A Persistence Based Decomposition of Macroeconomic and Financial Time Series

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A Persistence Based Classification of Shocks

- Traditional econometrics focuses on the information flow with respect to time evolution, here we explicitly take into account also the flow of information under a change of resolution.
- This paper proposes a linear decomposition of the economic factors as a linear combination of past uncorrelated innovations which are classified by the time of their arrival their level of persistence.
- This non-structural decomposition generalizes the Wold decomposition for stationary time series and the Beveridge-Nelson (1981) permanent transitory decomposition for non stationary integrated ones.

Background Literature

1. BN decomposition: Watson (1986), Morley, Nelson and Zivot (2003), Proietti (2006), Oh, Zivot and Creal (2006), and Morley (2011)
2. Low Frequency Structural Relations Detection: Muller and Watson (2008) and Muller and Watson (2009).
3. Time series multiresolution analysis in economics: Ramsey and Lampart (1998), Gencay and Fan (2008), Gencay and Gradojevic (2009), Gencay, Selcuk and Whitcher (2001), Daubechies (1990), Daubechies (1992), Mallat (1989a) and Mallat (1989b).
4. Long Run Risk and Asset Pricing: see the (in)complete list in Ortu Tamoni Tebaldi (2011).

What is a resolution?

- The time filtration: $\mathcal{F} = \{\mathcal{F}_{th}\}_{t=0,\dots,\lfloor T/h \rfloor}$, an increasing sequence of $\sigma-$algebras $\mathcal{F}_{t'} \subset \mathcal{F}_t$, $t' \leq t$.

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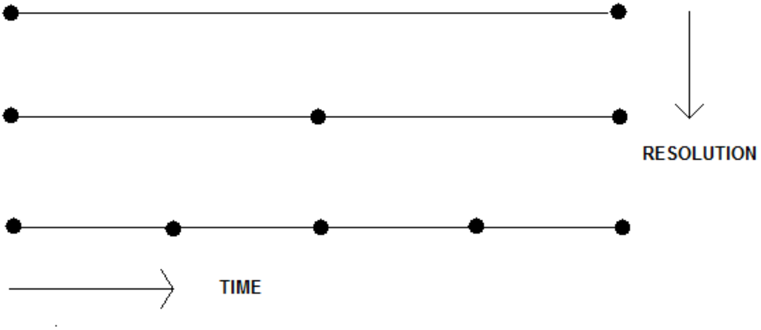
- Traditional time series representation approach: test the likelihood of a model specification on the finest grid (at the highest resolution) then a model is automatically specified at any coarser resolution by conditioning (temporal aggregation).

Illustration: Time vs Resolution filtration



Degree of persistence and changes of resolution.

- Conditioning under a change of resolution is equivalent to low pass filtering (averaging).
- A measure of persistence is implicitly defined by the resolution filtration.
- Fix minimum resolution scale $h_{\min} = 2^{-J_{\max}} h_{\max}$ then a sequence of resolution scales is defined by $h_j = 2^j h_{\min}$.
- DEFINITION: A shock has degree of persistence j if it is not measurable with respect to the $\sigma-$algebra $\mathcal{B}_{J_{\max}-j-1}$ but measurable at scale of persistence $\mathcal{B}_{J_{\max}-j}$, i.e. it is removed by $j+1$ applications of the filter.

Are we loosing some economic relevant information?

Good reasons for a closer look at the resolution dependence of our observations

- Macroeconomics: measurement of structural factors often rely on the definition of aggregate quantities which are observed at different frequencies and with different persistence properties (e.g. durability of consumption).

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- Decision Sciences: the preferences of the agents are often horizon dependent (e.g. recursive or hyperbolic preferences, rational inattention).
- Finance: the risk return tradeoff depends heavily on the holding period (long run risks literature)

A linear time series model accounting for persistence.

