On the importance of borrowing constraints for house price dynamics (preliminary)*

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Abstract

We study how a household borrowing constraint in the form of a down payment requirement affects house price dynamics in an OLG model with standard preferences. We find that in certain situations, the borrowing constraint shapes house price dynamics substantially. The importance of the borrowing constraint depends very much on whether house price changes are driven by interest rate or aggregate income shocks. Moreover, because of the borrowing constraint, house price dynamics display substantial asymmetries between large positive and large negative income shocks. These results are related to the fact that the share of borrowing constrained households is different following different shocks.

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1 Introduction

In this paper, we investigate the importance of borrowing constraints for house price dynamics. We analyze the issue using an OLG model with owner housing. In the model, young households need to borrow in order to finance their housing. Differences in household size create large differences in household leverage also among households of same age. We hit the economy with different aggregate shocks and solve for the house price dynamics following the shocks. We contrast two cases: one where household borrowing is unlimited and another where households can borrow only up to a certain fraction of the value of their house. We are particularly interested in situations where a substantial fall in house prices reduces the net worth of leveraged households dramatically. In such situations, the borrowing constraint may become binding for many households.

In order to be able to focus on the effect of the borrowing constraint, we make two key simplifying modelling assumptions. The first is that we assume perfect foresight. The second is that we abstract from transaction costs and other non-convexities in the household problem. These assumptions make it possible to solve for the fully non-linear dynamics very accurately.

In the analytical part of the paper, we study the behavior of borrowing constrained and unconstrained households when house prices change. The main aim of this exercise is to understand how the link between house prices and buyer liquidity works in a fully dynamic setting.

In the numerical part of the paper, we calibrate the model to Finnish household data and consider the dynamics following different shocks. We also compare the model dynamics to the recent experience in the Finnish housing market. The Finnish housing market is an interesting example since it has recently been hit by two big consecutive shocks, a credit market liberalization in the late 1980s that led to a drastic relaxation of household borrowing constraints and a severe depression in the early 1990s. Both episodes were associated with very large house price changes. Computing the house price dynamics in the model following shocks comparable to the Finnish case and comparing them to actual house price movements helps in understanding the quantitative relevance of the model.

To briefly summarize our results, we find, first of all, that the borrowing constraint can substantially shape house price dynamics especially following large negative aggregate income shocks. In particular, the borrowing constraint tends to speed up the convergence towards the new steady state price. The same does not happen after a positive income shock. Therefore, the borrowing constraint creates substantial asymmetries in the house price dynamics that follow large positive and large negative income shocks. The borrowing constraint is much less important for house price dynamics that are driven by interest rate shocks. These different reactions in the price dynamics can be explained by large differences in the share of borrowing constrained households after different shocks. Second, the model can explain a large part of the increase in Finnish house prices that coincided with the credit market liberalization as an equilibrium response to an empirically plausible relaxation of the borrowing constraint. This suggests that the model captures much of the actual relevance of borrowing constraints to aggregate housing demand.

In the next subsection, we discuss how our paper relates to the previous literature. In section 2, we describe the model and analyze the role of the borrowing constraint analytically. In section 3, we discuss the calibration and the initial steady state. In section 4, we analyze the dynamics of the model. We conclude in section 5.

Related literature and our contribution

Stein (1995) was the first to stress the importance of borrowing constraints for house price dynamics. To see the intuition behind the mechanism that he highlights, consider a household that has a house worth 100 000 euros and a mortgage loan of 70 000 euros. It has no other assets or debts, so its net worth is 30 000 euros. The household wants to move to a bigger house. Banks require a 20% down payment. Hence, the household could buy a house worth 150 000 euros, which is 50% bigger (in a quality adjusted sense) than its current one. Assume now that for some reason house prices fall by 10%. This reduces the net worth of the household to 20 000 euros. As a result, it can buy a house worth only 100 000 euros. Given that house prices have fallen, 100 000 euros will buy only 10% bigger than its current one. Hence, because of the borrowing constraint, a house price fall may induce the household to buy a smaller house compared to the one it would have bought had house prices remained constant. In Stein's model, this link between house prices and buyer liquidity can give rise to a multiplier mechanism and even multiple equilibria.

Stein's model is essentially static, as he assumes that all trade takes place in one period. Ortalo-Magné and Rady (1999, 2006) are able to characterize how the interplay between aggregate income shocks, homeowners' capital gains or losses and borrowing constraints affects house price dynamics and the transaction volume in a fully dynamic model where houses are available in two sizes, or 'property ladders'. Like Stein's analysis, their analyses are qualitative rather than quantitative in nature. For instance, in order to keep the model tractable, Ortalo-Magné and Rady assume preferences that rule out consumption smoothing: in their model, all non-housing consumption takes place in the last period of households' lives.

