This draft: October 2011

# Credit and business cycles: some stylised facts[1]

Feng Zhu[2]
Monetary and Economic Department
Bank for International Settlements

## Abstract

It has often been assumed that credit and business cycles are closely related, and credit market difficulties tend to cause, amplify and prolong cyclical downturns in real activity. We investigate the hypothesis using both time and frequency-domain methods, including cross-spectral density, coherency, gain, phase-to-frequency ratio and frequency-specific correlation and regression techniques proposed by Zhu (2005). The frequency-specific coefficient of correlation is real-valued, signed and normalised, and the frequency-specific regression coefficient has the advantage that conventional asymptotic theory continues to apply.

We show that the credit-output relationship varies across countries and over time. In particular, the correlation and linear association between US credit variables and real activity are quite weak in the business-cycle frequencies. Indeed the relationship varies across frequencies, and a significant correlation only exists in the very low frequencies, i.e. long run. Moreover, tests of causality between credit and real activity are inconclusive concerning the direction of causality, or indeed whether causality exists. Moreover, a different relationship emerges in the euro area and Japan, and it is also time-variant. Our analysis suggests that bank lending standards and private sector balance sheets might help predict credit growth.

[1] I thank Andy Filardo, Madhu Mohanty and BIS seminar participants for helpful comments. Jakub Demski provided able assistance. The views expressed in the paper belong to the author alone and do not represent the views of the Bank for International Settlements (BIS). All remaining errors are mine.

[2] Contact: feng.zhu@bis.org.

# I. Introduction

The recent global financial crisis has had a major impact on both advanced and emerging economies. The housing market downturn has been sharp and persistent in the US and several European economies. Global credit markets have gone through several turbulent years, marked by the failure of two Bear Stearns hedge funds in July 2007, the collapse of Lehman Brothers and the bailout of the American International Group in September 2008, and the still ongoing European sovereign debt crisis. Following the onset of the crisis, corporate credit spreads rose dramatically; equity prices fell rapidly, and implied volatilities in most market segments reached record high levels. Important segments of credit markets ceased to function and losses and write-downs amassed in major financial institutions. A wide-spread deterioration in the health of financial institutions and the lack of confidence in the viability of some of these institutions sharply reduced the availability of bank credit. Banks tightened lending standards and credit flows to the private sector slowed significantly. A major global recession ensued in 2008.

The virulent financial crisis and ensuing global recession evoked memories of and warranted comparisons to the Great Depression in the 1930s. One other oft-made comparison was with Japan's lost decade(s) following its asset market crash in the late 1980s and banking crisis in the 1990s. Economists confront the difficult questions of whether the global economy would have fallen into a depression if not for large-scale and coordinated government interventions, and whether the major advanced economies still face the prospects of Japan-style anaemic growth. In all three episodes, asset market booms and busts and the ups-and-downs in credit flows appeared to have played a major role. One implicit assumption is that credit market conditions are closely associated with real activity, and major financial sector disturbances are an importance source of macroeconomic fluctuations. In other words, credit and economic cycles are closed related and credit market events could trigger or prolong business-cycle fluctuations.

One strand of past literature focused on the longer-term causal link between financial market development and economic growth. A more developed banking sector and stock market are associated with faster growth: sound financial development encourages capital accumulation and facilitates technological innovations. Bagehot (1873) and Hicks (1969) argue that financial development was essential to jump-start England's industrialization by mobilising capital and facilitating investment. Schumpeter (1912) suggests that banks could stimulate technological innovations as they have the advantage of identifying capable entrepreneurs and channel funding to potentially lucrative innovative products and production processes. Causality could also run from the demand side, and economic activity could influence or shape financial development. As Robinson (1952) famously puts it, "where enterprise leads finance follows." Essentially, growth creates demand for specific financial arrangements, and a financial system evolves through its responses to such demand.

Nevertheless, not all economists are convinced of the importance of the finance-growth link. Lucas (1988) considers that the role of financial factors was "badly over-stressed" in the growth literature. Development economists have tended to be more sceptical about the role of finance in economic development, often opting to ignore it in their analysis of determinants of long-run growth. Discussions on financial systems are often short or even omitted outright in standard textbooks of development economics (e.g. Thirlwall 2011, Todaro and Smith 2005) or discounted in review articles (e.g. Stern 1989).

The emphasis of past work, theoretical or empirical, has been mostly focussed on the long run. Empirical evidence indicates a positive and quantitatively significant relationship between financial development and economic growth. Goldsmith (1969) asserts that "a rough parallelism can be observed between economic and financial development if *periods of* *several decades* are considered." King and Levine (1993a, 1993b, 1993c) use four measures of the level of financial development to analyse whether it would predict long-term growth. Based on a sample of 80 economies and using growth indicators and financial measures averaged over the period 1960–1989, they find positive and significant correlation between real per capita GDP and the extent to which loans are directed to the private sector; and a significant relationship between the initial level of financial development and growth.

One stand of theory considers credit expansion and contraction the main driver of economic fluctuations. Finance and banks play a key role in business cycles, so do speculative bubbles in asset markets. A notable contribution is the debt-deflation theory of Irving Fisher (1933), which was expanded by Hyman Minsky. Minsky's financial instability hypothesis places fluctuations in credit, interest rates and financial fragility at the centre of cycles. In expansion, interest rates are low and firms borrow excessively and over-invest. Once the process leads to very high levels of firm indebtedness, investment stops and the economy enters recession.

In the past few decades, the focus has shifted towards studying the interplay between credit market development and real activity over the business cycle.[3] If banks play a special role in credit creation due to financial market imperfections, and if credit is important to economic activity, then changes in bank lending could have significant real effects. Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999) emphasize the role of borrowers' balance sheets and point out that endogenous credit market developments could amplify and propagate shocks to the economy, and a "financial accelerator" could play a significant role in business-cycle dynamics. Kiyotaki and Moore (1997) and Kiyotaki (1998) also stress a borrower-lender agency problem which drives the external finance premium. Booms and busts of asset markets, real estate price dynamics in particular, affect borrowers' balance sheets. Fixed or durable assets play a dual role as factor of production and collateral for loans, interactions between credit limits and asset prices are a powerful transmission mechanism through which the effects of shocks persist and amplify.

In their recent work, Gertler and Kiyotaki (2010) focus on the balance sheet problems of financial intermediaries, which could limit their ability to obtain funds from depositors in retail markets, or from other intermediaries in wholesale or interbank markets. If businesses could only borrow from a limited number of intermediaries, then disruptions in interbank markets could also affect real activity.

Relying on existing theory and evidence on the long-term finance-growth link, causality and strong positive correlations between credit and output over economic cycles have often been assumed as "stylised facts" by economists and financial market participants. A systematic analysis is needed to investigate such assumed "empirical regularities". In this paper, we use both time and frequency-domain methods to determine whether the assumption of close correlations between credit cycles and fluctuations in real activity stands empirically.

We find that relationship between credit and real activity in the business-cycle frequencies varies both across countries and over time. In particular, the strength of correlation and linear association between US credit variables and real activity turns out to be rather weak over the business cycles. But credit is pro-cyclical and strongly correlated with output in the euro area. In Japan, the credit-output relationship changed over time, and a counter-intuitive correlation pattern emerges following the 1997 banking crisis. Moreover, tests of causality between credit and real activity are so far inconclusive concerning the direction of causality, or indeed whether causality exists. Our analysis also suggests that bank lending standards and private sector balance sheets might help predict credit growth.

[3] Gertler (1988) and Bernanke (1992) provide excellent reviews of past literature on the relation between financial intermediation and aggregate economic activity.

We examine the US credit-output relationship in the frequency-domain, and over the entire spectrum. We estimate a number of spectral indicators including spectral and cross-spectral densities, coherency, gain, phase-to-frequency ratio, and Zhu's (2005) frequency-specific correlation and regression coefficients. Our frequency-domain analysis confirms that the US credit-output relationship varies across frequencies, and significant correlation only exists in the very low frequencies, i.e. long run. The relationship is indeed quite weak over business cycles. This could have important implications for theories which rely heavily on the role of credit in business cycles.

Section II of the paper reviews a number of major episodes of credit and banking crises in the post-war period, examining interactions of credit market events, public policy and real activity. Section III presents evidence in the time domain, including causality test results. In Section IV, we consider further evidence on the credit-output relationship in the frequency domain. In Section V, we study the usefulness of lending standards and commercial bank balance sheets for predicting credit growth. Section VI concludes. The Annex provides technical details on the estimation of Zhu (2005) frequency-specific coefficients of correlation and regression.

## II. Financial crises and real activity: the post-War experience

Banks are special in their ability to attenuate problems of adverse selection, moral hazard and information free-riding arising from borrower-lender information asymmetry. Despite the secular trend of disintermediation and a dwindling share of banks in financial intermediation, bank lending retains a prominent role in the financial system, and it is particularly important for small and medium-sized firms with little access to funding through securities markets. A credit crunch could exert a large impact on the functioning of financial system, on credit flows, aggregate spending and ultimately on economic activity.

The past 50 years have witnessed a number of acute credit contractions, starting with the US credit crunch of 1966. Although credit downturns come in different forms, there have been striking similarities in terms of the underlying causes and ultimate financial and economic consequences. Credit contractions often go hand in hand with recessions, though the causal link may run in either direction. We examine the US 1989–1992 episode in the aftermath of the savings and loan (S&L) crisis, Japan's 1997–1998 banking crisis, the Nordic crises of the 1990s, and the recent credit downturn in Germany. Noteworthy were the roles played by: asset price fluctuations including real estate booms and busts; excessive risk-taking and over-investment in unfamiliar products driven by *de facto* deregulation, intense competition in the banking sector and a zeal for financial innovations; a prolonged period of easy monetary conditions.

### II.1. The US 1989–1992 credit crisis

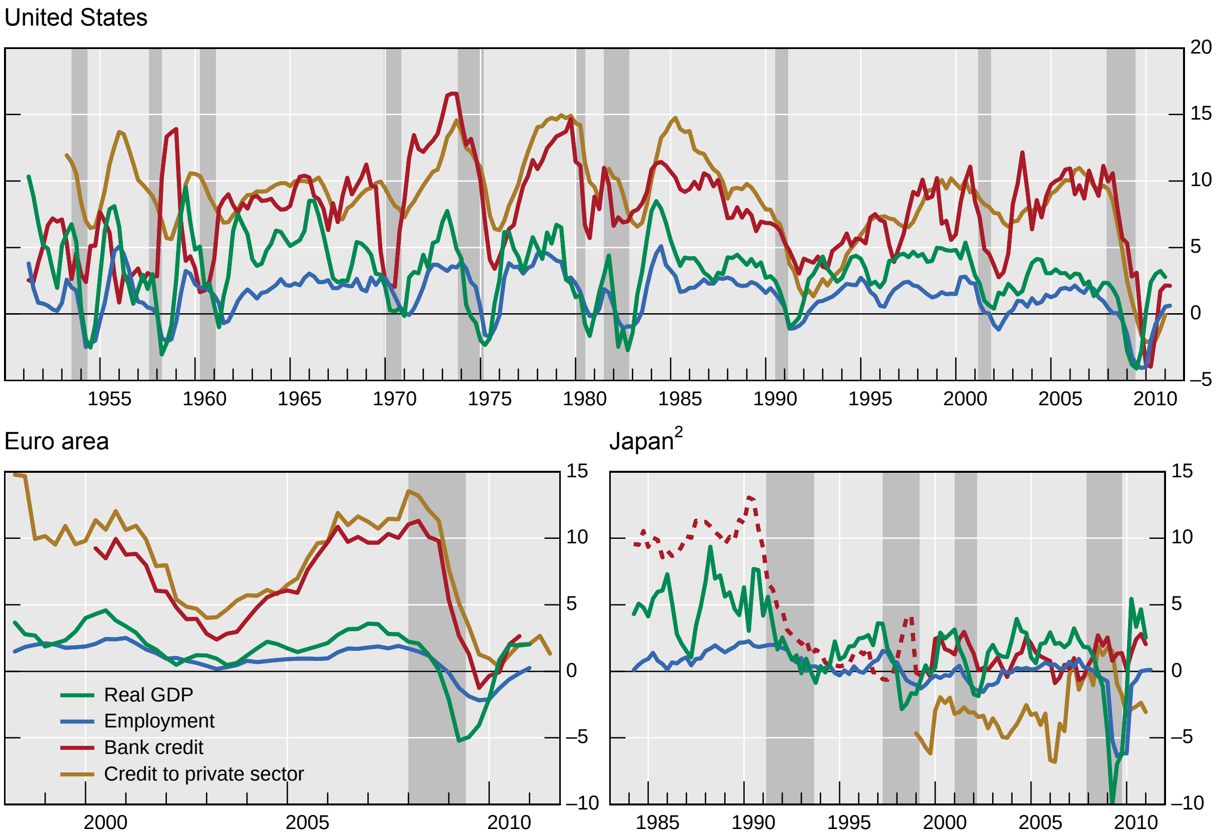
The 1989–1992 credit crisis in the United States occurred in the aftermath of the saving and loan (S&L) crisis in the 1980s and early 1990s, a period when banks greatly increased their exposure to the commercial real estate sector in response to intense competitive pressures. Several factors contributed to this: regulatory changes removed interest rate ceilings on deposits, and the S&L or thrift institutions were granted expanded lending and investment powers. With growing disintermediation, a sizeable portion of commercial and industrial lending business was lost to the commercial paper market. With maturity mismatch on banks' balance sheets, a sharp rise in interest rates in the early 1980s led to an erosion of banks' net worth, partly because they were constrained from increasing rates for long-term loans when deposit rates rose.