Definition

The dyadic mean operator acting on the time series of observations up to time t, $\mathbf{x}_t = \{x_{t-k}\}_{k\in 0,..,+\infty}$, is defined by:

$$\pi^{(1)}_{t-k} = \frac{x_{t-k} + x_{t-k-1}}{2}, \quad \pi^{(j)}_{t-2^j k} = \sum_{k'=0}^{2^{j-1}} \frac{x_{t-k'-2^j k}}{2^j}$$

The time series $\pi^{d(j)}_t = \left\{ \pi^{(j)}_{t-2^j k} \right\}_{k\in\mathbb{N}}$ is the $j-th$ (decimated) scale component. Define the detail at scale j, time $t - 2^j k$:

$$\delta^{(j)}_{t-2^j k} = \pi^{(j-1)}_{t-2^j k} - \frac{\pi^{(j-1)}_{t-2^j k} + \pi^{(j-1)}_{t-2^{j-1}(2k+1)}}{2} = \frac{\pi^{(j-1)}_{t-2^j k} - \pi^{(j-1)}_{t-2^{j-1}(2k+1)}}{2}$$

The time series $\delta^{d(j)}_t = \left\{ \delta^{(j)}_{t-2^j k} \right\}_{k\in\mathbb{N}}$ is called the j-th (decimated) detail component of the time series $\mathbf{x}_t$.

The PBD for a stationary process at fixed time t.

Theorem

Consider the PBD of a stationary time series $\{x_{t-k}\}_{k\in\mathbb{N}}$ *with Wold decomposition* $x_t = \mu + \psi(L)\varepsilon_t$*. Then:*

1. *the following decomposition holds for* x_t*:*

$$x_t = \sum_{j=1}^{+J} \delta_t^{(j)} + \pi_t^{(J)}$$

$$\pi_t^{(\infty)} \equiv \lim_{J\to+\infty} \pi_t^{(J)} = \mu$$

   the details $\left\{\delta_t^{(j)}\right\}_{j=1}^{+\infty}$ *define the term structure of shocks observed at time* t.

2. *the variance of the rescaled permanent component* $\pi_t^{(J)}$ *converges to the long run variance.*

Example: PBD of consumption growth

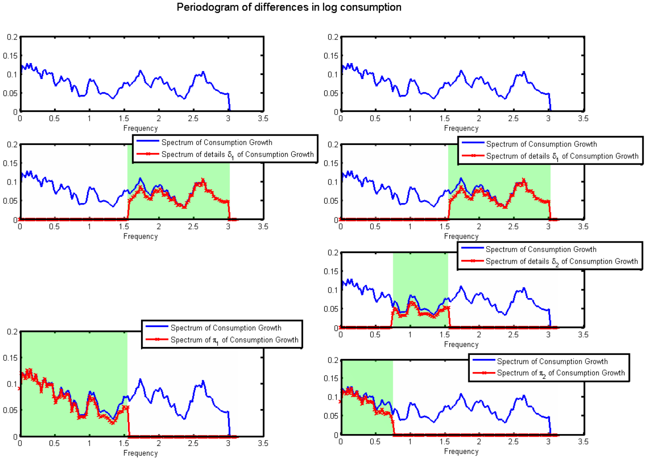

Figure: PBD of consumption growth.

Medium Term Business Cycle (Comin Gertler 2006)

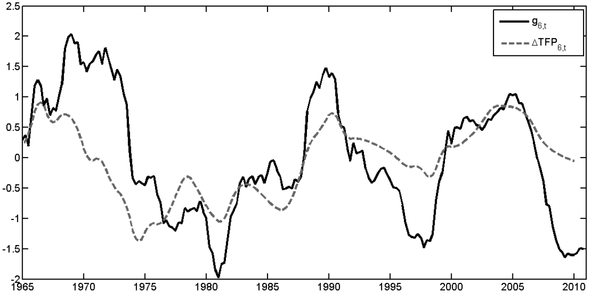

Figure: Component 6 of Consumption growth and TFP

Redundant vs Decimated PBD

- Shocks at scale of persistence j are naturally adapted on a grid with spacing 2^j.
- Forcing their detection at higher frequencies induces spurious correlation effects in the observation
- The instantaneous variance underweights the contribution to long run integrated variance of high persistence components