Some recent papers incorporate housing with a down payment constraint into quantitative business cycle models with standard preferences. Iacoviello (2005) and Iacoviello and Neri (2007) are good examples. In these models, which are designed to analyze monetary policy, there are two types of households: patient and impatient. In the steady state, the impatient households are borrowing constrained while the patient households are not. Dynamics are analyzed around such a steady state. Restricting the analysis to the neighborhood of a steady state is computationally convenient because one can then use a linearized version of the model. However, by construction, the share of borrowing constrained households then remains constant over time.[1]

To put it very briefly, we contribute to this literature by analyzing the importance of borrowing constraints for house price dynamics following large aggregate shocks. Two features of our model are particularly important in this respect. First, we use standard preferences with a consumption smoothing motive. Our analytical results show how the consumption smoothing motive is linked to the multiplier mechanism discussed above. Second, we solve for the fully non-linear dynamics. This means here that the fraction of households that are borrowing constrained may change over time. We show that this feature is quantitatively very important in the context of large shocks that create large capital losses or gains to highly leveraged households. We also believe that our analytical results are helpful in understanding more generally how the liquidity effect described by Stein (1995) works in a fully dynamic set-up.

However, it should also be stressed that in some ways, our model is simpler than the models of Ortalo-Magné and Rady (1999, 2006). In particular, we do not model the timing of the first home purchase, which plays an important role in their analyses. Instead, in our model, all households are assumed to buy some housing in the first period of their economically independent lives. Also, our model does not provide predictions about the transaction volume since households can costlessly adjust their housing stock every period.[2]

On the empirical side, Lamont and Stein (1999) relate U.S. city-level house price

[1] The multiplier mechanism discussed in Stein (1995) is also close to the 'credit cycles' -mechanism in Kiyotaki and Moore (1997). Cordoba and Ripoll (2005) have analyzed the quantitative importance of that mechanism with a linearized model.

[2] Recently, Ríos-Rull and Sanchez-Marcos (2008) have developed a model that has a property ladders -structure with two house sizes and that features aggregate uncertainty and preferences that exhibit a desire for consumption smoothing.

data to the data on household finances and Benito (2006) uses British Household Panel Survey. Both studies estimate the effect of income shocks on house price dynamics. They show that compared to other regions, house prices tend to overshoot or undershoot following aggregate income shocks more in regions where households are highly leveraged. These results are consistent with the multiplier mechanism à la Stein (1995), but do not testify to the importance of borrowing constraints for house price dynamics because households' asset positions may affect house price dynamics even in the absence of borrowing constraints. One purpose of this paper is to isolate the importance of borrowing constraints for house price dynamics in a theoretical set-up.

2 The model

We consider a model economy with overlapping generations of households. During the first J periods of their lives, households derive utility from non-housing consumption, c, and from their stock of owner housing, h. We follow Gervais (2002), Davis and Heathcote (2005), and others in assuming that housing services are proportional to housing capital. In period $J+1$, households derive a terminal utility that depends only on their remaining net worth. Each generation is of the same size so that population remains constant over time. The periodic earnings of households of age j is denoted by y_t^j.

The price of one unit of housing in period t is p_t. Housing involves some direct costs such as maintenance costs and property taxes. We assume that part of these costs are proportional to the size of the house and part of them (taxes in particular) are proportional to the value of the house.[3] We denote these two costs by η and κ. There

[3] We introduce these two types of costs because they have different implications for equilibrium house prices. For instance, if there are large maintenance costs that are proportional to the size of the house alone, a large part of the user cost is unrelated to the house price. As a result, a relatively large change in the steady state house price is needed in order to create a given percentage change in the total user cost of housing. In this case, a relatively small change in aggregate household income, for instance, implies a relatively large change in the equilibrium house price.

is also a financial asset, a. The interest rate that the financial asset earns from period $t-1$ to period t is $R_t - 1$. The net worth of household of age j in period t is given by

$$b_t^j = R_t a_{t-1}^{j-1} + p_t h_{t-1}^{j-1}.$$

Households face a borrowing constraint which means that they can borrow only against their housing and that they have to finance part of their housing with own equity. The fraction of the value of the house that the household has to finance itself is denoted by $\theta \leq 1$. This kind of borrowing constraint is often referred to as a down payment requirement. However, it can also be used to partly capture maturity constraints. In particular, if households can only take mortgages with a very short maturity, they have to pay a relatively large fraction of their housing during the first period.

In each generation, there are I different household types, indexed by $i = 1, 2, .., I$. The intragenerational heterogeneity stems from households getting children at different ages. Children affect household savings behavior by changing the household size over the life cycle. As we will see, differences in the age at which households get children result in large differences in household leverage. The mass of households of type i is denoted by m_i. We normalize the size of each generation to one. That is $\sum_{i=1}^{I} m_i = 1$. We denote the household size by s.

