregular roughly 12 year cycle, with peaks appearing just before recessions, which as inflation is procyclical.might be expected. IMF7 is obviously a very strong cycle as well, and has a length of roughly 22 years, but since the 1990s, it is curious here that IMF7 has essentially disappeared. A very weak 30 year cycle is also apparent in IMF8.

[11] The EEMD parameters were set at 30% of standard deviation and 500 iterations were completed.

[12] In the signal processing literature these short bursts or packets of volatile movements in the series are often called "chirps".

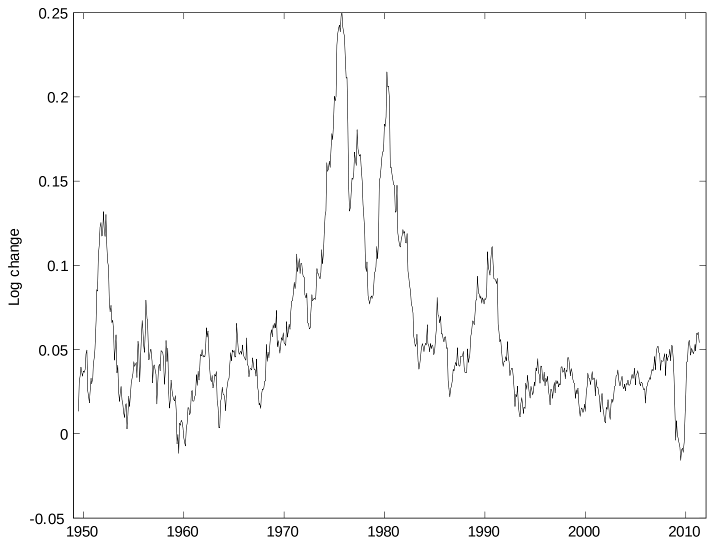

Figure 17: UK log differenced RPI

In terms of frequency, figure 19 suggests that IMFs are well separated for the most part, but that IMFs 5 and 6 mode mix in the 1960s and 1980s and IMFs 3 and 4 mode mix in the 1960s and 1990s. IMF6 operates at roughly a 5 year cycle while both IMF6 and 7 operate at business cycle frequencies from the mid-1980s onwards. IMF8 operates at very long cycles of around 40 years. Figure 20 offers a significance test of the IMFs, and once again all the IMFs are significant in energy against white noise. Hilbert spectra are relegated to an appendix.

3.2.3 UK M0 monetary aggregate

Given that the previous section looked at inflation, it is informative to look at a UK monetary aggregate to see if we get any similar frequency fluctuations evident in the data. Figure 21 shows the M0 series from 1949 through to 2006 when the monthly series was discontinued. The peaks in the series in the 1970s have a striking correspondence to the peaks seen in the UK retail price index. Figure 22 shows that EEMD yields 7 active IMFs, with IMF8 having very little fluctuation. In the higher frequency IMFs, there appears to be less volatility post-early-1980s whereas for the lower frequency IMFs although amplitudes have reduced since the 1980s they do not appear to be much different from the 1950s or 1960s. In terms of frequencies shown in figure 23, the IMFs appear to be fairly well resolved,