Definition

The decimated PBD truncated at level J of $\mathbf{x}_t$ is given by the vector $\left(\delta_t^{d(J)}, \pi_t^{(J)}\right)$ where $\delta_t^{d(J)} \equiv \left\{\delta_{t-2^j k_j}^{(j)}\right\}_{j=1,..,J,\ k_j=0,..,2^{J-j-1}}$ and $\pi_t^{(J)}$.

The PBD for an integrated time series

Theorem

Consider $\mathbf{y}_t = \{y_{t-k}\}_{k \in 0,..,+\infty}$ *such that* $E\left[y_0^2\right] < +\infty$ *and* $x_t = \Delta y_t$ *admit the Wold representation* $x_t = \mu + \psi(L)\varepsilon_t$ *with* $\sum_{j=0}^{+\infty} j\psi_j < +\infty$. *Then:*

$$y_t - y_0 = \tilde{\pi}_t^{(\infty)} + \sum_{j=1}^{+\infty} \widetilde{\delta}_t^{(j)} \tag{1}$$

where the (stationary) details $\widetilde{\delta}_t^{(j)}$ *at any level of persistence* j *and the scale component are given by*

$$\begin{aligned} \widetilde{\delta}_t^{(j)} &= -\sum_{k_j=0}^{+\infty} \left(\mathcal{T}_{Haar}^{(\infty)} \widetilde{\psi}\right)_{j,k_j} \varepsilon_{j,t-2^j k_j} \\ \tilde{\pi}_t^{(\infty)} &= \mu t + \psi(1) \sum_{s=1}^{t} \varepsilon_s. \end{aligned}$$

Wold Theorem and the PBD.

- Let the Hilbert space (same role of the $\mathcal{H}^{\gamma}(\mathbf{x}_t)$ space):

$$\mathcal{H}(\mathbf{x}_t) = \left\{ Z = \sum_{k \in \mathbb{N}} \alpha_k x_{t-k},\ \langle Z^1, Z^2 \rangle = \sum_{k \in \mathbb{N}} \alpha_k^1 \alpha_k^2 \right\}$$

metric definition neglects temporal correlations!

- Consider the rescaling operator, $R = D \circ M$ the composition of the dyadic dilation operator D with the dyadic mean M (same role of the L the lag operator). Then:

$$\begin{aligned} \mathcal{H}(\mathbf{x}_t) &= \bigoplus_{j=1}^{+\infty} R^j \mathcal{W}_t^R \oplus \mathcal{H}_{t,R}^{(\infty)} \\ R^j \mathcal{W}_t^R &= \left\langle \delta_t^{(j)} \right\rangle, \quad R\mathcal{H}_{t,R}^{(\infty)} \subseteq \mathcal{H}_{t,R}^{(\infty)} \end{aligned} \tag{2}$$

The Scale Component is the Beveridge Nelson Trend

The permanent component (stochastic trend) p_t^{BN} of a unit root non stationary process $x_t = \Delta y_t$ process is that component whose effect is not expected to decay but "persists" at any horizon (resolution scale).

$$
\begin{aligned}
p_t^{BN} &= y_t + \lim_{h \to +\infty} + E\left[\sum_{k=1}^{h} \Delta y_{t+k} - \mu h \mid \Omega_t\right] \\
&= y_t + \lim_{j \to +\infty} E\left[\Delta_{2^j} y_{t+2^j h_{\min}} - \mu 2^j h_{\min} \mid \mathcal{B}_j \cap \mathcal{F}_t\right]
\end{aligned}
$$

hence by CLT for stationary processes:

$$
\Delta p_t^{BN} \in \mathcal{H}_{t,R}^{(\infty)} = \cap_{j=0,..,+\infty} R^j \mathcal{H}_t\left(\mathbf{x}\right)
$$