The periodic utility function for $j = 1, ..., J$ is denoted by $u(c, h; s)$ and terminal utility is denoted by $v(b; s)$. The subjective discount factor is β. We use superscripts to denote household age and subscripts to denote household type and time period so that $c_{i,t}^j$, for instance, denotes non-housing consumption of a household of age j and type i in period t.

The problem of a household of age $j = 1$ and type i in period t is the following:

$$\max_{\{c_{i,t+j-1}^j, h_{i,t+j-1}^j\}_{j=1}^J} \sum_{j=1}^{J} \beta^{j-1} u(c_{i,t+j-1}^j, h_{i,t+j-1}^j; s_i^j) + \beta^J v(b_{i,t+J}^{J+1}; s_i^{J+1}) \tag{1}$$

subject to

$$
\begin{aligned}
c^j_{i,t+j-1} + g_{t+j-1} h^j_{i,t+j-1} + a^j_{i,t+j-1} &= y^j_{t+j-1} + b^j_{i,t+j-1} && (2)\\
a^j_{i,t+j-1} &\geq -(1-\theta)\, p_{t+j-1} h^j_{i,t+j-1} && (3)\\
h^0_{i,t} &= a^0_{i,t} = 0, && (4)
\end{aligned}
$$

where

$$
g_t = p_t + \kappa p_t + \eta.
$$

The first constraint is the periodic budget constraint. The second constraint is the periodic down payment constraint. The third constraint states that the household starts its life without initial assets or debt.

We consider a small open economy in the sense that the interest rate and the wage level are exogenously given. The only aggregate consistency condition is the market clearing condition for the housing market. We assume that the supply of housing is fixed at $\overline{H}$.[4] The market clearing condition reads as:

$$
\sum_{i=1}^{I} \sum_{j=1}^{J} m_i h^j_{i,t} = \overline{H}. \tag{5}
$$

The Lagrangian for the household's maximization problem is (we drop here the type index):

$$
\begin{aligned}
L \;=\; & \sum_{j=1}^{J} \beta^{j-1} u(c^j_{t+j-1}, h^j_{t+j-1}; s^j) + \beta^J v(b^{J+1}_{i,t+J}; s^{J+1}) && (6)\\
& + \sum_{j=1}^{J} \lambda^j_{t+j-1} [y^j_{t+j-1} + b^j_{i,t+j-1} - c^j_{t+j-1} \\
& - g_{t+j-1} h^j_{t+j-1} - a^j_{t+j-1}] \\
& + \sum_{j=1}^{J} \gamma^j_{t+j-1} (a^j_{t+j-1} + (1-\theta)\, p_{t+j-1} h^j_{t+j-1}),
\end{aligned}
$$

[4]Without loss of generality, the aggregate supply of housing can be normalized to any strictly positive level. We choose the aggregate supply of housing so that the house price is equal to 1 in the initial steady states.

where λ_t^j and γ_t^j are the Lagrange multipliers for the budget constraint and the borrowing constraint for a household of age j at time t.

We now discuss the importance of some simplifying assumptions we have made. There is no aggregate uncertainty in the model which means that we can consider only perfect foresight dynamics following completely unanticipated shocks. Clearly, this limits the way we can compare house price dynamics in the model to the data. However, with aggregate uncertainty, the model would become very difficult to solve since we would then have to use recursive methods with the distribution of households over their asset positions (or at least some moments describing it) as a state variable. Perfect foresight dynamics are the easiest way of illustrating how the borrowing constraint affects house price dynamics.

We also assume that there are no transaction costs. Again, transaction costs would make it much more difficult to solve the model since the household problem would then become non-convex and since we would then need a model with a continuum of households in different situations (in order to get a smooth aggregate demand function).[5] The absence of transaction costs means that households generally adjust their housing position every period, which is not realistic if the model period is relatively short. It also means that we cannot consider the dynamics of the transaction volume. However, as we show below, since the demand for housing in our model is affected by changes in household size, the model nevertheless has the realistic feature that households undertake major adjustments to their housing only a few times in their life.

We take the supply of housing as fixed and hence our focus is entirely on the demand side. In any case, we believe that the supply side is not the key to understand the drastic house price movements that we have recently observed in Finland (which we describe below). While construction volume varies a lot over the business cycle, the level of investment is so small compared to the aggregate stock of housing that the aggregate stock changes very slowly. Related to this, we abstract from growth. An extended

[5]Technically, we could handle convex transaction costs but not realistic non-convex transaction costs.

version of the model with income growth would have a steady state with constant house prices assuming that the supply of housing increases at the same rate as income.