Changes in tax rules in the real estate sector in the 1980s (e.g. the 1981 Economic Recovery Tax Act) had a direct impact on real estate lending and prices and spurred investment by S&L institutions. Credit terms loosened significantly. Many banks abandoned their traditional focus on local-area lending and ventured into real estate projects with which they had little experience and expertise. A lending and building boom ensued and real estate prices skyrocketed in areas like New England. Bank balance sheets grew and loan portfolios became riskier. Between 1980 and 1990, as a ratio to total assets, total loans and leases increased from 55% to 63%, and total real estate loans from 18% to 27%. At the same time, loan quality deteriorated, with the share of non-performing loans rising from 2.9% in 1984 to 5.2% in 1991.

Graph 1

**Credit and economic cycles**[1]

In per cent




1 Annual growth of quarterly data. Shaded areas represent recession dates: for United States specified by NBER, for Euro area specified by CEPR and for Japan defined by output cycle. 2 Dashed line represents bank credit spliced by loans by private financial Institutions.

Sources: Bank of Japan; IMF; national data.

The Tax Reform Act of 1986 repealed many advantageous provisions, dampening demand for commercial real estate and softening prices. Tighter limits on the thrift institutions' lending activity and new capital regulations led banks to reduce lending and reallocate assets from loans to holdings of government securities. The bust resulted in an unprecedented level of bank losses and business failures, leaving many regions distressed. Losses with commercial real estate loans were pronounced. In 1980, the ratios of commercial real estate loans to total assets of banks that eventually failed and non-failed banks were about 6%, but rose to 30% and 11% in 1993, respectively. The financial distress was seen to have set off and aggravated the recession that began in July 1990. The decline in bank lending affected small businesses the most. Credit to the private sector continued to fall even as the economy began to recover and output growth picked up quickly (Graph 1, upper panel). Nevertheless, Bernanke and Lown (1992) were sceptical on whether the credit crunch played a major role in the recession, and demand effects could be more important.

In fact, US credit and business cycles have not always been well synchronised (Graph 1, upper panel). The credit-output relationship was time-varying, and the synchronisation was apparently weak in the 1950s, early 1960s and also 1990s. The synchronisation was stronger during the recessions. The credit-output relationship involved leads or lags, although it was not always clear which variables were leading. A formal cross-correlation analysis could be useful in uncovering the timing of cyclical synchronisations.

### II.2. Japan's banking crisis of 1997–1998

The second half of the 1980s was characterised by a fast rise in asset prices, expansion in money and credit, and an overheated economy (Graph 1, lower right panel). Although financial liberalisation and disintermediation proceeded in measured pace since the early 1980s, concerns with a loss of customers and profit pressures led to aggressive lending behaviour among banks. Large unrealised capital gains from stockholdings and banks' own equity financing boosted banks' capital base, encouraging banks to lend even more.

Monetary policy might also have played a role: after the 1985 Plaza Agreement, the Bank of Japan (BoJ) eased policy substantially to counter yen appreciation. From the summer of 1987 onwards, the BoJ sought to tighten its policy stance but found it difficult to proceed. Much emphasis was given to maintaining exchange rate stability preventing further yen appreciation, and reducing current account imbalances by boosting domestic demand. Large increases in money supply and in asset prices caused concerns, and in May 1989 the BoJ shifted to much needed monetary tightening. In the meantime, stock prices continued to rise till the end of 1989 and then plummeted, falling to half of the peak level by August 1990.

Declines in Japanese equity and commercial real estate prices in the late 1980s and early 1990s had a large impact on Japan's banking sector and the economy. From their peaks in 1989 and 1992, stock prices lost almost 60% of value within three years, and commercial land prices fell by roughly 50% in the next 10 years. The collapse of the asset price bubble negatively affected firms' balance sheets and banks' capital positions, and consequently banks' willingness to lend. Problems emerged in the banking sector in the early 1990s but only blew to crisis proportions towards the end of the decade, leading to a credit crunch in early 1997. Several high-profile banks and securities firms failed in October and November 1997, and the situation worsened in 1998. The government was, therefore, forced to inject a large sum of public funds into the deposit insurance fund and to recapitalise troubled banks.

Japan's banking crisis was marked by severe non-performing loan problems, which weighed heavily on banks' balance sheets and depressed own capital ratio, curtailing bank's ability and willingness to lend. At the end of March 1999, non-performing assets of major banks, including direct write-offs, stood at 9.0% of GDP. The crunch prolonged and deepened the ongoing recession. By some estimates, it shaved 1.6 percentage points off Japan's GDP growth during the crisis period.

However, the credit and output cycles became little synchronised since the mid-1990s, about the time when the banking crisis erupted (Graph 1, lower-right panel). Japanese banks were affected by new capital regulations: since any attempt to call in non-performing loans would imply a possible write-off of existing capital, many banks continued to lend to "zombies" or unprofitable and problematic borrowers receiving subsidised credit either to stay above minimum capital requirement or maintain ongoing business relationships with long-time clientele. Such bank forbearance collided with regulator forbearance. In fact, credit appeared to have been largely available for large but not small firms, and the bank lending channel of monetary policy was severely damaged.

### II.3. Nordic banking crises in the 1980s

Financial liberalisation in the 1980s led to greater competition in the banking system, which led to aggressive lending activities in Finland, Norway and Sweden. Bank loan to GDP ratio rose from 40%, 55% and 41% in 1984, to 65% in 1988 in Norway, and to 98% and 58% in 1990 in Finland and Sweden. The lack of market discipline, insufficient and inadequate risk management, and implicit government guarantee might have played a role in the rapid rise in lending to real estate, construction and services. Foreign currency denominated borrowing also surged, representing about half of the corporate debt in Finland in the late 1980s. Loan demand by household and corporate sectors increased rapidly, with the household debt to net disposable income ratios increasing in Finland and Norway from 45% and 90% in 1980 to about 90% and 175% by the end of 1990s.

Excessive lending boosted asset prices in all three economies. Compared to 1980 levels, real estate prices in Norway quadrupled in 1987 and rose by 800% and 300% in Sweden and Finland before reaching the peak in 1989. In the same period, bank share prices rose even more in Sweden and Finland. Excessive lending also had negative impact on banks. First, as banks relied increasingly on money markets and foreign lenders instead of deposits, funding costs and operational risks rose. Second, bank asset quality deteriorated and profit margins became compressed.

Real estate prices later collapsed, falling to 1980 levels in 1994–1995; declines in bank share prices were even more dramatic. Since the onset of recessions, bank losses rose quickly in the early 1990s. The share of bank loan losses in total loans jumped from 0.7% in 1987 to 6% in 1991 in Norway; 0.7% in 1898 to 4.7% in 1992 in Finland, and 0.3% in to 7% in 1992. In Sweden, banks' before-tax profits became negative in 1991 and fell further in the following years. A significant portion of losses pertained to real estate loans, and banks carried a large amount of non-performing loans to households. In Sweden, commercial real estate loans created serious problems, while in Finland foreign-currency denominated debt played a special role.

Falling asset prices and increasingly unfavourable economic conditions led households and firms to cut down consumption and investment, further depressing asset prices and reduce aggregate spending. The collapse of trade with members of the Council for Mutual Economic Assistance in 1990-1992, and depreciations of the Finnish markka and Swedish krona in the early 1990s dealt a blow to the regional economies.

Monetary and fiscal actions might also be useful in improve economic performance. In the Nordic economies, economic recovery prompted loan losses to fall since 1994, and the banking sector returned to profitability in 1993 in Norway, and 1994 in Sweden. In Finland where the crunch and recession had been most severe, banks only became profitable again in 1996.

### II.4. The 2002–2003 German credit crunch

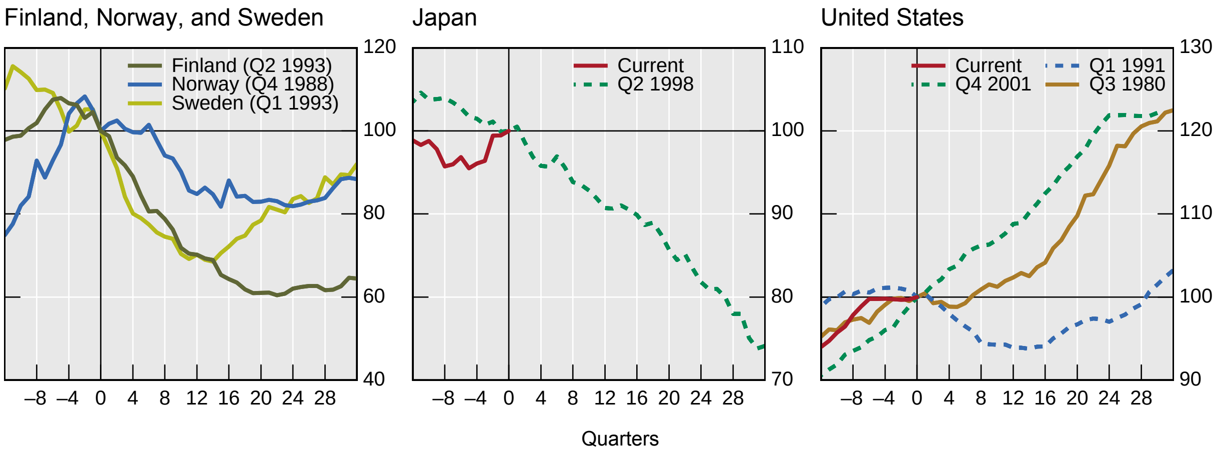
The 2002–2003 German credit crunch was mainly attributed to a sharp decline in stock prices in the early 2000s, which weakened bank balance sheets and capital positions. The CDAX composite index fell from its February 2000 peak of over 500 points to about 160 in March 2003. The impact on German banks was large, with *Grossbanken* the hardest hit. Substantial supply-side factors contributed to the credit downturn: banks were relatively thinly capitalised at the time; a substantial part of bank capital consisted in shares and bonds; and German banks already started to adjust their lending policies in anticipation of the implementation of new risk-based capital requirements in 2007.

Business loans began to slow in 2000 and contracted from the second quarter of 2002 onwards, after years of 6–9% growth. The economy contracted and did not start to recover until 2004. Already high at 9.2% in May 2001, German unemployment rate rose again and only started to fall after reaching a peak of 12.1% in March 2005. The economic and credit cycle reinforced each other and delayed the recovery. In fact, euro area credit and output cycles also appeared to be well synchronised since 1999 (Graph 1, lower-left panel).

Importantly, past experience points to very different patterns of credit recovery following a normal recession and following a major banking crisis. For instance, in the United States after the recessions in the early 1980s and 2000s, the credit-to-GDP ratio was little affected, growing almost immediately at the end of the downturn (Graph 2). By contrast, following the recession in the early 1990s that was preceded by a combination of the savings and loan crisis and wider stress among commercial banks, it took more than 7 years for debt-to-GDP to regain to its pre-recession peak. Similarly, in the Nordic countries and Japan, during the 1980s and 1990s banking crisis, the decline in the credit-to-GDP ratio was long-lasting.

**Graph 2**

Credit recovery over selected business cycles[1]




[1] Period zero and dates in the panel legends refer to the trough of the output cycle; for the United States, trough dates for 1980, 1991 and 2001 are from the National Bureau of Economic Research (NBER); domestic credit to the private sector as a percentage of GDP, rebased on the trough of the business cycle; for the current cycle the trough is assumed to be second quarter of 2009.

Sources: IMF; national data.

Unlike other post-war episodes, the most recent credit market crisis, originated in a number of major industrial economies, was more severe and quickly spread to other economies. The turmoils seemed to have led to a generalised credit squeeze in the global economy, with large consequences for global trade and global economic activity. A sharp and widespread downturn followed: orders fell, factories closed, output collapsed and massive job losses followed. Notably, the impact was less severe and the subsequent recovery more rapid and robust in most emerging economies, which were less affected by the credit market meltdown.

Past financial crisis episodes illustrate the important role played by the boom and bust of asset markets, and the most recent credit cycle reinforced the idea that real estate market developments was a key link between credit and real activity. First, real estate constitutes a substantial part of household wealth and firm assets, fluctuations in property prices directly affect household and firm spending decisions; Second, as pointed out by Kiyotaki and Moore (1997), fixed or durable assets such as real estate play a dual role as factors of production and collateral for loans, and interaction between credit limits and asset price dynamics are a powerful transmission mechanism through which the effects of shocks persist and amplify; Third, real estate market developments could influence the behaviour of both lenders and borrowers, and could have an impact on the functioning of monetary policy transmission. We examine correlations between asset prices, bank lending and real activity.