Co-integration of Dividends and Prices

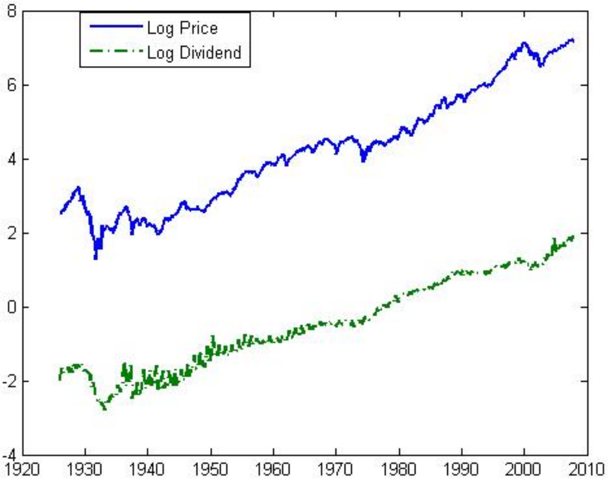

Figure: Time series of dividends and prices

Illustration: The scale component and the co-integration of dividends and prices.

Campbell and Shiller log-linear approximation: the permanent components of p_t and d_t are co-integrated with $\beta = [1 \ -1]$

- The detail component $\delta_t^{(1)}$ accounts for most of the transitory component.
- Given an integrated process y_t the quantity $\pi^{(J)}_{\lfloor 2^J r \rfloor}/\sqrt{2^J} \Rightarrow \psi(1) \int_0^1 W(r)dr$, we estimate $\pi^{(J)}_{\lfloor 2^J r \rfloor}/\sqrt{2^J}$ using $\frac{1}{T^{\frac{3}{2}}} \sum_{t=1}^{T} y_{t-1}$.
- The estimated coefficients are consistent with the co-integration relation ....

BN cycle and level 1 detail

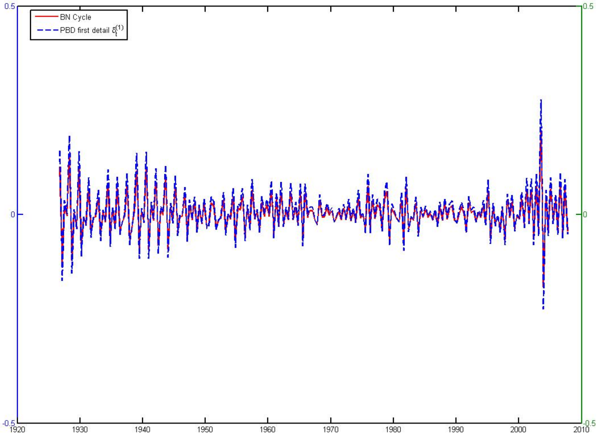

Figure: Comparison between BN cycle and the first detail component

BN Trend and Stochastic Scale Component

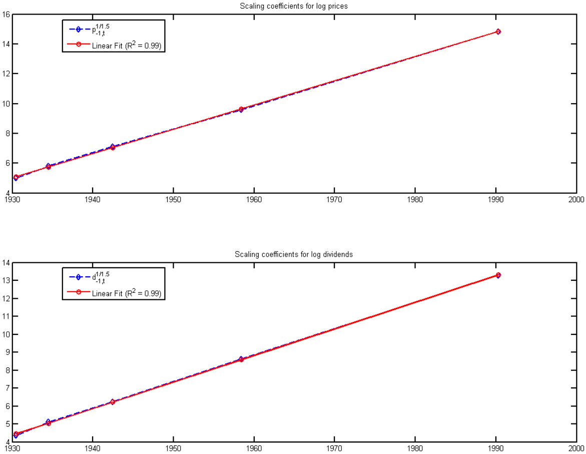

Figure: A test on the scaling properties of the scale component

GDP and Inflation forecasting

- A test on the predictive content of the transitory components we run the OLS regression of