2.1 Solving the model

We solve for the transitional dynamics of the economy following different completely unexpected shocks. We assume that it takes up to T periods for the economy to converge to a new steady state after a shock. Using the household first-order conditions, the budget constraints and the borrowing constraint together with the housing market equilibrium condition for each period, we get the following system of equations for $i = 1, 2, .., I$ and $t = 1, 2, ..., T$.

$$\beta^{j-1} u_{h^j_{i,t}} + p_{t+1}\lambda^{j+1}_{i,t+1} = g_t \lambda^j_{i,t} - \gamma^j_{i,t}(1-\theta)\, p_t \text{ for } 1 \leq j < J \quad (7)$$

$$\beta^{J-1} u_{h^J_{i,t}} + \beta^J p_{t+1} v_{b^J_{i,t}} = g_t \lambda^J_{i,t} - \gamma^J_{i,t}(1-\theta)\, p_t \quad (8)$$

$$\beta^{j-1} u_{c^j_{i,t}} = \lambda^j_{i,t} \quad (9)$$

$$-\lambda^j_{i,t} + R_{t+1}\lambda^{j+1}_{i,t+1} + \gamma^j_{i,t} = 0 \text{ for } 1 \leq j < J \quad (10)$$

$$\beta^J R_{t+1} v_{b^J_{i,t}} - \lambda^J_{i,t} + \gamma^J_{i,t} = 0 \quad (11)$$

$$\gamma^j_{i,t}\left(a^j_{i,t} + (1-\theta)\, p_t h^j_{i,t}\right) = 0 \quad (12)$$

$$\gamma^j_{i,t} \geq 0,\ a^j_{i,t} + (1-\theta)\, p_t h^j_{i,t} \geq 0 \quad (13)$$

$$c^j_{i,t} + g_t h^j_{i,t} + a^j_{i,t} = y^j_{i,t} + b^j_{i,t} \quad (14)$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} m_i h^j_{i,t} = \overline{H} \quad (15)$$

This set of equations fully characterizes the dynamics of the economy. With a multi-period life cycle, this is a relatively large system of non-linear equations. In our calibrated model, it consists of about 2000 equations. We solve this system using the broydn's algorithm. When solving the system, we impose a very strict error tolerance (10^{-5}). Hence, we solve for the dynamics very accurately. (We also have to check that the solution is not affected by our guess for T.)

2.2 The borrowing constraint and housing demand

As we discussed in the introduction, the multiplier mechanism in Stein (1995) is essentially a link between house prices and buyer liquidity. In this section, we will analyze in more detail how this link works in a fully dynamic set-up. We do this by determining analytically the effect of a marginal change in current house price on current and future housing demand. We determine the demand effect separately for borrowing constrained and unconstrained household and then compare their behavior. The main purpose of this exercise is to disentangle the different channels through which current price affects housing demand in the presence of binding borrowing constraints. The results will be useful when developing intuition for our numerical results.

Let us consider a household of age $1 < j < J$. For notational convenience, we drop here time, age, and type indices. We denote housing and financial assets of the household in the beginning of the current period by h^{-1} and a^{-1} and its housing and financial assets in the beginning of the next period by h and a. We further denote the current house price by p, and the next period house price by p'. We assume that the interest rate is constant.

The problem of the household can now be formulated as:

$$\max_{c,h,b} \{u(c,h) + \beta V(b)\} \tag{16}$$

subject to

$$\begin{aligned} c + (p + \kappa p + \eta)\, h + a &= y + ph^{-1} + Ra^{-1} \quad (17) \\ a &\geq -(1-\theta)\, ph. \quad (18) \end{aligned}$$

where $V(b)$ denotes remaining life time utility and $b = Ra + p'h$. As long as the household has a consumption smoothing motive, $V_{bb} < 0$. As for the periodic utility function, we assume here, for simplicity, that it is separable between consumption and housing, that is $u_{ch} = 0$.

We first ask how the current housing demand depends on the current house price, given a^{-1} and h^{-1}. The Appendix shows that in the unconstrained case, the effect of a marginal change in the current house price on current housing demand is given by

$$\frac{\partial h}{\partial p} = \frac{1}{D}\left[\underbrace{\frac{(1+\kappa)\,u_c u_{cc}}{R}}_{\text{negative}} + \beta R\left(\underbrace{V_{bb}\,(1+\kappa)\,u_c}_{\text{negative}} + \underbrace{PV_{bb}u_{cc}\left(h^{-1}-(1+\kappa)\,h\right)}_{\text{negative if }(1+\kappa)h>h^{-1}}\right)\right]$$

where $D > 0$ and $P = p + \kappa p + \eta - \frac{p'}{R}$. We assume here that $P > 0$.