## III. Credit and economic cycles in the time domain

We first examine the relationship between output and credit cycles in the United States, euro area and Japan in the time domain, relying on simple cross-correlation estimates and tests of causality.

### III.1. Output and credit cycles: cross-correlations

Using cross-correlation estimates to study business cycles has been a time-honoured tradition in macroeconomic analysis.[4] We follow Blanchard and Waston (1986) and Kydland and Prescott (1990) and examine the comovement between the cyclical component of real GDP and those of other important real and financial variables, with a focus on credit flows. Following Kydland and Prescott (1990), we define *cyclicality* and *lead-lag* relationships based on cross-correlation estimates as follows:

- Let $Y$ be real GDP. The series $X$ is said to be ***procyclical*** if $Corr(X, Y_t)$'s the coefficients of correlation between $X$ and $Y_t$ are positive and close to 1 around period $t$; the series $X$ is ***countercyclical*** if $Corr(X, Y_t)$'s are negative and close to −1 around period $t$; and the series $X$ is ***acyclical*** if $Corr(X, Y_t)$'s are all small.
- With respect to phase shift between two economic time series, we say the series $X$ ***leads*** the output cycle by $i$ quarters if $Corr(X, Y_t)$'s peak at $X_{t-i}$ with $i>0$; the series $X$ ***lags*** the output cycle by $j$ quarters if $Corr(X, Y_t)$'s peak at $X_{t+j}$ with $j>0$; the series $X$ ***coincides with*** the output cycle if $Corr(X, Y_t)$'s peak at $X_t$.

We first estimate the cyclical components for all macroeconomic variables, logged whenever necessary before the use of the Hodrick-Prescott (HP), Band-pass (BP), and first difference (FD) filters. For BP-filtered estimates of cyclical components, the business-cycle frequency is defined to be between six and 32 quarters. For HP-filtered estimates, the smoothing parameter (lambda) is set to 1,600. Second, we calculate correlation coefficients between the current-period cyclical component of the real GDP and the led, current and lagged cyclical components of several real and nominal variables including bank credit. The maximum number of leads and lags is each fixed to five. We use quarterly data ending in Q4 2009. The original full sample sizes are listed in Table 1.

Table 1 **Raw data sample periods**

| | United States | Euro area | Japan |
|---|---|---|---|
| Real GDP | 1950 Q1 – 2009 Q4 | 1991 Q1 – 2009 Q4 | 1955 Q2 – 2009 Q4 |
| Nominal bank credit | 1950 Q1 – 2009 Q4 | 1997 Q4 – 2009 Q3 | 1997 Q4 – 2009 Q3 |
| Employment | 1950 Q1 – 2009 Q4 | 1980 Q1 – 2009 Q3 | 1953 Q2 – 2009 Q4 |
| Real private fixed investment | 1995 Q1 – 2009 Q4 | 1991 Q1 – 2009 Q3 | 1955 Q2 – 2009 Q4 |
| Real private consumption | 1950 Q1 – 2009 Q4 | 1991 Q1 – 2009 Q3 | 1955 Q2 – 2009 Q4 |

The exact sample size for correlation estimates varies from one country to another, and from one pair of variables to another. We use the longest sample available for each pair of variables, for several reasons. First, it is notoriously difficult to accurately estimate the trend and cyclical components of any macro variable given the available sample size; second, estimation of lead-and-lag relationships implies that ten observations would be lost in the final cross-correlation estimates; third, in computing BP-filtered cyclical components, the first

[4] A recent application of cross-correlation analysis is Comin and Gertler (2006).

and last six estimates have to be trimmed. It is also known that end-of-sample HP-filtered estimates of cycle components are very imprecise, we therefore trim four last estimates.

According to Abel, Bernanke and Croushore (2007), financial variables are "sensitive to the cycle", in particular, "stock prices are generally procyclical and leading", "nominal interest rates are procyclical and lagging", while "real interest rate doesn't have an obvious cyclical pattern". Therefore in this section we focus on the estimates of correlations between the cyclical components of financial variables with the cyclical component of real GDP, using correlations estimates between the cyclical components of real variables with that of real GDP as a benchmark. We are especially interested in whether money, credit and asset prices are strongly procyclical and whether they lead or lag the output cycle.

### *III.1.1. Output and credit cycles in the United States*

| Table 2 | **Cyclical Behaviour of US Output Components** Deviations From Trend of Product Variables* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Variable X** | **Cross Correlation of Real GDP With** | | | | | | | | | | |
| | **x(t-5)** | **x(t-4)** | **x(t-3)** | **X(t-2)** | **x(t-1)** | **x(t)** | **x(t+1)** | **x(t+2)** | **x(t+3)** | **x(t+4)** | **x(t+5)** |
| Real GDP | -0.50 | -0.31 | 0.06 | 0.50 | 0.86 | 1.00 | 0.86 | 0.51 | 0.06 | -0.32 | -0.52 |
| Real private consumption | -0.36 | -0.13 | 0.22 | 0.60 | 0.85 | 0.89 | 0.71 | 0.36 | -0.03 | -0.35 | -0.51 |
| Nondurables | 0.17 | 0.44 | 0.66 | 0.81 | 0.85 | 0.79 | 0.66 | 0.48 | 0.28 | 0.11 | -0.03 |
| Durables | 0.31 | 0.45 | 0.59 | 0.66 | 0.60 | 0.38 | 0.07 | -0.22 | -0.36 | -0.29 | -0.12 |
| Real private fixed investment | 0.00 | 0.18 | 0.38 | 0.58 | 0.75 | 0.86 | 0.88 | 0.80 | 0.64 | 0.42 | 0.17 |
| Structures | 0.02 | 0.15 | 0.29 | 0.41 | 0.51 | 0.58 | 0.61 | 0.59 | 0.52 | 0.40 | 0.21 |
| Non-residential | -0.17 | -0.01 | 0.18 | 0.41 | 0.62 | 0.80 | 0.90 | 0.88 | 0.77 | 0.56 | 0.31 |
| Residential | 0.60 | 0.67 | 0.64 | 0.56 | 0.46 | 0.34 | 0.19 | 0.05 | -0.07 | -0.14 | -0.18 |
| Equipment | -0.02 | 0.17 | 0.38 | 0.61 | 0.80 | 0.92 | 0.94 | 0.84 | 0.64 | 0.39 | 0.14 |
| Government Purchases | -0.02 | -0.05 | -0.06 | -0.02 | 0.05 | 0.11 | 0.12 | 0.09 | 0.04 | 0.03 | 0.10 |
| Govt consumption | 0.19 | 0.11 | -0.01 | -0.17 | -0.38 | -0.59 | -0.76 | -0.83 | -0.78 | -0.62 | -0.38 |
| Govt investment | 0.13 | 0.22 | 0.27 | 0.26 | 0.16 | -0.02 | -0.24 | -0.49 | -0.64 | -0.60 | -0.41 |
| Net trade | -0.26 | -0.29 | -0.30 | -0.30 | -0.29 | -0.25 | -0.19 | -0.12 | -0.08 | -0.09 | -0.12 |
| Exports | -0.21 | -0.15 | 0.00 | 0.23 | 0.50 | 0.72 | 0.80 | 0.70 | 0.44 | 0.11 | -0.18 |
| Imports | 0.03 | 0.10 | 0.22 | 0.41 | 0.64 | 0.80 | 0.83 | 0.67 | 0.38 | 0.06 | -0.19 |
| Employment | -0.49 | -0.44 | -0.22 | 0.11 | 0.48 | 0.76 | 0.86 | 0.77 | 0.51 | 0.18 | -0.12 |
| Ind. production | -0.38 | -0.28 | -0.03 | 0.30 | 0.61 | 0.80 | 0.79 | 0.60 | 0.28 | -0.05 | -0.30 |
| Retail sales | -0.38 | -0.06 | 0.28 | 0.56 | 0.72 | 0.73 | 0.63 | 0.47 | 0.31 | 0.20 | 0.14 |
| New orders, manufacturing | -0.22 | -0.09 | 0.12 | 0.35 | 0.55 | 0.67 | 0.68 | 0.55 | 0.33 | 0.08 | -0.12 |

* Quarterly data. We extracted cyclical components using the Hodrick-Prescott (HP) filter, the Band-Pass (BP) filter and the first-difference (FD) filter. Results are shown for the BP-filtered cycles.

We examine correlations for the US economy. The focus can be justified by that fact that it is by far the largest economy in the world; the US credit-output relationship has been subject to intense academic and policy analyses; and US data are longer, of higher quality and more consistent. The results can be compared with a rich pool of existing work on US business or credit cycles. In the next section, we investigate the credit-output relationship over time and across three major economies: the United States, euro area and Japan.

| Table 3 | **Cyclical Behaviour of US Nominal Variables**<br>Deviations From Trend of Nominal Variables* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Variable X** | **Cross Correlation of Real GDP With** | | | | | | | | | | |
| | **x(t-5)** | **x(t-4)** | **x(t-3)** | **X(t-2)** | **x(t-1)** | **x(t)** | **x(t+1)** | **x(t+2)** | **x(t+3)** | **x(t+4)** | **x(t+5)** |
| M2 | 0.23 | 0.37 | 0.48 | 0.51 | 0.44 | 0.28 | 0.07 | -0.14 | -0.30 | -0.39 | -0.39 |
| Bank credit | -0.01 | 0.15 | 0.27 | 0.32 | 0.29 | 0.20 | 0.11 | 0.04 | 0.02 | 0.02 | 0.03 |
| Business loans | -0.40 | -0.46 | -0.42 | -0.28 | -0.05 | 0.22 | 0.45 | 0.60 | 0.63 | 0.54 | 0.35 |
| Real estate loans | -0.31 | -0.20 | -0.04 | 0.15 | 0.33 | 0.46 | 0.52 | 0.48 | 0.37 | 0.21 | 0.04 |
| Commercial real estate | | | | | | | | | | | |
| Mortgage | 0.02 | -0.01 | -0.04 | -0.03 | 0.02 | 0.12 | 0.21 | 0.25 | 0.21 | 0.11 | -0.02 |
| Consumer loans | -0.39 | -0.39 | -0.27 | -0.04 | 0.26 | 0.52 | 0.68 | 0.70 | 0.59 | 0.38 | 0.13 |
| Credit to private sector | -0.49 | -0.50 | -0.40 | -0.20 | 0.09 | 0.38 | 0.62 | 0.74 | 0.72 | 0.59 | 0.41 |
| Bank credit (FoF) | -0.20 | -0.05 | 0.12 | 0.28 | 0.37 | 0.40 | 0.35 | 0.27 | 0.18 | 0.10 | 0.06 |
| Bank loans (FoF) | -0.53 | -0.49 | -0.35 | -0.10 | 0.20 | 0.48 | 0.68 | 0.75 | 0.67 | 0.49 | 0.27 |
| Asset prices | | | | | | | | | | | |
| Residential property prices | -0.23 | -0.13 | -0.02 | 0.06 | 0.12 | 0.15 | 0.16 | 0.15 | 0.14 | 0.13 | 0.12 |
| Commercial property prices | -0.37 | -0.40 | -0.36 | -0.23 | -0.02 | 0.21 | 0.42 | 0.55 | 0.56 | 0.48 | 0.33 |
| Share prices, overall | 0.00 | 0.09 | 0.18 | 0.23 | 0.24 | 0.19 | 0.12 | 0.05 | -0.01 | -0.03 | -0.02 |
| Share prices, banks | 0.36 | 0.23 | 0.06 | -0.09 | -0.19 | -0.19 | -0.11 | 0.02 | 0.12 | 0.17 | 0.17 |
| Interest rates | | | | | | | | | | | |
| Long rate | -0.43 | -0.41 | -0.28 | -0.07 | 0.14 | 0.30 | 0.36 | 0.30 | 0.15 | -0.01 | -0.14 |
| Short rate | -0.64 | -0.57 | -0.35 | -0.02 | 0.32 | 0.57 | 0.66 | 0.58 | 0.38 | 0.14 | -0.05 |
| IR spread | 0.47 | 0.39 | 0.21 | -0.04 | -0.28 | -0.47 | -0.55 | -0.51 | -0.38 | -0.22 | -0.08 |
| Prime rate | -0.50 | -0.54 | -0.46 | -0.27 | 0.00 | 0.27 | 0.47 | 0.56 | 0.51 | 0.37 | 0.19 |
| Fed funds rate | -0.62 | -0.66 | -0.55 | -0.31 | 0.03 | 0.38 | 0.63 | 0.71 | 0.63 | 0.42 | 0.18 |
| Loan spread | 0.31 | 0.33 | 0.27 | 0.13 | -0.07 | -0.27 | -0.41 | -0.43 | -0.35 | -0.18 | 0.00 |

* Quarterly data. We extracted cyclical components using the Hodrick-Prescott (HP) filter, the Band-Pass (BP) filter and the first-difference (FD) filter. Results are shown for the BP-filtered cycles.