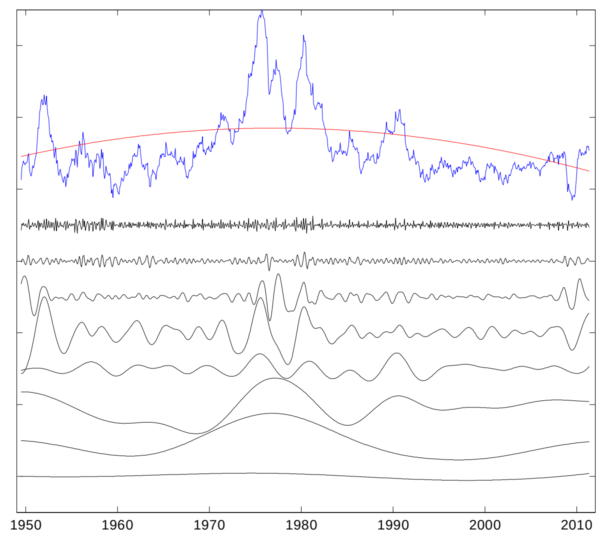

Figure 18: IMFs for UK RPI

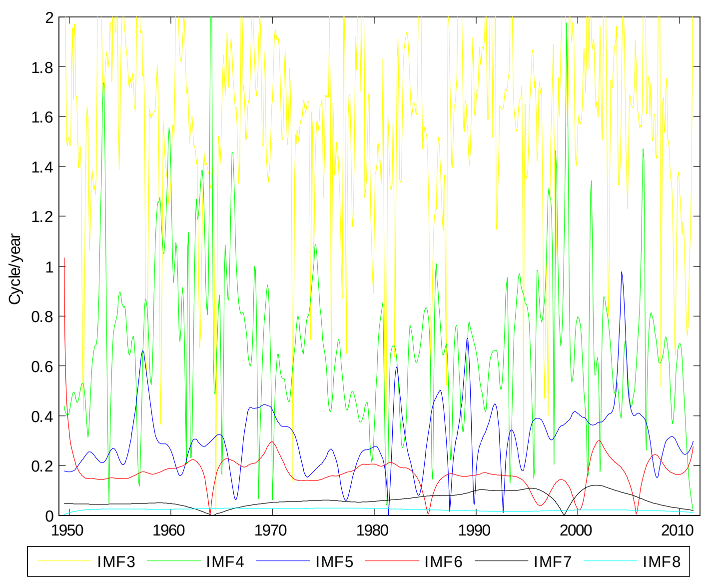

Figure 19: Instantaneous Frequency of IMFs for UK RPI

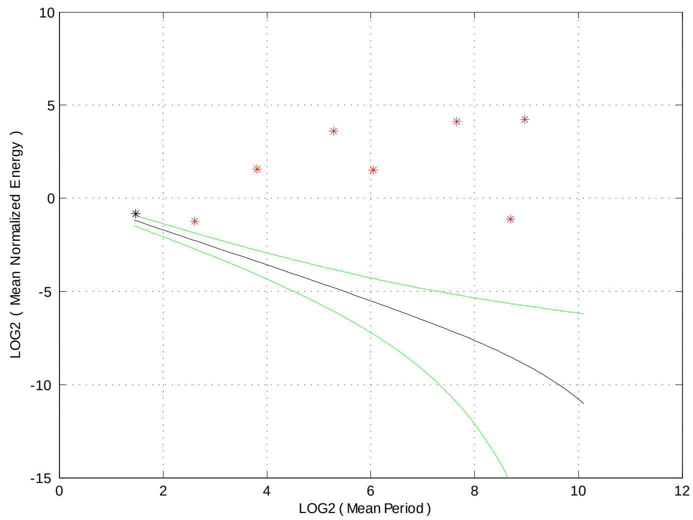

Figure 20: Significance of UK RPI IMFs vs white noise

although there is a similar amount of mode mixing compared with the UK retail price index. The business cycle appears to be contained in IMF5 and to a lesser extent IMF6 given their synchronicity around certain downturns in economic activity.

Figure 23 shows instantaneous frequencies for each IMF, and there is good resolution except for the latter half of the 1980s where there is substantial mode mixing. It is also clear that IMFs 5 and 6 appear to operating within the limits of the business cycle, but they appear to operate at either end of the band that would be expected to contain the business cycle. Figure 24 the IMFs are tested against white noise, and in this instance IMF2 appears to contain only white noise, whereas IMFs 3 to 8 contain cyclical activity that is significant.

One interesting corollary of this analysis relates to the long run relationship between money and prices in the UK. There does appear to be long cycles in both inflation and the monetary aggregate for the UK, and with the very long cycle (IMF6) in both cases, inflation appears to lag movements in the monetary aggregate by approximately 2 to 3 years. This mirrors results recently found by Benati (2009). The relationship between other IMFs in the two variables is much less clear though, and merits further investigation.