The overall effect consists of three terms that reflect the standard substitution and income effects. The first term is independent of V. Hence, it is related to the intratemporal resource allocation alone. It is always negative: An increase in the current house price makes current housing more expensive relative to current non-housing consumption. The other two terms depend on V. They are therefore related to the intertemporal resource allocation. The first of these terms is also always negative: An increase in the current house price makes current housing more expensive relative to future non-housing and housing consumption. The third term depends on whether $(1+\kappa)\,h$ is smaller or larger than h^{-1}. Intuitively, this term is related to an endowment effect: An increase in the current house price makes the household "wealthier" if $h^{-1} > (1+\kappa)\,h$, that is, if it is downsizing fast enough. In that case, the third term works to increase housing demand when the house price increases. Note that the last two terms would both go to zero if V_{bb} goes to zero. Without a consumption smoothing motive, current housing demand would depend only on the relative price of current housing and current consumption.

When the household faces a binding borrowing constraint, the effect of a price change on current housing is given by the following expression (see the Appendix):

$$\begin{aligned}\frac{\partial h}{\partial p} &= \frac{1}{D^c}\left[\underbrace{-u_c\,(1+\kappa)}_{\text{negative}} + \underbrace{u_{cc}T\left((1+\kappa)\,h - h^{-1}\right)}_{\text{negative if }(1+\kappa)h>h^{-1}}\right] \\ &\quad + \frac{1-\theta}{D^c}\left[\underbrace{-u_{cc}Th}_{\text{positive}} + \underbrace{u_c}_{\text{positive}}\underbrace{-R\beta V_b}_{\text{negative}}\underbrace{-S\beta V_{bb}Rh}_{\text{positive}}\right]\end{aligned} \tag{19}$$

where $D^c > 0$ and $T = p + \kappa p + \eta - (1-\theta)\,p > 0$ and $S = p' - R\,(1-\theta)\,p$. We assume here $S > 0$.

The overall effect now consists of six terms. The first two terms have the same interpretation as the first and the third term in the unconstrained case: The first term is related to the intratemporal resource allocation and is negative. The second term is related to the endowment effect, which is also negative as long as $(1+\kappa)\,h > h^{-1}$. The second term of the unconstrained case is missing here. This is because, with the borrowing constraint binding, the household does not want to substitute future non-housing or housing consumption for current housing consumption.

Compared to the unconstrained case, there are hence four additional terms which depend on the borrowing constraint parameter, θ. The first term shows the direct link with the borrowing constraint: as the current house price goes up, the household can borrow more which increases the demand for housing.[6] This effect creates the multiplier effect in Stein's (1995) model. The second term is also positive and closely related to the first one: Recall that a borrowing constrained household can only increase its housing demand by giving up more current consumption. When the house price increases, for each unit of housing the household can borrow more and hence must give up less current consumption. This induces the household to buy more housing. We will refer to these two terms together as the *liquidity effect*.

When the household is borrowing constrained, its current housing demand directly determines its future savings. The last two terms show how a price change affects the incentive to save through housing. The first of them is a direct substitution effect: an increase in the current house price makes saving more expensive which reduces savings. The second term is related to the fact that as the current price increases, for a given housing demand, the household has less savings in the future. With a consumption smoothing motive (that is, $V_{bb} < 0$), the household wants to partly compensate this by increasing its housing demand.

[6] Note that $T = 0$ if $\theta = 0$ and $\eta = \kappa = 0$. If the all the costs related to the purchase of new house can be entirely financed by mortgage, a change in current house price does not have a direct effect on housing demand through the borrowing constraint.

In equilibrium, the current house price depends on both current and future housing demand. In order to analyze the effect of a change in the current house price on future housing demand, we note first that future housing demand must depend positively on household's next period net worth. Hence, we now consider how next period's net worth is affected by current house price changes.

The Appendix shows that in the unconstrained case we have

$$\frac{\partial b}{\partial p} = \frac{1}{D}\left[\underbrace{-u_{cc}P\left(1+\kappa\right)u_c}_{\text{positive}} + \underbrace{u_{cc}u_{hh}\left(h^{-1}-\left(1+\kappa\right)h\right)}_{\text{negative if } (1+\kappa)h>h^{-1}}\right] \tag{20}$$

where again $D > 0$. As the above expression shows, a sufficient condition for a price increase to increase savings is that $(1+\kappa)\,h < h^{-1}$. It is straightforward to show that with $u\left(c,h\right) = \log c + \log h$, for instance, a higher current house price always increases savings for unconstrained households.

When a household faces a binding borrowing constraint, next period's net worth is directly determined by its housing demand. Hence, it follows that

$$\frac{\partial b}{\partial p} = S\frac{\partial h}{\partial p} - R\left(1-\theta\right)h, \tag{21}$$

where $\frac{\partial h}{\partial p}$ is given by (19). Since $S > 0$, this expression will be negative if $\frac{\partial h}{\partial p} < 0$, that is, if the *liquidity effect* is not strong enough to dominate in the demand response of the borrowing constrained households. In that case, the fall in house price will induce them to save more. This result has two sources: Increased housing demand in its own right increases future net worth. In addition, when house price drops, for each unit of housing they are able to borrow less. Hence, for each unit of housing, they must give up more current consumption. Through this effect, a decrease in the current house price works to increase next period housing demand. The existence of this "delayed demand" turns out to be important in shaping the price dynamics after certain type of shocks.