Table 2 presents cross-correlation coefficient estimates between the BP-filtered cyclical components of real variables and of the real GDP, for the US economy. HP and FD-filtered series provide similar results. Our results are largely in line with previous findings in the real business cycle literature. In particular, real variables appear to be strongly pro-cyclical so that business cycles are indeed "real". The exceptions are government consumption and investment, which are counter-cyclical, as expected, and they lag the output cycle. Real non-durable and durable consumption and residential investment lead the output cycle, while real investment including non-residential investment, exports, imports and employment all lag the output cycle. Interestingly, several "forward-looking" variables usually considered indicators of output cycle actually do not lead output cycle. In fact, new manufacturing orders lag the output cycle while consumption, industrial production and retail sales are contemporary with the output cycle.

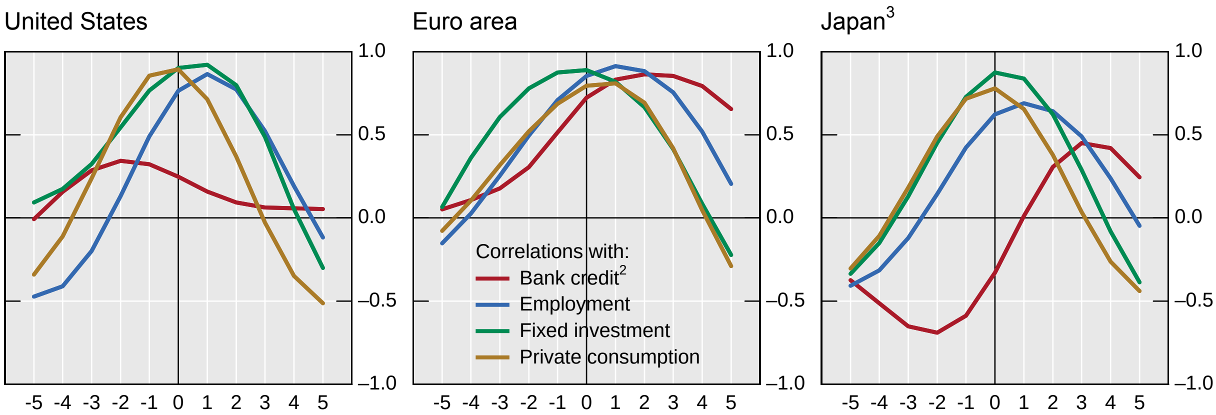
Cross-correlation coefficient estimates between the BP-filtered cyclical components of the financial variables and of the real GDP are provided in Table 3. A rather different picture emerges. Nominal variables are largely acyclical, which essentially suggests essentially very weak or no correlations. M2 appears to lead the output cycle, but the correlation is not strong, to some extent this invalidating the monetarist claim that money has important real effects. Results for the credit variables are mixed. While the credit to the private sector, bank loans, business and consumer loans are apparently pro-cyclical and appear to lag the output cycle, bank credit and mortgage are acyclical. In addition, correlation of credit variables with output turns out to be much weaker than real variables. Therefore the strong bank credit-output nexus suggested by, e.g. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), might not exist.

Moreover, except for commercial property prices which are weakly pro-cyclical and lag the output cycle, other asset prices, namely housing prices, share prices and bank equity prices, are all basically uncorrelated with real GDP. In addition, the correlation patterns are such that it is rather difficult to determine whether key interest rates in the US economy are pro- or counter-cyclical. Moreover, such correlations are weak, particularly for long rates and loan spreads. The results suggest that business cycles are apparently "real", and the monetary or credit explanations indeed seem to be "myths". Both the credit and asset price transmission channels look pretty impotent.

### *III.1.2. Comparing credit-output relationships in G3 economies*

Graph 3

**Cross-correlations of GDP with other variables**[1]




1 Refer to data up to Q4 2010; for Euro area, bank credit refers to data up to Q3 2010; all variables have been logged and de-trended using band-pass filter; x-axis: number of lags (leads) of quarters of the variables with which contemporaneous GDP has been correlated; y-axis: correlation coefficient. 2 In nominal terms. 3 Bank credit defined as non-financial corporations' liabilities plus housing loans plus consumer credit plus liabilities of government from the Japanese Flow of Funds for the period Q4 1997–Q4 2010.

Sources: Bank of Japan; national data; BIS calculations.

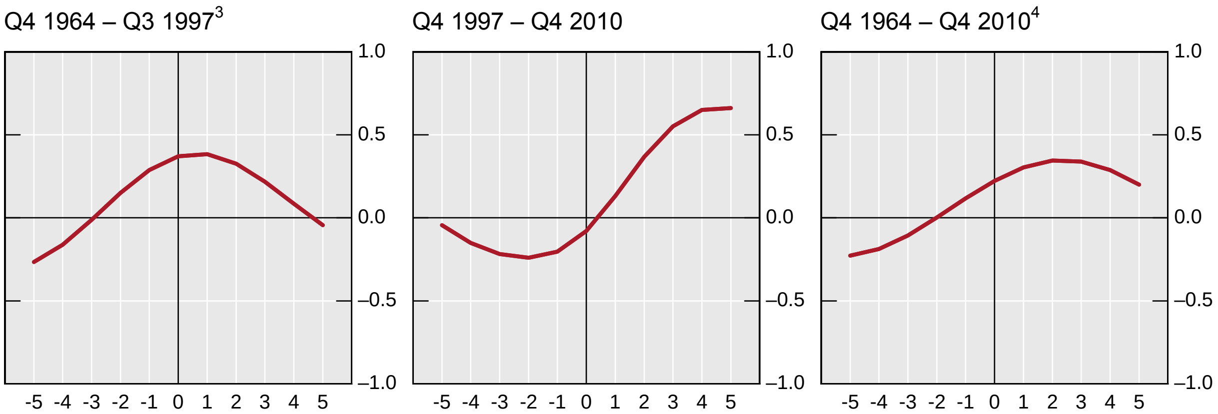
Does this result of rather weak credit-output correlations also hold in the euro area and in Japan? Graph 3 depicts cross-correlation coefficient estimates for G3 economies, based on BP-filtered cyclical components. First, real variables tend to be strongly pro-cyclical in G3 economies, with employment typically lagging output and consumption and investment being coincident with it. Second, the bank credit-output relationship is substantially different in G3 economies: the US credit-output correlations are weak with bank credit leading output by about two quarters. In euro area, bank credit appears to be strongly pro-cyclical but lags the output cycle by two quarters. In Japan, output is positively correlated with lagged credit but positively with led credit, and the strength of correlation varies depending on the filter used.

We also examine cross-correlations between output and CPI inflation-deflated bank credit in G3 economies, based on BP, HP and FD-filtered cyclical components. Qualitatively, the results remain largely unchanged from those for nominal bank credit.

In addition, cross-correlation estimates could be sensitive to varying sample periods. Japan's case is illustrative. Estimates presented in panel 3 of Graph 3 are based on bank credit data which at the quarterly frequency are only available for Q4 1997 – Q3 2009. Graph 4 provides estimates based on credit defined as loans by private financial institutions, a longer series. We divide the sample into two, the first ranging from Q4 1964 to Q3 1997, the second from Q4 1997 to Q3 2009. The date Q4 1997 is chosen because, first, it marked the onset of a major banking crisis in Japan, and second, estimates can be compared to those in Graph 3.

Graph 4

**Cross-correlations of Japanese GDP with nominal credit**[1, 2]




1 All variables have been logged and de-trended using band-pass filter; x-axis: number of lags (leads) of quarters of the variables with which contemporaneous GDP has been correlated; y-axis: correlation coefficient. 2 Credit defined as loans by private financial Institutions. 3 Discontinued series. 4 Credit data backdated using the discontinued series.

Sources: Bank of Japan; BIS calculations.

Notice first that the results based on two different credit series for the post-Q4 1997 period are qualitatively similar (Graph 3 right panel, and Graph 4 centre panel). Second, estimates based on the Q4 1964 - Q3 1997 sub-sample and the full-sample are similar but differ from the post-Q4 1997 sample estimates. Credit is weakly pro-cyclical and lags the output cycle. Credit pro-cyclicality is weaker for the full sample, for which the correlation estimate may be "contaminated" by the post-Q4 1997 period, when the credit-output relationship was severely disrupted. Several factors could have contributed to this: zombie lending and chronic non-performing loan problems for banks; slow growth and deflation; severely impaired monetary transmission as Japan faced the zero lower bound on nominal interest rates. Eventually, a "quantitative easing" policy was put in place between March 2001 and March 2006.

### III.2. Output and credit cycles: causality

Many economists believe that sound credit flows are essential to solid and sustained output growth. This belief relies on a causal relationship on which our estimates of credit-output correlations shed little light. A formal econometric test is in order. To carry out the Granger

(1969) non-causality test, first one needs to be clear about the exact meaning or definition of an economic or credit "cycle".

The definition of business cycles has changed over time. In the early years, cycles were classified according to duration or periodicity, e.g. Juglar's (1862) fixed investment cycle of 7 to 11 years; Kitchin's (1923) inventory cycle of 3 to 5 years; Kuznets' (1930) infra-structural investment cycle of 15 to 25 years, and the Kondratiev (1935) wave or technological cycle of 45 to 60 years. Schumpeter (1954) argues that a Juglar cycle has four stages: expansion, crisis, recession and recovery. Mitchell (1913, 1927) and Burns and Mitchell (1946) defines business cycles as "a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises", with a reference cycle consisting of expansion, recession, contraction, and revivals. The duration of a business cycle varies from over one year to ten or twelve years and are not divisible into shorter cycles of similar characteristics with amplitudes approximating their own. Their work, however, was criticised by Koopmans (1947) as measurement without theory.

Modern business cycle theory arrived with Lucas' (1977, 1987) proposal of "equilibrium cycles" as deviations from trend, emphasizing the importance of uncovering regularities in the comovements of such cyclical components. The idea has led to the Real and the New Keynesian business cycle theories. Kydland and Prescott (1981, 1990, 1997) use cycles uncovered with a Hodrick-Prescott filter (1981, 1997) to study such comovements.

**Table 4: Granger non-causality tests***

| **Variable *X*** | **Variable *Y*** | ***X→Y*** | ***Y→X*** |
|---|---|---|---|
| Real GDP | Bank credit | No | Yes |
| | Credit to private sector | Yes | Yes |
| Employment | Bank credit | Yes** | Yes |
| | Credit to private sector | Yes | Yes |
| Industrial production | Business loans | Yes | Yes |
| | Bank loans (FoF) | Yes | Yes |
| New orders, manufacturing | Business loans | Yes | Yes |
| Bankruptcy filings | Business loans | Yes | Yes** |
| Real private fixed investment | Bank loans (FoF) | No | No** |
| Structures | Real estate loans | No | No |
| Non-residential | Commercial real estate loans | No | No |
| Residential | Mortgage loans | Yes** | Yes** |
| Real private consumption | Consumer loans | Yes | No |
| Durable goods | Consumer loans | No | No |
| Non-durable goods | Consumer loans | No | No |
| Retail sales | Consumer loans | No | No |

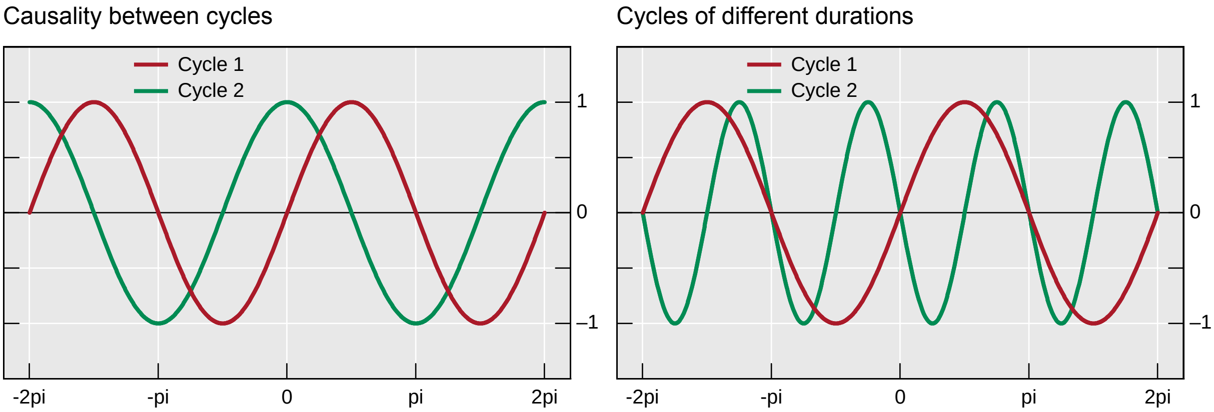
* All variables are in terms of year-on-year growth rates. ** Weak evidence.