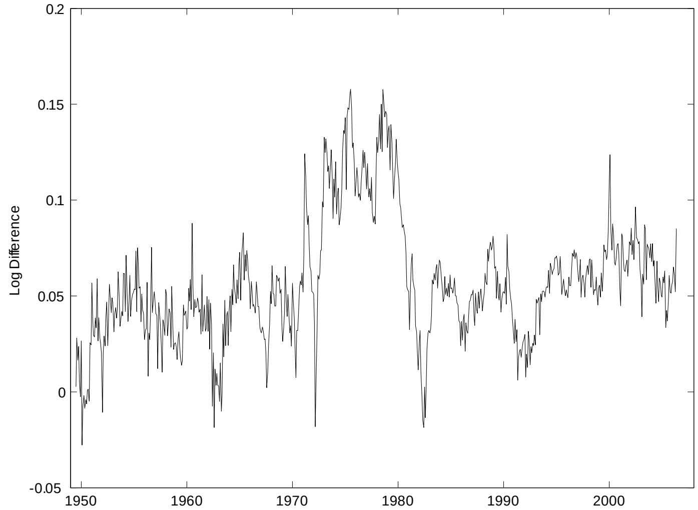

Figure 21: Log change in UK M0 monetary aggregate

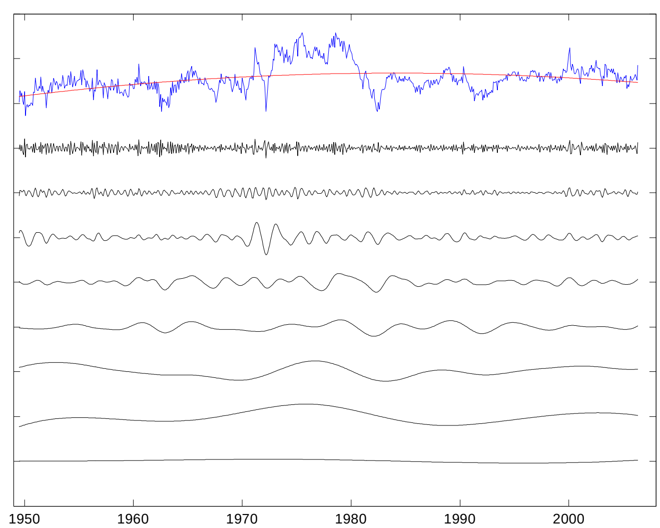

Figure 22: IMFs for UK M0

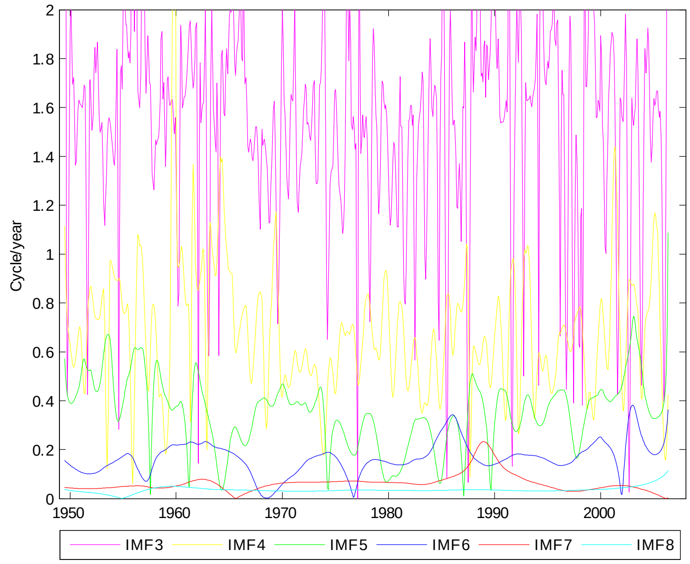

Figure 23: Frequency of IMFs for UK M0

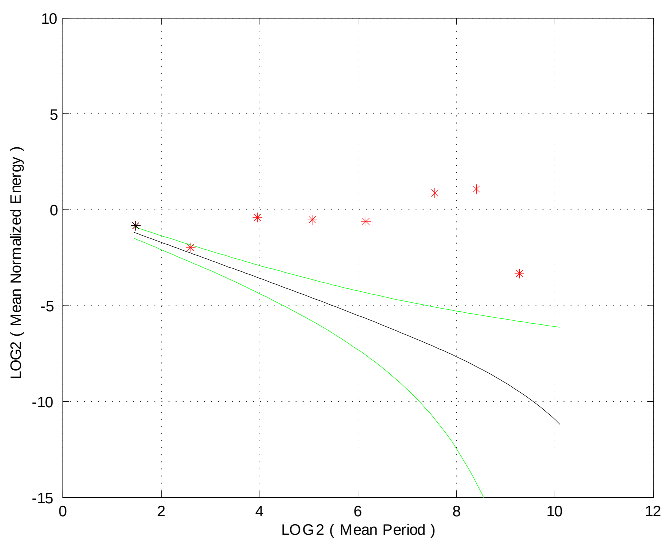

Figure 24: IMF significance for UK M0 vs white noise

4 Conclusions

EMD and the HHT are relatively new techniques which were introduced just over a decade ago. The approach offers a new technique for use in frequency domain analysis by using an adaptive data algorithm which accurately extracts cycles embedded in the data. The HHT consists of two stages - first sifting the data using empirical mode decomposition, which extracts the different embedded frequency series (known as IMFs) - and then transforming the data using the Hilbert transform so as to analyse the data in terms of frequency domain measures. The main advantages of the method are that it does not assume stationarity (either globally or locally), and does not assume any data generating process, so can cope with non-linear data. Spectral analysis, the traditional workhorse of frequency domain analysis assumes both stationarity and a linear data generating process so is not suitable for the analysis of many economic variables. The method is still in development, with a new variant, the EEMD introduced in 2008, but nevertheless is available for use by economic and financial researchers using widely available software.

Several examples using both economic and financial variables were presented using the software currently available to researchers. The technique revealed several interesting results:

i) with Dow-Jones industrial average stockmarket data, there appears to be a 30-40 year cycle, as well as a cycle operating at or near the business cycle;

ii) with the US dollar-British pound exchange rate, there appears to be a 15-20 year cycle operating;

iii) with US industrial production data, there appears to have been a waning of a 25 year cycle, but a roughly 10 year cycle persists, even though higher frequency cycles have become much less volatile since the 1980s. There is also the possibility of a much longer 80 year cycle in the data, but it is very weak;

iv) with UK inflation and M1 data, there appears to be a long cycle of around 20-25years operating in the data - with the monetary aggregate leading the inflation rate by a period of roughly 2 to 3 years.

In terms of future research, there is clearly much that can be done. One forthcoming paper by this author concerns application of the method to US growth data spanning more than a hundred years in order to confirm the lack of a long cycle in growth. Other possibilities are clearly evident - the relationship of prices and money as well as the relationship between consumption and investment, for example.