3 Calibration and the steady state

In this section, we describe the household data we use in the calibration, the calibration procedure, and the steady state of the model economy.

3.1 Household leverage in the data

We base our calibration on 2004 Wealth Survey conducted by Statistics Finland, which includes portfolio information from about 2500 Finnish households. We consider only homeowners. In the survey, they were asked an estimate of the current market value of their house.

The importance of borrowing constraints should crucially depend on household leverage. We characterize household leverage with the net worth-to-house value ratio (NWHV). Net worth is defined as the sum of the market value of household's residential property and its financial assets less all debt. Hence, the lower the NWHV of a household is, the more highly leveraged it is in the sense that it has more debt or less assets relative to the value of its house. The distribution of the NWHV ratios in the data is shown in figure 1.

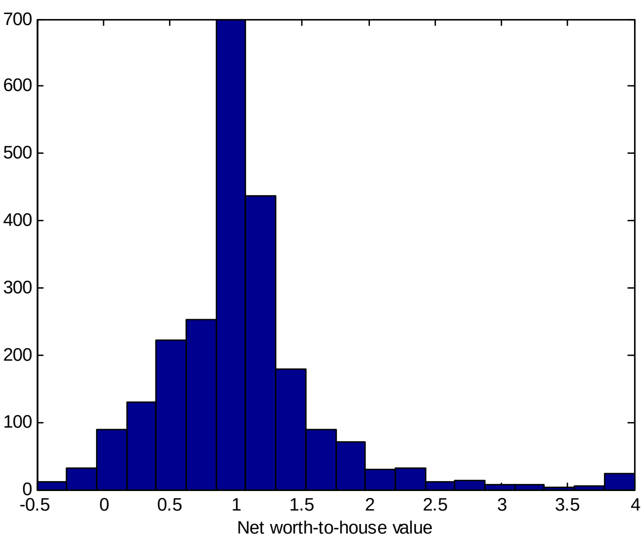

Figure 1. The distribution of net worth-to-house value ratios in the data.

Figure 2 shows the median NWHV ratio in different age groups. Young households are much more leveraged than older households. The median NWHV ratio increases from about 0.25 among households of age 25-29 to about 1.1 among households of age 70-74.

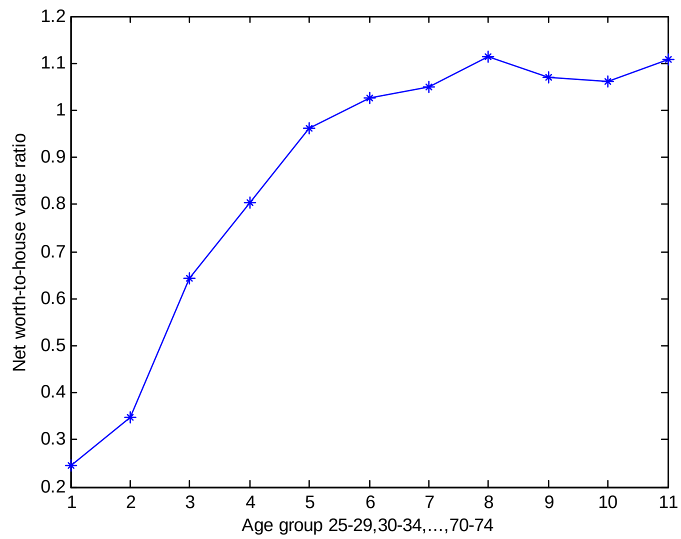

Figure 2. The median net worth-to-house value ratio in different age groups in the data.

3.2 Calibration

We take one model period to correspond to four years and assume that households' economically independent life lasts for 12 periods, that is $J = 12$. We interpret model age 1 as real ages 25-28. Model age 12 then corresponds to real ages 69-72.

These choices are somewhat arbitrary, of course. Given that the model does not feature transaction costs related to moving, a relatively long model period seems more natural than a model period of, say, one year. A relatively long model period also allows us to partly capture maturity constraints with the borrowing constraint. On the other hand, a model period of four years suffices to describe the main price changes we have observed in Finland: It took four years for house prices to fall from peak to bottom in the early 90s.