Our causality tests are applied to the growth rates of a range of US real and credit variables, the results are mixed (Table 4). First, there is evidence that bank credit growth precedes real GDP growth, but not the other way round. In addition, growth in US employment, industrial production, new manufacturing orders seem to precede and be preceded by credit or loan growth. Second, growth in real investment and in its main components seems to have no causal relationship with growth of loan in the corresponding categories. Similarly, growth in real consumption seems to drive consumer loan growth, but the latter has no causal relationship with growth in retail sales, and durables and non-durables consumption.

Indeed, a careful analysis suggests that testing for causality in the cyclical components of credit and real variables might well be a mathematical impossibility. This is because the Granger-type causality tests are based on the idea that causes should precede effects on the time scale. Graph 5 is illustrative: the left panel shows the "correct" time sequence cannot be pinned down by such tests for two infinite cycles of equal magnitude and duration, but of different location in time. The right panel shows that such tests become even less useful once we have cycles of different duration. In practice, credit and economic cycles are far more complicated and it may be impossible to determine the correct causal relationship solely on the basis of econometric tests.

Graph 5

**Causality between cycles**




## IV. Credit and economic cycles in the frequency domain

Is there a stable credit-output relationship which holds true in the short, medium and long run? That is, is the process generating interactions between credit and real activity consistent across the frequency spectrum? Empirically, the question is whether there is one single economic model which fits data well at all frequencies, or there are distinct data generation processes in different frequency bands. Engle (1974) stresses the importance of the issue by pointing out that "there is little discussion of whether the same model applies to all frequencies. It may be too much to ask of a model that it explain both slow and rapid shifts in the variables, or both seasonal and non-seasonal behavior. It is at least reasonable to test the hypothesis that the same model applies at various frequencies."

The issue is of particular importance to the credit-output relationship: does it vary significantly in low, business-cycle and high frequencies? Large frequency-wise variations can invalidate pure time domain analysis, which effectively aggregates or averages a relationship across the whole frequency range by implicitly assuming a frequency-invariant relationship. Indeed there is no reason why the credit-output relationship should stay unchanged in the short, medium and long run. Significant cross-frequency differences in the credit-output relationship could have serious implications for both the theoretical underpinning and empirical analysis of the relationship.

Frequency-domain analysis of time series was well understood before the introduction of the Box-Jenkins approach. It is appealing as covariance stationary processes can be uniquely decomposed into mutually uncorrelated components, each associated with a specific frequency (band). It is a time-honoured practice to study low-frequency characteristics of time series to understand certain long-run economic behaviour. While high-frequency seasonal or irregular components are of particular importance in finance, medium-frequency components are of great interest to business cycle analysts.

There are two main approaches to frequency-domain time series analysis. The first relies on carefully designed filters to decompose data into trend and cycle components corresponding to different frequency bands. Filters frequently used in economic analysis include the linear detrending filter, the ARIMA filter, the first difference filter, the Lucas (1980) exponential smoothing filter, the Hodrick-Prescott (1980, 1997) or Whittaker-Henderson filter, and the Baxter-King (1999) band-pass filter. Using low and band-pass filters, King and Watson (1994) uncover a pronounced and stable negative inflation-unemployment correlation at the business-cycle frequencies, but very unstable comovements at lower frequencies. Due to finite data length, these filters are only approximations to the ideal filters, therefore filter leakage, compression and exacerbation are inevitable. [5] Furthermore, averaging (e.g. correlation and regression) within each frequency band could mask possible large variations within any pre-specified band.

The other approach conducts time series analysis solely in the frequency domain. Early work in economics relies on simple statistics, e.g. gain, phase, spectral and cospectral densities, and coherence. In a seminar paper, Granger (1966) uncovers the "typical spectral shape" of economic variables. Other applications are Granger and Morgerstern (1963), Granger and Rees (1968) and Granger (1969), and more recently Pakko (2000), Estrella (2003), Cogley and Sargent (2001, 2004) and Cogley and Sbordone (2004). Spectral regression analysis was pioneered by Hannan (1963) and introduced to economics by Engle (1974, 1978, 1980) and Harvey (1978). Spectral regression allows one to focus on specific frequency bands, and permit a nonparametric treatment of regression errors. Phillips (1991) applies it to integrated time series to obtain asymptotically median unbiased estimates of cointegrating coefficients. Summers (1983) uses the technique to examine whether the Fisher equation is a proposition valid only at low frequencies. Other macroeconomic applications include Thoma (1994) and Tan and Ashley (1997, 1998, 1999).

In this paper, we take a more direct approach to examine the behaviour of the credit-output relationship in the entire spectrum. First, we examine a number of simple spectral indicators: spectral and cross-spectral densities, coherency, gain and phase-to-frequency ratio. Then, we estimate Zhu's (2005) frequency-specific coefficients of correlation (FSCC) and regression (FSCR) for the relationship.[6] The coefficients are obtained using a simple data extraction procedure based on Fourier and inverse Fourier transforms. The procedure is linear so the conventional inferential theory applies. The FSCC is superior to traditional indicators, such as coherence and cospectrum, by providing a real-valued, normalized and signed measure of the strength of multiple correlations. Unlike coherence, the FSCC makes it transparent the sign of frequency-wise correlations between any two series. Compared to cospectrum, the FSCC is standardized and takes values only in the [-1,1] range, therefore delivering a clear indication of the strength of correlation which is independent of the scale of data. When the distribution theory for cospectral density and coherence is complicated, $p$-values and confidence intervals for FSCC estimates are straightforward to obtain. The FSCR has the advantage of targeting any specific frequency and the conventional OLS inference remains valid.

Graphs 6 and 7 present the spectral estimates for the US credit-output relationship between Q1 1951 and Q1 2009. The sample period includes all post-War US recessions, times when the credit-output relationship is apparently strongest. The upper panels of Graph 6 contain estimates of spectral densities for real GDP, credit to the private sector and bank credit, and the cross-spectral densities between output and credit, presented in natural logarithm. Spectral densities show contributions of a specific frequency (band) to the total variance of a stochastic process. Our estimates indicate that low-frequency components are of great importance, indeed trends matter; also all variables have the "typical spectral shape" suggested by Granger (1966). Cross-spectral density or cospectrum represents covariance

[5] See Baxter and King (1999) and Zhu (2005) for details.

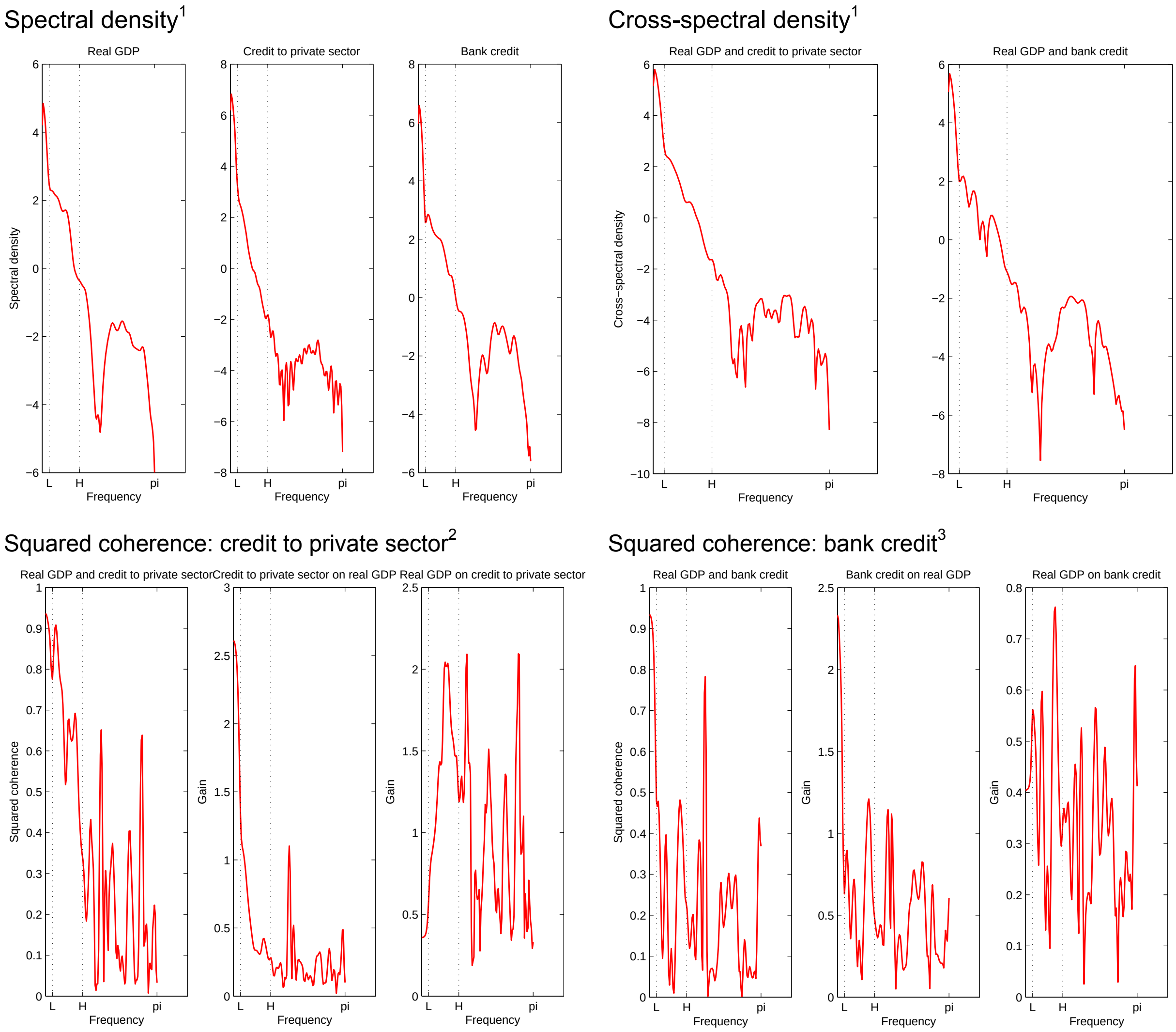
[6] Details on the estimation procedures are provided in the Annex.

between in-phase components of two time series at a given frequency (band). Our cross-spectral density estimates indicate that much of the strength of the credit-output correlation stays in the very low frequency (beyond 32 quarters); it declines sharply and becomes rather weak in the business cycle frequency (8 to 32 quarters); the correlation is even weaker, and more volatile in higher frequencies.

The lower two panels of Graph 6 presents coherency (i.e. squared coherence) and gain estimates for US credit and real output. Coherency varies in [0,1], it is the frequency-domain analogue to the correlation of determination ( $R^2$ ). And gain or transfer is analogous to a standardised regression coefficient at a given frequency (band). Again the US credit-output relation seems to be stronger in the lower frequencies, and it tends to fluctuate strongly across frequencies, often with large swings. This suggests that a time-domain regression or even a band spectrum regression could mask significant differences in the credit-output relation across frequencies.

Graph 6

**Spectral analysis of US credit-output relationship**




[1] In natural logarithm. [2] Growth in domestic credit to the private sector. [3] Growth in domestic bank credit.

Sources: Author's own calculations.

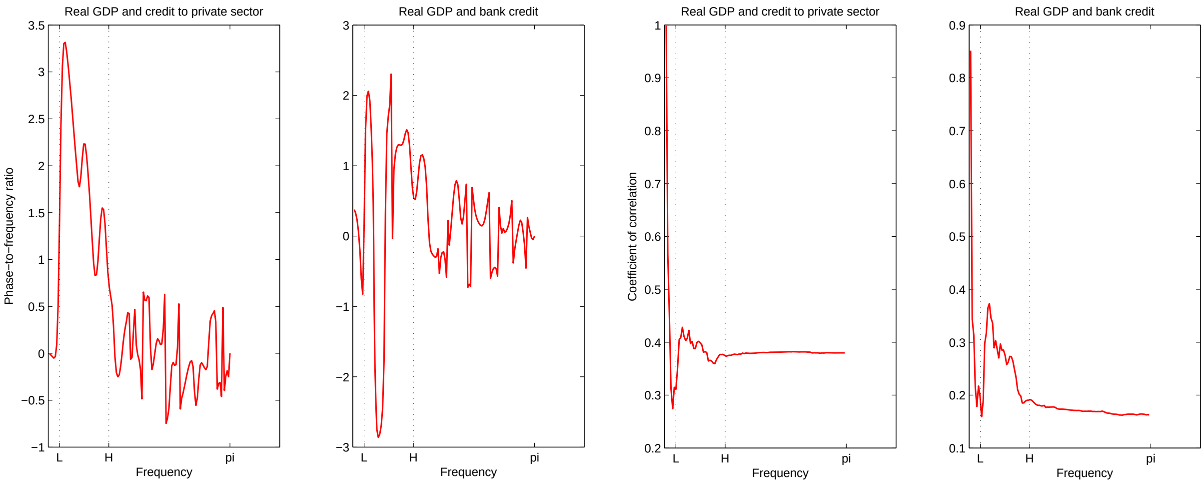
The upper left panel of Graph 7 contains estimates of the phase-to-frequency ratios between real GDP and US credit. The phase measures the timing or average phase lead of one series over another at different frequencies, and it incorporates all relevant information about leads and lags. Our estimates suggest that the lead-and-lag relationship between credit and output varies significantly across frequencies, with large swings above and below zero most obvious in the business-cycle and higher frequencies.