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Appendices

A Data sources

The Dow Jones Industrial Average was sourced from the Bank of Finland stockmarket database.

The US dollar - British pound exchange rate was sourced from the Bank of Finland exchange rate database.

US industrial production was sourced from the Bureau of Economic Analysis, Dept of Commerce.

UK RPI was sourced from the National Statistics Office, UK.

UK M0 was sourced from the Bank of England monetary database.

B Software resources

Software for HHT exists from a variety of different sources:

i) The US National Aviation and Space Agency (NASA) Goddard Space Flight Center has three issued patents, one published patent application, and one copyright on this method. A MATLAB based package is available for use through a contracter at http://www.dynadx.com/index.htm. This only executes EMD.

ii) Alan Tan has contributed code to MATLAB central which requires both the MATLAB signal processing and spline toolboxes, and this is located at

http://www.mathworks.nl/matlabcentral/fileexchange/19681

iii) Patrick Flandrin has MATLAB/C code on his website at

http://perso.ens-lyon.fr/patrick.flandrin/emd.html

iv) EEMD code is available from the Research Center for Adaptive Data Analysis which is lead by Dr. Norden Huang in Taiwan. The website is at

http://rcada.ncu.edu.tw/research1.htm

B.1 Hilbert spectra

B.1.1 US dollar-British pound exchange rate

The results of using EMD are very similar to EEMD as shown in the main text. The Hilbert spectrum for the IMFs is shown in figure 25 and as might be expected, shows that the large amplitudes of the longer cycles dominate movements in the series while higher frequency IMFs only appear to gain power over relatively short periods of time (such as in the early 1990s). Figure **??** shows the marginal power spectrum and shows that the two low frequency IMFs hold most of the energy in the series, and dominate the shorter frequencies over the life of the series.

B.1.2 US Industrial Production

Lastly, figures 27 and 28 show the Hilbert spectrum for the IMFs and the Marginal Hilbert power spectrum for the IMFs. As expected, lower frequencies contain most energy in the series, and the most noticeable aspect of the Hilbert spectrum is the waning of power in higher frequencies in 1960 and then in the early 1980s. What is also very noticeable is that the energy in the lower frequency cycles has not changed much over time. This has possibly important implications for economic policymakers.- it implies that better economic policymaking can dissipate the energy that resides in high frequency cycles, but to date, it doesn't seem to have affected the energy contained in low frequency cycles, and particularly the IMFs where the business cycle resides.