$$\Delta_h x_{t+h} = \alpha + \sum_{j=1}^{J} \beta_j x_{t,j} + \epsilon_{t+h} \qquad (3)$$

  where $\Delta x_{t+1} = x_{t+1} - x_t$ is the next period change in US GDP and $x_{t,j}$ is the transitory component at level of persistence j.
- Select $x_t = 100 \times \log(GDP_t)$ 1947.q1-2010.q4.
- Select $x_t = \log(CPI_t / CPI_{t-1})$ CPI_t consumer price index seasonally adjusted, January 1947 - December 2008.
- We measure the adjusted R^2 and

$$G(h) = 100 \times \left(1 - \frac{\widehat{MSFE}(h)_{PBD}}{\widehat{MSFE}(h)_{TC}}\right)$$

  percent gain in forecast accuracy arising from our suggested decomposition compared to alternative TC (trend-cycle) measures.

Horse race

Panel A: Final cycle estimates vs. Extended BN

C_1	0.568	2.564	0.574	0.186	0.591
	(3.62)	(1.29)	(3.64)	(1.758)	(4.34)
C_2	0.228	0.227	0.218	0.630	0.462
	(1.85)	(1.84)	(1.74)	(4.79)	(4.19)
C_3	-0.172	-0.174	-0.191	0.325	0.050
	(-2.32)	(-2.34)	(-2.22)	(1.382)	(0.73)
C_4	0.031	0.029	0.017	0.273	0.127
	(0.60)	(0.56)	(0.28)	(0.66)	(2.79)
C_5	-0.028	-0.031	-0.031	0.255	-0.020
	(-0.78)	(-0.85)	(-0.84)	(0.826)	(-0.664)
C_6	0.004	0.002	0.006	-0.154	-0.013
	(0.16)	(0.09)	(0.24)	(-0.500)	(-0.637)
C_7	-0.008	-0.008	-0.008	0.557	0.004
	(-0.87)	(-0.90)	(-0.895)	(1.83)	(0.489)
C_8					
Beveridge — Nelson		1.723			
		(1.01)			
Clark			0.341		
			(0.43)		
Hodrick-Prescott 1-sided				0.006	
				(0.062)	
Hodrick-Prescott 2-sided					-0.388
					(-9.06)
$\bar{R}^2$	0.15	0.15	0.15	0.15	0.35

Table: Predictive regressions for real GDP growth using lag of cycle estimates.