Graph 7

**Spectral analysis of US credit-output relationship**

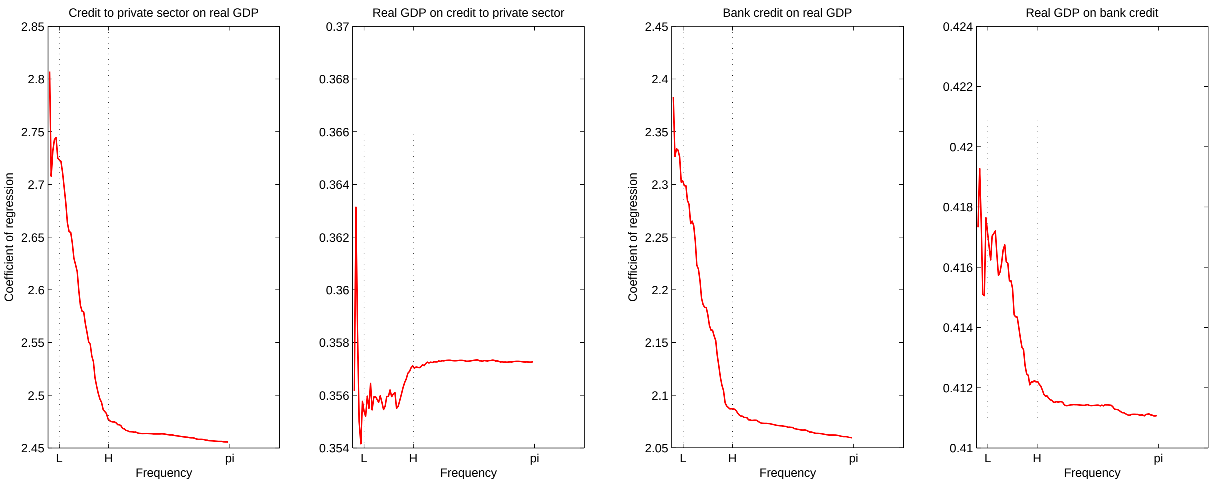
Phase-to-frequency ratio[1]

Coefficient of correlation[1]




Coefficient of regression credit to private sector[2]

Coefficient of regression: bank credit[3]




[1] In natural logarithm. [2] Growth in domestic credit to the private sector. [3] Growth in domestic bank credit.

Sources: Author's own calculations.

The upper right panel and the lower panels of Graph 7 present estimates of Zhu's (2005) frequency-specific coefficients of correlation (FSCC) and regression (FSCR). Clearly, the US credit-output correlation is very strong and concentrates mostly in the very low frequencies, i.e. long run. The business-cycle-frequency correlation is rather weak and often lacks a clear pattern. Notably, averaging the correlation estimates across the business-cycle frequencies yield numbers which are not too different from the cross-correlation estimates seen in Table 3. Also confirmed is the fact that US credit to private sector has a more significant correlation with output than bank credit. Frequency-specific regression coefficient estimates provided in the lower panels reveal a similar story. The coefficient estimates are large in the low frequencies, decline rapidly when moving into business-cycle frequencies, becoming smaller and more stable in the higher frequencies (less than two years). Frequency-domain analysis confirms that the US credit-output relationship is indeed valid in the long run, but quite weak over business cycles.

## V. How to better predict credit developments?

Given the rather weak US credit-output correlation, one natural question is whether output growth would remain a good indicator of credit demand and hence a reliable predictor of future credit growth. Or would there be variables other than output which could better predict credit growth? Although we have not found conclusive evidence to support a strong role of credit in business cycles, evolution of asset prices and credit could still be good indicators of potential financial instability. Market participants and policymakers alike remain focused on credit developments particularly in cyclical downturns.

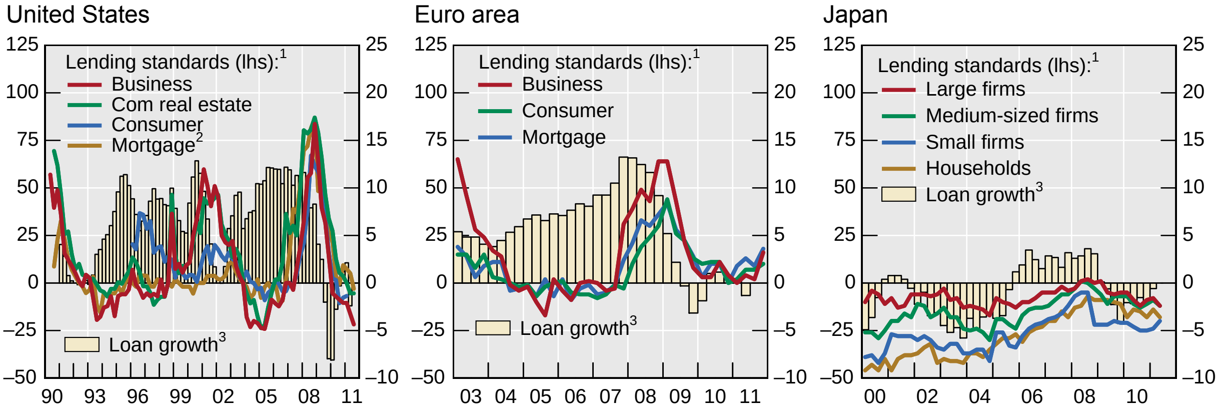
We examine two possibilities. First, bank lending standards provided in senior loan officer surveys appear to be a good proxy for credit supply, and its usefulness has been suggested by Lown, Morgan and Rohatgi (2006) and Lown and Morgan (2006). Second, the literature on the transmission of monetary policy and business cycle emphasises the role of balance sheets of borrowers, commercial banks, and investment banks and dealers. We examine the predictive value of credit standards and commercial banks' balance sheets in this section.

### V.I. Credit developments and bank lending standards

One feature common to the past economic downturns in G3 economies is a sharp tightening of credit standards by banks (Graph 8). In the United States, lending standards in all sectors tightened in the early 1990s and in the early and late 2000s. Euro area lending standards also tightened sharply following the onset of the recent crisis in 2007. Tightenings seem to occur after a period of sustained growth in bank loans, and precede a sharp slowdown in loan growth or a contraction.

Graph 8

**Credit standards and bank loan growth**




[1] Net percentage of banks reporting tightening standards. [2] From Q2 2007 refers to the simple average of prime, subprime and non-traditional mortgage loans. [3] Annual changes, in per cent (right-hand scale).

Source: National data.

Past work shows that bank lending standards could be a good proxy for credit supply and often lead credit growth. Lown and Morgan (2006) suggest that changes in credit standards have a significant impact on bank loan growth and output growth. Credit standards provide information about banks' lending intensions and could help predict future credit growth.

Yet it is difficult to assess the ultimate impact and predictive power of lending standards, for several reasons. First, there may be considerable lags between banks' lending intentions and actual decisions. Second, during a cyclical slowdown credit demand and supply tend to fall in tandem, it is difficult to disentangle their impact and measure the relative importance. Third, even when supply factors dominate credit flows, it is difficult to ascertain whether banks are reacting to deteriorating own balance sheets, or to a weakening of borrowers' balance sheets and creditworthiness. Fourth, in a cyclical downturn, firms may initially raise loan demand to meet cash flow problems which may be considered temporary, and eventually cut borrowing as demand weakens.

Following Lown, Morgan and Rohatgi's (2000) approach, we regress US credit growth on: bank lending standards; real GDP growth, a proxy for credit demand; and loan spread (bank prime minus effective federal funds middle rate). The maximum number of lags is set to 8 for each variable. Based on US data from Q2 1990 to Q3 2009, the following "optimal" model is selected using the Akaike information, Schwarz information and final prediction error criteria:

$$y_t = \alpha + \sum_{i=1}^{4} \beta_i y_{t-i} + \delta_1 LEND_{t-1} + \sum_{k=0}^{1} \phi_k g_{t-k} + \gamma LS_t + \varepsilon_t$$

where *y* is US credit growth; *LEND* is the lending standards for commercial and industrial (C&I) loans, expressed as the net percentage of banks reporting tightening standards; *g* is real GDP growth; and *LS* is a measure of the loan spread. Table 5 contains the main results.

**Table 5: Credit growth and lending standards 1990Q2 – 2009Q1***

| Variable *X* | Independent variable $\Delta Y_t$ (credit growth) | | | |
|---|---|---|---|---|
| | **Credit to private sector** | **Bank credit** | **Bank loans** | **Business loans** |
| ***Constant*** | 0.76 | 1.16 | −0.46 | 0.88 |
| $\sum \Delta Y_{t-i}$ | 0.94 (0.00) | 0.77 (0.00) | 0.82 (0.00) | 0.93 (0.00) |
| $Standards_{t-1}$ | −0.12 (0.06) | −0.09 (0.01) | −0.15 (0.00) | −0.66 (0.00) |
| $\Delta$ $Real\ GDP_{t-1}$ | −0.14 (0.89) | 0.43 (0.39) | 1.35 (0.06) | 2.18 (0.34) |
| $Loan\ spread_{t-1}$ | −1.82 (0.37) | 0.54 (0.64) | 2.55 (0.14) | −3.16 (0.55) |
| | | | | |
| **Adjusted $R^2$** | 0.97 | 0.72 | 0.86 | 0.96 |
| ***DW* statistic** | 1.70 | 1.93 | 1.73 | 1.92 |

* All credit variables are in terms of year-on-year growth rates. Reported values are long-run regression coefficient estimates (with *p*-values included in parentheses). We use the Akaike Information Criterion (AIC), Bayesian information criterion (BIC) and Final Prediction Error (FPE) to choose the optimal lag combination. For variables "Credit to private sector" and "Business loans", the optimal number of own lags were four and two, respectively.

Our results indicate that, first, credit growth is persistent; second, both lending standards and output growth have the right signs, but while lending standards are highly significant, coefficient estimates for real GDP growth and loan spread are statistically insignificant. Lending standards contain valuable information, and if they faithfully reflect credit supply conditions, then credit supply could be the driving force of credit growth; third, our focus on the optimal combination of lag lengths in the estimated regression model reveals interesting dynamics: credit growth responds quickly to developments in lending standards, precisely in just one quarter.

### V.II. Credit developments and commercial bank balance sheets

Bank lending standards provide useful input as proxy for credit supply, but could there be other supply-side factors which could serve as good or even better proxies? In this section, we consider private sector balance sheets. While the previous literature on bank lending channel e.g. Bernanke and Gertler (1989, 1990), Bernanke, Gertler and Gilchrist (1996, 1999) and Kiyotaki and Moore (1997) emphasised borrowers' balance sheets, a recent literature, e.g. Kiyotaki and Gertler (1997) examine the role of lenders' balance sheets. Instead of looking at the balance sheets of investment banks and dealers, we focus on the balance sheets of traditional commercial banks.

The estimation results are provided in Table 6. Applying the same estimation strategy, our selection criteria yield an "optimal" model which has a very similar lag structure. Again credit growth responds quickly to commercial bank leverage, defined as the assets-to-equity ratio, and to real GDP growth. But commercial bank leverage turns out to be less robust than lending standards, as it is statistically insignificant in regressions with bank loans and credit to the private sector despite having the expected sign. Credit growth is persistent and loan spread remains statistically insignificant. Real activity is now a main driver of credit growth, significant both economic and statistically. Commercial bank leverage is therefore a less useful predictor for credit growth compared to bank lending standards.

**Table 6: Credit growth and commercial bank leverage 1988Q1 – 2008Q4***

| Variable $X$ | Independent variable $\Delta Y_t$ (credit growth) | | | |
|---|---|---|---|---|
| | **Credit to private sector** | **Bank credit** | **Bank loans** | **Business loans** |
| ***Constant*** | 1.11 | 3.68 | 1.29 | 3.38 |
| $\sum \Delta Y_{t-1}$ | 0.91 (0.00) | 0.71 (0.00) | 0.79 (0.00) | 0.90 (0.00) |
| $CBLEV_{t-1}$ | −0.005 (0.33) | −0.01 (0.04) | −0.01 (0.18) | −0.024 (0.06) |
| $\Delta$ ***Real Activity***$_{t-1}$ | 0.42 (0.53) | 0.84 (0.01) | 1.81 (0.00) | 2.11 (0.01) |
| ***Loan spread***$_{t-1}$ | −0.64 (0.64) | −0.08 (0.93) | 1.25 (0.37) | −2.05 (0.58) |
| | | | | |
| **Adjusted $R^2$** | 0.96 | 0.70 | 0.85 | 0.96 |
| ***DW* statistic** | 1.69 | 1.77 | 2.06 | 1.97 |

* All credit variables are in terms of year-on-year growth rates. Reported values are long-run regression coefficient estimates (with *p*-values included in parentheses). We use the Akaike Information Criterion (AIC), Bayesian information criterion (BIC) and Final Prediction Error (FPE) to determine the optimal lag combination. The commercial bank leverage is defined as the assets-to-equity ratio.