We assume that there are three different household types, so that $I = 3$. Households consist of two adults who get two children in model age 1, 2 or 3 depending on their type. Children live within the household for five model periods (or 20 years). We compute the corresponding household sizes using the OECD scale for household consumption units. For instance, for households of type 2 (that get children in model age 2), this means that $s_2^1 = 1.7$, $s_2^j = 2.7$, for $j = 2, 3, 4, 5$,6, and $s_2^j = 1.7$ for $j \geq 7$. We assume $m_1 = m_2 = m_3$. That is, all household types are equally common in the population.

We take the income profile directly from the data. We first compute the average annual non-capital income in age groups 25-28, 29-32,..., 57-60, that is for the first 9 model periods. We assume that after model age 9, households receive a pension which is 60% of the average income over the life cycle. This figure is close to the actual replacement rate of the Finnish pension system. The resulting income profile is

$$\{y^j\}_{j=1}^J = \{0.67, 0.82, 1.04, 1.12, 1.11, 1.11, 1.10, 1.02, 0.98, 0.60, 0.60, 0.60\}.$$

We assume that the periodic utility is determined by a CES-CRRA utility function:

$$u(c, h; s) = s\frac{[\alpha_c(c/s)^\gamma + (1-\alpha_c)(h/s)^\gamma]^{\frac{1-\sigma}{\gamma}}}{1-\sigma}, \text{ for } \sigma > 1,\ \gamma < 1 \text{ and } \gamma \neq 1.$$

The terminal utility is

$$v(b; s) = s\alpha_b\frac{(b/s)^{1-\sigma}}{1-\sigma}, \text{ for } \sigma > 1.$$

The case of $\sigma = 1$ corresponds to the logarithm function and $\gamma = 0$ to the Cobb-Douglas function.

We set the interest rate term at $R = 1.08$. This corresponds closely to the average yearly real after tax interest rate on mortgage loans during the period 2000-04, which was 1.95%. We set the housing related cost parameters at $\kappa = 0.04$ and $\eta = 0.08$. Hence, in the initial steady state with the house price equal to one, these costs are annually about 1% and 2% of the house value.

Finally, we must determine the borrowing constraint. We assume that the borrowing constraint takes the value $\theta = 0.25$. This means that a household is required to make a down payment of 25% of the value of the house. We think of this as a realistic borrowing constraint in Finland *after* the credit market liberalization. For comparison, we also consider the case where there is no borrowing constraint, which is equivalent to the case where θ is negative and large in absolute value.

We are then left with the preference parameters, γ, σ, β, α_c, and α_b. In the benchmark calibration, we set $\sigma = 2$ and $\gamma = -1$. The latter parameter value implies an intratemporal elasticity of substitution between housing and consumption equal to 0.5. We later consider different values for these two parameters. We calibrate the remaining parameters separately with and without the borrowing constraint. Alternatively, when comparing the dynamics with and without the borrowing constraint, we could keep all other parameter values fixed and start from the same calibration. This would mean, however, that the initial distributions of household leverage in the model would be very different in the two cases. We choose parameters β, α_c, and α_b so that:

i) Average NWHV is 0.95.

ii) Average NWHV in age J is 1.1.

iii) Average net worth-to-income ratio is 0.8.

These targets are based on the 2004 wealth survey. The first target is the median NWHV for households of age 25-72. The second target is the ratio of net worth for households of age 69-72 to the median net worth of households of age 25-72. The third target is based on the median net worth-to-annual income ratio being 3.2. Since the model period is four years, we divided this ratio by four.

The resulting parameter combinations are shown in table 1. Note that in order to get the same NWHV without the borrowing constraint as with it, we have to choose a higher discount factor. This reflects the fact that the borrowing constraint limits household borrowing.

	β	α_c	α_b
$\theta = 0.25$	0.98	0.85	0.91
$\theta = -\infty$	1.01	0.85	0.88

Table 1: Parameter combinations.

3.3 The steady state

Figure 3 displays the steady state housing profiles, h_i^j, for the three different household types in the case with logarithmic preferences (these profiles are not scaled by household size, s). Consumption profiles (not shown) are similar. Consider first households of type 3. These households get children at model age 3. They are never borrowing constrained and hence their housing follows closely household size. They move to a bigger house when they get children at age 3, and move to a smaller house at age 7. In contrast, households of type 1, who get children at age 1, are borrowing constrained until model age 5. This distorts their housing (and consumption) profiles over the life cycle. Households of type 2 are an intermediate case: they are borrowing constrained at ages 2 and 3. The share of borrowing constrained households is 7/36 in steady state (in all periods, there are three times 12 different household groups).

Because of the absence of transaction costs, housing does not remain exactly constant between any two periods. These profiles are realistic, however, in the sense that in any time period, only a small fraction of households wish to make *major* adjustments in their housing.