GDP forecast over multiple horizons

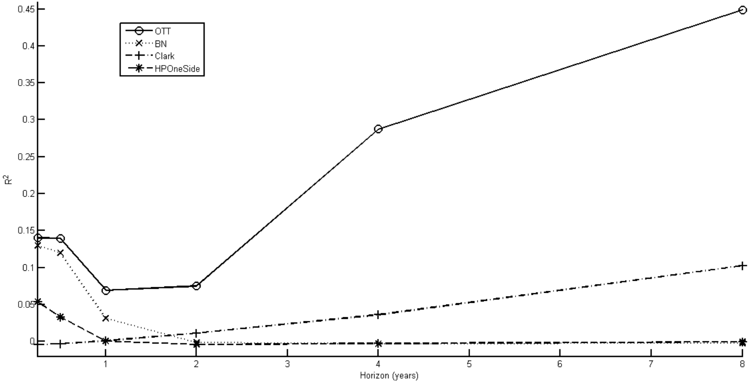

Figure: Adjusted R^2 for GDP forecast over multiple horizons

Gain in GDP forecast accuracy

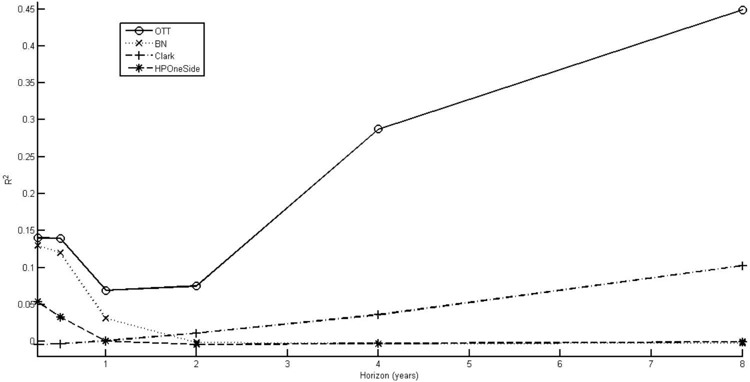

Figure: Percent gain in GDP forecast accuracy

Inflation forecast over multiple horizons

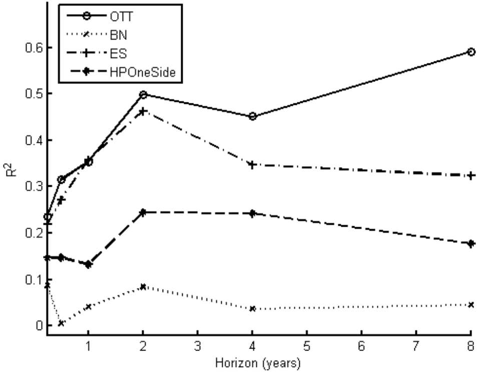

Figure: Adjusted R^2 for Inflation forecast over multiple horizons

Gain in Inflation forecast accuracy

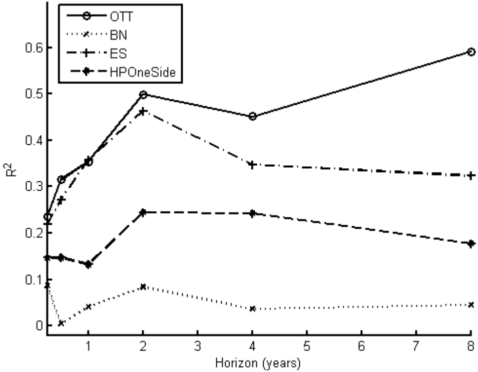

Figure: Percent gain in Inflation forecast accuracy

A comparison with Core Inflation Cogley (2002)

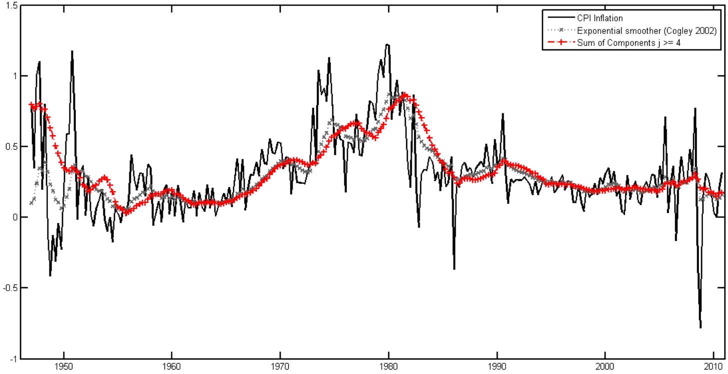

Figure: Percent gain in Inflation forecast accuracy

Motivation.

- In macroeconomics the study of business cycles begins with the problem of measurement: how to separate macroeconomic data into trends and cycles.
- In finance the risk-return trade-off profile which describes efficient investment opportunities in the market is strongly dependent on the investor's holding period.
- These two areas often address the same issues from different perspectives and languages. Their reconciliation is non trivial and multiresolution approaches seem to be a promising direction!