## VI. Conclusion

Many economists hold the view that credit market developments are intimately associated with sustained economic growth, both in the short and longer run. Market participants and policymakers alike often view in credit and business cycles a strong cause-effect relationship. In this paper, we use a range of simple time and frequency-domain techniques to examine whether such views are validated by data, with a special focus on the US economy.

We first calculate estimates of cross-correlations between the output cycle and the cyclical components of a large number of real and nominal variables. The results suggest that indeed US business cycles are "real", as advocated by the real business cycle literature. Beyond their suggestion of a "monetary myth", we also find that credit, asset prices various interest rates and spreads tend to be weakly correlated with output, indeed there is also a "credit" or "financial myth". However, the cyclical credit-output relationship varies across countries and over time: it is strong in the euro area but relatively weak in Japan; and it changed significantly in Japan following the 1997-1998 banking crisis.

Our estimates of cross-correlations between output and credit cycles are robust to the different filters used, and to the use of real or nominal credit variables. Credit appeared to have led output in euro area, but not so in the United States. In the US case, the cyclical components of output and credit were weakly correlated, and credit lagged output. In Japan, the output-credit relationship was complicated, with a distinct pattern for the sample period starting from Q4 1997. However, these results should be interpreted with caution. In particular, our results do not provide strong evidence on the existence of a “causal” relationship between output and credit. Furthermore, bank lending standards could be a useful indicator for credit growth and should be included in central bankers’ information set.

We further conduct Granger non-causality tests for growth in credit and output components of the US economy. Our results are mixed, and there is no evidence of loan growth preceding real consumption or investment growth, or the other way round.

To verify the results obtained in the time-domain analysis, we examine the US credit-output relationship in the frequency-domain, and over the entire spectrum. We estimate a number of spectral indicators including spectral and cross-spectral densities, coherency, gain, phase-to-frequency ratio, and Zhu’s (2005) frequency-specific correlation and regression coefficients. Our frequency-domain analysis confirms that the US credit-output relationship is indeed valid in the long run, but quite weak over business cycles. This could have important implications for theories which rely heavily on the role of credit in business cycles.

## References:


Abel, Andrew B., Ben S. Bernanke and Dean Croushore (2007): *Macroeconomics*, 6th Edition, Prentice Hall.

Baxter, Marianne and King, Robert G. (1999), "Measuring business cycles: Approximate band-pass filters for economic time series" Review of Economics and Statistics, Vol. 81:4, pp. 575-593.

Bernanke, Ben and Gertler, Mark (1989): “Agency Costs, Net Worth, and Business Fluctuations,” *American Economic Review*, Vol. 79(1), pp. 14-31.

Bernanke, Ben and Gertler, Mark (1990): “Financial Fragility and Economic Performance,” *Quarterly Journal of Economics*, Vol. 105(1), pp. 87-114.

Bernanke, Ben S., Mark Gertler and Simon Gilchrist (1996): “The Financial Accelerator and the Flight to Quality,” *Review of Economics and Statistics*, Vol. 78(1), pp. 1-15.

Bernanke, Ben S., Mark Gertler and Simon Gilchrist (1999): “The financial accelerator in a quantitative business cycle framework,” Chapter 21, *Handbook of Macroeconomics*, edited by J. B. Taylor and M. Woodford, Vol. 1, pp. 1341-1393, Elsevier.

Blanchard, Olivier J. and Mark W. Watson (1986): “Are Business Cycles All Alike?,” *The American Business Cycle: Continuity and Change*, edited by Robert J. Gordon, National Bureau of Economic Research. Chicago: University of Chicago Press, Chapter 2, pp. 123-179.

Burns, Arthur F. and Wesley C. Mitchell (1946): *Measuring business cycles*, New York: National Bureau of Economic Research.

Burns, Arthur F. (1951): “Introduction,” in *What happens during business cycles: A progress report*, edited by Wesley C. Mitchell, New York: National Bureau of Economic Research.

Cogley, Timothy and Sargent, Thomas J. (2001), "Evolving Post-War II U.S. Inflation Dynamics," NBER Macroeconomics Annual, Vol. 16, pp. 331-373.

Cogley, Timothy and Sargent, Thomas J. (2004), "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US," Manuscript, Department of Economics, University of California, Davis.

Cogley, Timothy and Sbordone (2004), "A Search for a Structural Phillips Curve," Manuscript, Department of Economics, University of California, Davis.

Comin, Diego and Mark Gertler (2006): “Medium-Term Business Cycles,” *American Economic Review*, vol. 96:3, pages 523-551.

Corbae, D, Ouliaris, S., and Phillips, P. C. B. (2002), "Band Spectral Regression with Trending Data," *Econometrica*, Vol. 60:3, pp. 413-435.

Engle, R. F. (1974), "Band Spectrum Regression," International Economic Review, Vol. 15:1, pp. 1-11.

Engle, R. F. (1978), "Testing Price Equations for Stability across Spectral Frequency Bands," Econometrica, Vol. 46, pp. 869-881.

Engle, R. F. (1980), "Exact Maximum Likelihood Methods for Dynamic Regressions and Band Spectrum Regressions," International Economic Review, Vol. 21:2, pp. 391-407.

Estrella, Arturo (2003), "Productivity, Monetary Policy, and Financial Indicators," Manuscript, Federal Reserve Bank of New York.

Geweke, J. (1982), "Measure of Linear Dependence and Feedback Between Multiple Time Series," Journal of the American Statistical Association, Vol. 77, pp. 304-313.

Granger, C. W. J. (1966), "The Typical Spectral Shape of An Economic Variable," Econometrica, Vol. 34:1, pp. 150-161.

Granger, C. W. J. (1969), "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," Econometrica, Vol. 37:3, pp. 424-438.

Granger, C. W. J. and Rees, H. J. B. (1968), "Spectral Analysis of the Term Structure of Interest Rates," Review of Economic Studies, Vol. 35:1, pp. 67-76.

Hodrick, Robert J. and Edward C. Prescott (1981): “Postwar U.S. Business Cycles: An Empirical Investigation,” *Discussion Papers* 451*,* Northwestern University.

Hodrick, Robert J. and Edward C. Prescott (1997): “Postwar U.S. Business Cycles: An Empirical Investigation,” *Journal of Money, Credit and Banking*, Vol. 29:1, pages 1-16.

Fisher, Irving (1933): “The Debt-Deflation Theory of Great Depressions,” *Econometrica*, Vol. 1:4, pages 337-357.

Gertler, Mark (1988): “Financial Structure and Aggregate Economic Activity: An Overview,” *Journal of Money, Credit and Banking*, Vol. 20:3, pages 559-588.

Gertler, Mark and Nobuhiro Kiyotaki (2010): “Financial Intermediation and Credit Policy in Business Cycle Analysis”, Chapter 11, *Handbook of Monetary Economics*, Volume 3, pages 547-599.

Juglar, Clement (1862): *Des Crises commerciales et leur retour periodique en France, en Angleterre, et aux Etats-Unis*. Paris: Guillaumin.

Kitchin, Joseph (1923): “Cycles and Trends in Economic Factors,” *Review of Economics and Statistics*, Vol. 5:1, pages 10–16.

Kiyotaki, Nobuhiro (1998): “Credit and business cycles,” *Japanese Economic Review*, Vol. 49, pages 18–35.

Kiyotaki, Nobuhiro and John Moore (1997): “Credit Cycles,” *Journal of Political Economy*, Vol. 105:2, pages 211-48.

Kocherlakota, Narayana R. (2000): “Creating Business Cycles Through Credit Constraints,” Federal Reserve Bank of Minneapolis *Quarterly Review*, Vol. 24:3, pp. 2–10.

Kondratieff, N. D. and W. F. Stolper (1935): “The Long Waves in Economic Life,” *Review of Economics and Statistics*, Vol. 17:6, pages 105–115.

Koopman, Siem Jan, Roman Kräussl, André Lucas and André B. Monteiro (2009): “Credit cycles and macro fundamentals,” *Journal of Empirical Finance*, Vol. 16:1, pages 42-54.

Kuznets, Simon (1930): *Secular Movements in Production and Prices. Their Nature and their Bearing upon Cyclical Fluctuations*. Boston: Houghton Mifflin.

Kydland, Finn E. and Edward C. Prescott (1982). "Time to Build and Aggregate Fluctuations". Econometrica, vol. 50:6, pages 1345–1370.

Kydland, Finn E. and Edward C. Prescott (1990): “Business Cycles: Real Facts and a Monetary Myth,” Federal Reserve Bank of Minneapolis *Quarterly Review*, vol. 14, pp. 3-18.

Lown, Cara and Donald P. Morgan (2006): “The Credit Cycle and the Business Cycle: New Findings Using the Loan Officer Opinion Survey,” *Journal of Money, Credit and Banking*, vol. 38:6, pp. 1575-1597.

Lown, Cara and Donald P. Morgan and Sonali Rohatgi (2000): “Listening to Loan Officers: The Impact of Commercial Credit Standards on Lending and Output,” Federal Reserve Bank of New York *Economic Policy Review*, vol. 6, pp. 1-16.

Lucas, R. E., Jr. (1973), "Some International Evidence on Output-Inflation Tradeoffs," American Economic Review, Vol. 63, pp. 326-334.

Lucas, R. E., Jr. (1980), "Two Illustrations of the Quantity Theory of Money," American Economic Review, Vol. 70:5, pp. 1005-1014.

McCallum, B. T. (1984), "On Low-Frequency Estimates of Long-Run Relationships in Macroeconomics," Journal of Monetary Economics, Vol. 14, pp. 3-14.

Pakko, M. R. (2000), "The Cyclical Relationship between Output and Prices: An Analysis in the Frequency Domain," Journal of Money, Credit and Banking, Vol. 32:3, pp. 382-399.

Phillips, P. C. B. (1990), "Spectral Regression for Cointegrated Time Series," in W. A. Barnett, J. Powell and G. E. Tauchen, eds., Nonparametric and Parametric Methods in Econometrics and Statistics, Cambridge: Cambridge University Press, pp. 413-435. .

Schumpeter, J. A. (1954): *History of Economic Analysis*, London: George Allen & Unwin.

Sims, Christopher A. (1972), "Money, Income and Causality," American Economic Review, Vol. 62:4, pp. 540-552.

Tan, Hui Boon and Ashley, Richard (1997), "An Elementary Method for Detecting and Modeling Regression Parameter Variation across Frequencies with an Application to Testing the Permanent Income Hypothesis," *Working Paper* No. 97-02, Virginia Polytechnic Institute and State University.

Tan, Hui Boon and Ashley, Richard (1998), "On the Inherent Nonlinearity of Frequency Dependent Time Series Relationships," *Working Paper* No. 97-07, Virginia Polytechnic Institute and State University.

Tan, Hui Boon and Ashley, Richard (1999), "Detection and Modeling of Regression Parameter Variation across Frequencies," Macroeconomic Dynamics, Vol. 3, 69-83.

Thirlwall, A. P. (2011): *Economics of Development: Theory and Evidence*, 9th edition, Palgrave-Macmillan.

Thoma, Mark (1994), "The Effects of Money Growth on Inflation and Interest Rates across Frequency Bands," Journal of Money, Credit and Banking, Vol. 26:2, pp. 218-231.

Todaro, Michael P. and Stephen C. Smith (2009): “Finance and Fiscal Policy for Development,” Chapter 15, *Economic Development*, 10th Edition, Prentice Hall.

Zhu, Feng (2005): “The fragility of the Phillips curve: A bumpy ride in the frequency domain,” *BIS Working Papers* 183, Bank for International Settlements.

## Annex: Frequency domain analysis

In this annex, we illustrate the method we use to estimate frequency-wise correlation and regression estimates, which is based on a simple frequency-specific data extraction procedure.

### A.1. A frequency-specific data extraction procedure

Consider a time series vector $x = [x_1, x_2, ..., x_T]^T$ . For $s = 1,...,T$ , define the fundamental frequencies as $\omega_s = 2\pi s/T$ . The discrete Fourier transform of $x$ at frequency $\omega_s$ is

$$w_s x = T^{-1/2} \sum_{t=1}^{T} x_t e^{(t-1)i\omega_s}$$

where

$$w_s = T^{-1/2} \begin{bmatrix} 1 & e^{i\omega_s} & \cdots & e^{(T-1)i\omega_s} \end{bmatrix}$$

Define

$$W = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{T-1} \end{bmatrix}$$

$W$ is a unitary matrix such that $W^*W = WW^* = I$ , where $*$ indicates the Hermitian conjugate (i.e., transpose of the complex conjugate). Then $\tilde{x} = Wx$ is the vector of discrete Fourier transform of time series $x$ at all fundamental frequencies $\omega_s$ , $s = 1,...,T-1$ .