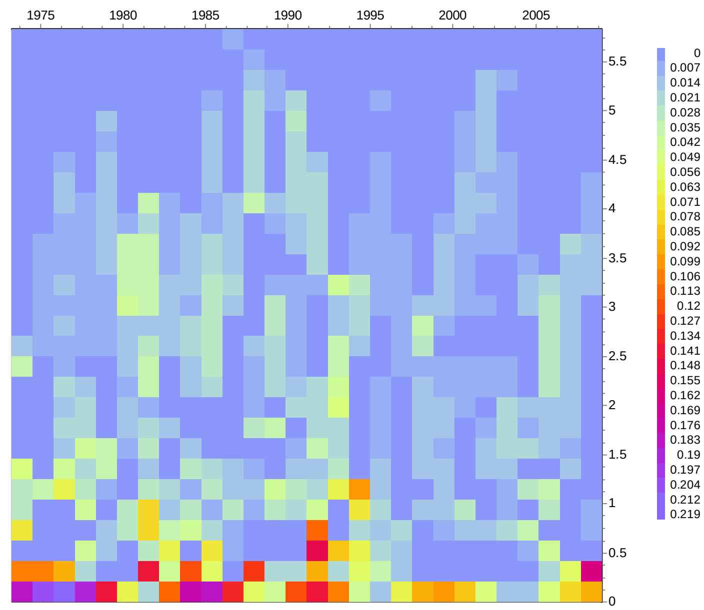

Figure 25: Hilbert spectrum for IMFs of US dollar-British pound exchange rate

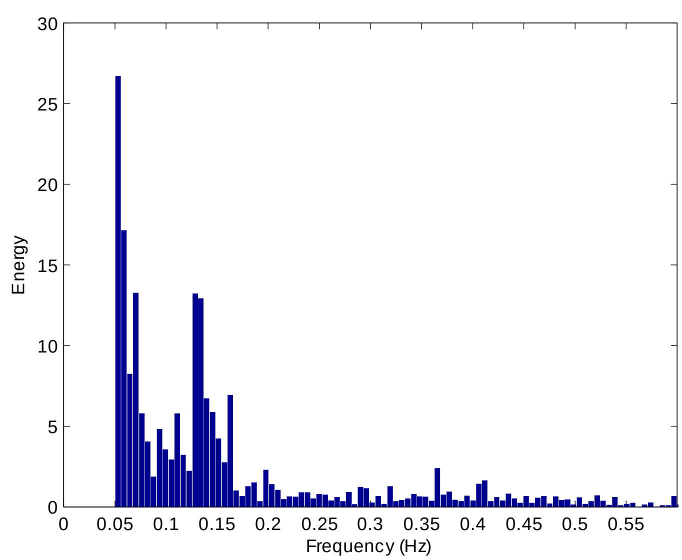

Figure 26: Marginal Hilbert power spectrum for IMFs of the US dollar-British pound exchange rate

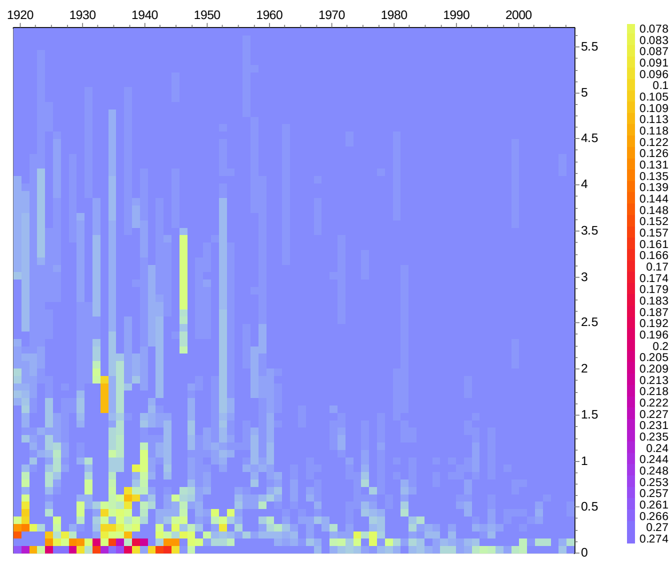

Figure 27: Hilbert spectrum for IMFs of US industrial production

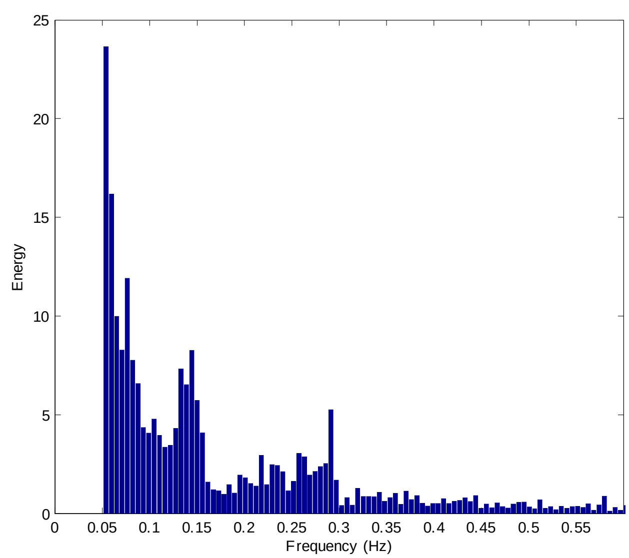

Figure 28: Marginal Hilbert power spectrum for IMFs of US industrial production

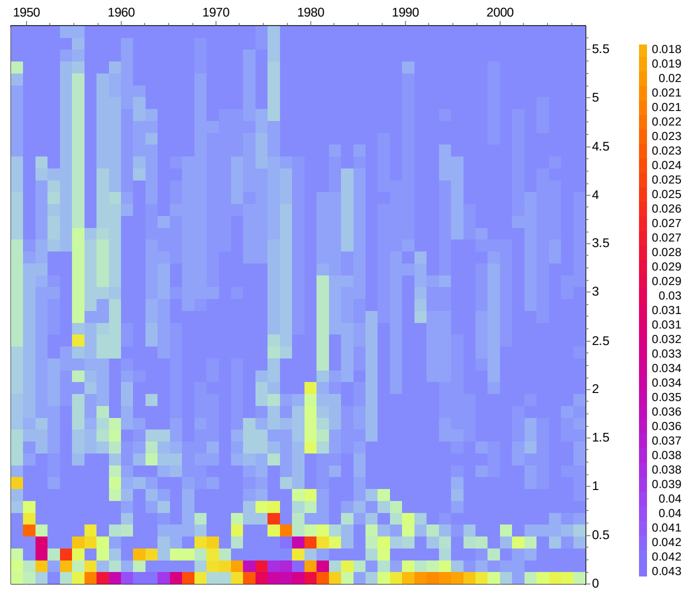

Figure 29: Hilbert spectrum for IMFs of UK RPI

B.1.3 UK Retail Price Index

The spectra for UK retail price inflation are shown in figures **??** and **??**. Clearly the two lower frequency cycles contain most energy but then there is also a two year cycle that is still evident in the data, but the Hilbert spectrum clearly indicates that the cyclical properties of the data are weaker now than they were in the past.

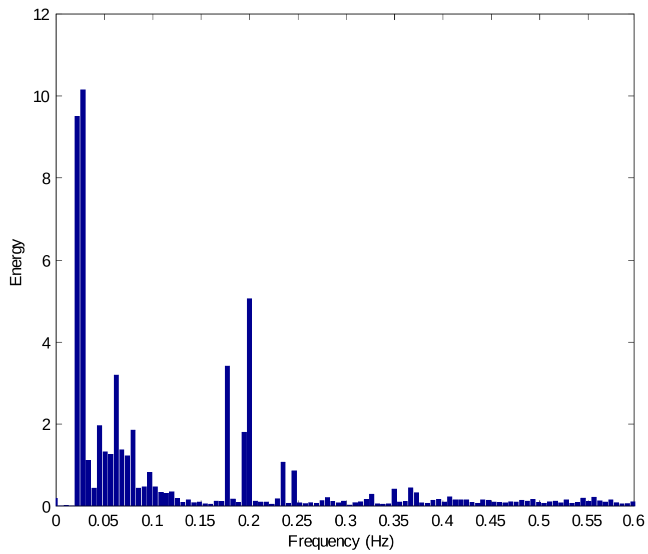

Figure 30: Marginal Hilbert power spectrum for IMFs of UK RPI