Define $A_s$ as a $T \times T$ selection matrix which selects the $s$ -th element or row from any data vector or matrix, respectively. It has 1 as the $s,s$ -th element and zeros elsewhere.[7] The data vector of the discrete Fourier transform of time series $x$ at the $s$ -th frequency $\omega_s$ is

$$A_s\tilde{x} = A_s Wx$$

So there are $T$ data vectors $A_s\tilde{x}$ , extracted from the original time series $x$ , each of length $T$ . All but the $s$ -th elements of the $s$ -th data vector $A_s\tilde{x}$ are zero. We then use inverse Fourier transform to convert the complex data vector $A_s\tilde{x}$ back into the time domain. Write the frequency- $\omega_s$ inverse Fourier transform of the time series $x$ as

$$\tilde{x}(\omega_s) = L_s x = W^* A_s Wx$$

[7] To select a frequency band $[\omega_s, \omega_t]$, let the $s$ -th to $t$ -th diagonal elements of $A$ be one.

where $L_s = W^* A_s W$ is a linear operator. Using Fourier and inverse Fourier transforms and the selection matrix $A_s$, from any data vector $x$, we can extract $T$ time series $x(\omega_s) = [x_1(\omega_s), x_2(\omega_s), ..., x_T(\omega_s)]^T$, each corresponding to a specific frequency $\omega_s$, where $s = 1,...,T$. Based on these frequency-specific data, we can then design frequency-wise correlation and regression coefficients in a conventional way.

For any time series, the data extraction procedure retrieves frequency-specific data and transforms them back into the time domain. Given that the procedure is linear in nature, all conventional time domain techniques and the related finite sample and asymptotic apparatus remain valid and can be applied to the extracted time series without any modification. Furthermore, by an appropriate change in the selection matrix $A_s$, one can also extract band-specific data for any frequency bands. This method is flexible, and it is superior to conventional data filtering methods by avoiding the effects of filter leakage, compression and exacerbation.

### A.2. Correlation analysis

For bivariate stochastic processes $z_t = [x_t \quad y_t]^T$, which are assumed to be jointly weakly stationary with continuous spectra, we write the corresponding spectral density matrix as[8]

$$f_{zz}(\omega) = \begin{bmatrix} f_{xx}(\omega) & f_{xy}(\omega) \\ f_{yx}(\omega) & f_{yy}(\omega) \end{bmatrix}$$

where the spectral densities $f_{xx}(\omega)$, $f_{yy}(\omega)$ and the cross-spectral density $f_{xy}(\omega)$ determine the relationship between $x_t$ and $y_t$ at frequency $\omega$. In Cartesian form, the cross-spectral density $f_{xy}(\omega)$ can be written as

$$f_{xy}(\omega) = c_{xy}(\omega) - iq_{xy}(\omega)$$

where $c_{xy}(\omega)$ and $q_{xy}(\omega)$ are real-valued functions known as cospectrum (or cospectral density) and quadspectrum (or quadrature spectral density), respectively. The cospectrum $c_{xy}(\omega)$ represents the covariance between coefficients of the in-phase components of two time series, while the quadspectrum $q_{xy}(\omega)$ represents the covariance between coefficients of the out-of-phase components. Cospectrum estimation is equivalent to studying the off-diagonal elements of the variance-covariance matrix between two time series, which are uniquely related to cospectra by Fourier and inverse Fourier transforms.

In polar form,

$$f_{xy}(\omega) = |f_{xy}(\omega)| \exp(i\varphi_{xy}(\omega))$$

where

[8] All concepts described in the Annex for bivariate time series can be easily generalised to multivariate stochastic processes, where exogenous variables can also be introduced.

$$\varphi_{xy}(\omega) = -\arctan\left(\frac{q_{xy}(\omega)}{c_{xy}(\omega)}\right)$$

The phase $\varphi_{xy}(\omega)$ measures the average phase lead of $x_t$ over $y_t$, and $\varphi_{xy}(\omega)/\omega$ indicates the extent of time lag. The gain $G_{xy}(\omega)$ is defined as

$$G_{xy}(\omega) = \frac{|f_{xy}(\omega)|}{f_x(\omega)}$$

which is a standardized version of the regression coefficient of $y$ on $x$ at frequency $\omega$. A small $G_{xy}(\omega)$ indicates that $x$ has little effect on $y$ at frequency $\omega$.

Define the complex coherence $ccoh_{xy}(\omega)$ at frequency $\omega$ as

$$ccoh_{xy}(\omega) = \frac{f_{xy}(\omega)}{[f_{xx}(\omega)f_{yy}(\omega)]^{1/2}}$$

The complex coherence $ccoh_{xy}$ is the frequency domain analogue of the coefficient of correlation, but since $f_{xy}$ and $ccoh_{xy}$ are complex, it is hard to interpret this indicator in terms of the overall strength of linear correlation between $x$ and $y$. Real coherence $rcoh_{xy}$, which we define as the real part of $ccoh_{xy}$, is the cospectrum $c_{xy}$ standardized by the square root of the product of $f_{xx}$ and $f_{xy}$. It is the coefficient of correlation between coefficients of the in-phase components of two time series $x$ and $y$. However, a true frequency-specific correlation coefficient needs to account for both the real and complex parts of the complex coherence $ccoh_{xy}$.

One alternative is the coefficient of coherence of $x_t$ over $y_t$ at frequency $\omega$, defined as $cohe_{xy}(\omega) = |ccoh_{xy}(\omega)|$.[9] But although it delivers a real number, it fails to reveal the sign of linear correlation at frequency $\omega$. The coherency $ccoh_{xy}(\omega)$ of $x_t$ over $y_t$ at frequency $\omega$ is

$$coh_{xy}(\omega) = cohe_{xy}(\omega)^2 = |ccoh_{xy}(\omega)|^2$$

Analogous to the coefficient of determination (i.e. $R^2$) in the time domain, the coherency $coh_{xy}$ is the standardized modulus of cross spectral density. It measures the strength of linear association between two or more variables of interest across frequencies. By Schwarz Inequality, $\forall \omega$, $coh_{xy}(\omega) \in [0,1]$. At frequencies for which $f_{xx}(\omega)f_{yy}(\omega) = 0$, we define $coh_{xy}(\omega) = 0$, so the two series $x_t$ and $y_t$ are completely unrelated at frequency $\omega$. If $coh_{xy}(\omega) = 1$, then one series is an exactly linearly filtered version of the other at frequency $\omega$. In general, $coh_{xy}$ varies with frequency $\omega$, indicating the changing pattern of linear association across frequencies. Regions of high coherence are of particular interest.

[9] Extending the conceptual construct of bivariate coherence to multiple time series, we have multiple and partial coherences.

Focussing on estimated cospectra $c_{xy}$, Pakko (2000) found important differences in the low and higher-frequency components of the output-inflation tradeoff, and gave examples of how cospectra can be used for model evaluation. However, without appropriate standardization, the magnitude of cospectra estimates depends on the scale of data and they do not provide an appropriate measure of correlative strength. Indeed they may produce a misleading picture of cross-frequency variations in bivariate correlations when relationships are put to comparison.

On the contrary, estimates of the coefficient of coherency $coh_{xy}$ are scale-independent, being normalized to fall in the [0,1] interval. By definition, it has the disadvantage of taking only non-negative values, therefore failing to differentiate positive and negative correlations between two variables at different frequencies. Aggregating a bivariate relationship across frequencies, when the sign of correlation changes from frequency to frequency, may lead to wrong conclusions as positive and negative correlations cancel each other out and leave an aggregated value of correlation close to zero. Indeed, time domain methods, which aggregate over the entire frequency domain, and methods based on arbitrarily pre-defined frequency bands that aggregate over each band, are not robust when cross-frequency variations are large, and when sign reversals are frequent.

What we need is a frequency-domain analogue of the time-domain coefficient of correlation, corresponding either to a specific frequency $\omega$, or to a frequency band $[\omega_l, \omega_m]$, where $0 \le \omega_l \le \omega_m \le 2\pi$. One natural choice would be the complex coherence $ccoh_{xy}$. But although $ccoh_{xy}$ is signed and normalized, since in general $f_{xy}$ is complex-valued, so is $ccoh_{xy}$. There is no easy way to graphically illustrate, and to interpret the interplay between the real and complex parts of the complex coherence, even if we are able to represent the indicator in a three-dimensional diagram. Our solution is to take advantage of the proposed simple frequency-domain data recovery procedure, and we define $\rho(\omega)$, the frequency-specific coefficient of correlation (FSCC) at frequency $\omega$, as follows

$$\rho(\omega) = \frac{Cov(\hat{x}(\omega), \hat{y}(\omega))}{\sqrt{Var(\hat{x}(\omega))}\sqrt{Var(\hat{y}(\omega))}}$$

where $\hat{x}(\omega)$ and $\hat{y}(\omega)$ are frequency-specific time series extracted from data vectors $x$ and $y$, and $Cov(\bullet)$ and $Var(\bullet)$ stand for covariance and variance, respectively. The confidence interval for $\rho(\omega)$ can be computed in the conventional way.

The frequency-specific coefficient of correlation is normalized to take values in the [-1,1] interval. Unlike cospectral density $c_{xy}$, the FSCC $\rho$ is free from data scale, hence a true measure of the strength of frequency-ω correlation between $x$ and $y$. Comparing to coherence $coh_{xy}$, the FSCC $\rho$ signs the direction of correlation existing in the data. The FSCC estimate is a clear improvement upon cospectrum and coherence estimates, and we use it as the main indicator of strength of bivariate correlation for Phillips relations. When the distribution theories for the cospectral density $c_{xy}$ and the coherence $coh_{xy}$ are complicated, $p$-values and confidence intervals for the FSCC estimates $\hat{\rho}$'s can be provided in the usual way. In fact, these are often supplied automatically in an econometric or statistical software package.

### A.3. Frequency-Specific Spectral Regression

Consider a simple model for two time series $y = [y_1, y_2, ..., y_T]^T$ and $x = [x_1, x_2, ..., x_T]^T$

$$y = \beta x + \varepsilon$$

where $\varepsilon \sim iid\left(0, \sigma^2 I\right)$ and $x$ is uncorrelated with $\varepsilon$. The periodogram of $x$ and the cross-periodogram between $x$ and $y$ at frequency $\omega_s$ are, respectively

$$I_x(\omega_s) = |w_s x|^2$$

$$I_{xy}(\omega_s) = (w_s x)^* (w_s y)$$

where $w_s$ is defined as before. The $s$-th frequency spectral regression is

$$A_s \tilde{y} = \beta_s A_s \tilde{x} + A_s \tilde{\varepsilon}$$

where $\tilde{q} = Wq$, for any $q$, and $W$ is defined as before. The $s$-th frequency spectral regression coefficient is

$$\tilde{\beta}_s = \left(\tilde{x}^* A_s \tilde{x}\right)^{-1} \tilde{x}^* A_s \tilde{\varepsilon} = I_x(\omega_s)^{-1} I_{xy}(\omega_s)$$

Since the unsmoothed periodogram $I_x$ and the cross-periodogram $I_{xy}$ are not consistent estimators of the spectral and cross-spectral densities, and we are interested in frequency-specific regression coefficients that do not involve averaging over a frequency band to obtain consistent estimates of the sums of periodogram and cross-periodogram ordinates, we may instead use smoothed spectral estimates to estimate $\beta_s$:

$$\hat{\beta}_s = \hat{f}_x(\omega_s)^{-1} \hat{f}_{xy}(\omega_s)$$

In general, the estimator $\hat{\beta}_s$ will be complex-valued. To obtain a real-valued estimate, one can take the real part of $\hat{\beta}_s$, but typically, both the real and complex parts of $\hat{\beta}_s$ matter. Or we may simply use the gain, i.e., the modulus $\left|\hat{\beta}_s\right|$, which has the drawback of not allowing us to discern the sign of spectral regressions. We take advantage of the proposed data extraction procedure and run OLS regressions with frequency-specific data. Since the Fourier transform and inverse Fourier transform are linear operations, conventional asymptotic theory continues to apply, and the confidence interval for $\hat{\beta}_s$ can be computed in the usual way (except at the zero frequency). Write the inverse Fourier transform as

$$L_s y = \beta_s L_s x + L_s \varepsilon$$

Simple OLS spectral regressions lead to the frequency-specific coefficient of regression (FSCR) $\hat{\beta}_s$ corresponding to frequency $\omega_s$

$$\hat{\beta}_s = \left(x^T L_s^T L_s x\right)^{-1} x^T L_s^T L_s y = \left(\hat{x}_s^T \hat{x}_s\right)^{-1} \hat{x}_s \hat{y}_s$$

The great advantage of the data extraction procedure is that it is linear in nature, therefore all inferential apparatus in the conventional OLS regression theory can still be used as usual.