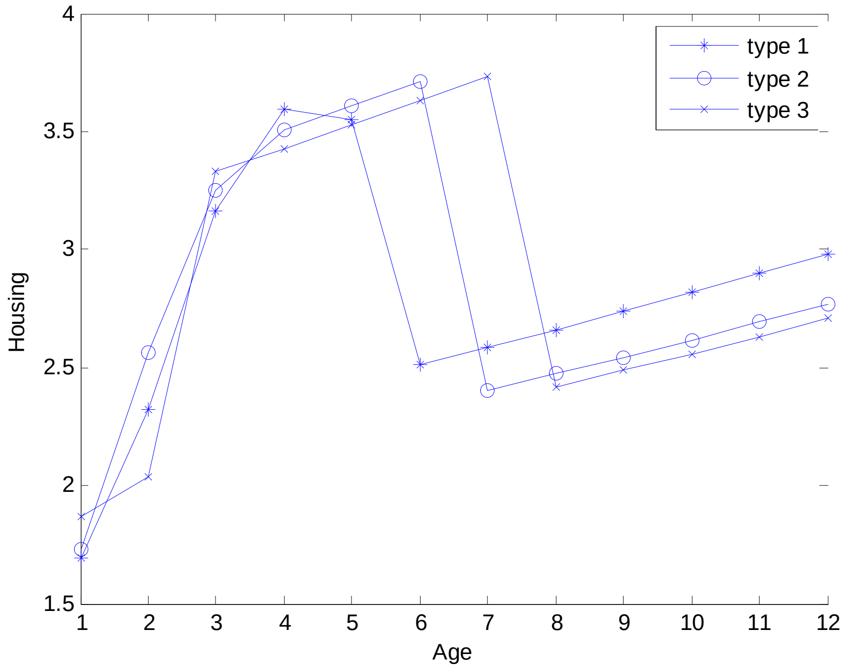

Figure 3. Housing profiles over the life cycle in steady state.

Table 2 compares the distribution of household leverage in the data to the model when $\theta = 0.25$. For the table, we have divided the households into four groups according to their NWHV ratio and calculated the share of households in each group. As the table shows, this distribution is more dispersed in the data than in the model. In the data, some households report to have NWHV less than 0.25, which is the lowest NWHV we allow for in the model. On the other hand, the model economy also features too few households with NWHV larger than one.

	Net worth-to-house value ratio			
	< 0.25	$0.25 - 0.5$	$0.5 - 1.0$	> 1.0
Data	6.7%	8.6%	28%	57%
Model ($\theta = 0.25$)	0%	25%	32%	43%

Table 2: Share of households with different net worth-to-house value ratios.

4 Dynamics

In this section, we analyze numerically the dynamics of the model following different shocks. In subsection 4.1, we consider different income and interest rate shocks with the benchmark calibration. In subsection 4.2, we consider how the dynamics are affected by the preference parameters. In subsection 4.3, we look at how a marginal change in the current house price affects housing demand at individual level. There, our aim is to provide further intuition for the results in subsection 4.2. In addition, this exercise allows us to discuss the possibility of multiple equilibria. Finally, in subsection 4.4, we consider a similar relaxation of the borrowing constraint and similar income and interest rate shocks that were associated with the Finnish credit market liberalization in the late 1980s and the Finnish depression in the early 90s. Our aim is to see to what extent the model can replicate the dramatic house price movements experienced in Finland.

4.1 Income and interest rate shocks

We now consider large income and interest rate shocks. In order to get a good overall picture of the importance of borrowing constraints in different cases, we consider both permanent and temporary shocks as well as both positive and negative shocks.

We assume that the income shocks affect all households equiproportionally. Specifically, in the case of a permanent shock, we multiply y_t^j by 0.85 or 1.15 from period 1 onwards. In the case of a temporary shock, we assume that the aggregate income first drops or increases sharply and then converges back to its initial level in four periods and multiply y_t^j by either 0.80, 0.86, and 0.93 or 1.2, 1.13, and 1.06 in periods 1, 2, and 3, respectively. Households make their period 1 decisions after learning about the shock. In all cases, we compute the house price dynamics with and without the borrowing constraint. We assume that the economy is initially in a steady state in discussed in section 3.3 (with $\theta = 0.25$ or $\theta = -\infty$).

Figure 4 displays the house price dynamics following the different income shocks.

Top-left graph relates to a temporary negative shock, top-right graph to a permanent negative shock, bottom-left graph to a temporary positive shock and bottom-right graph to a permanent positive shock. By construction, the initial house price in period 0 is equal to 1 in all cases.

The first thing to note from figure 4 is that the borrowing constraint shapes the price dynamics substantially only following negative income shocks: following positive shocks, the price dynamics are remarkably similar with and without the borrowing constraint. The reason is that few households are borrowing constrained just after a large positive income shock. For instance, following the permanent positive shock considered here, the share of borrowing constrained households decreases from $7/36$ in the initial steady state to $2/36$ in period 1. Therefore, a marginal increase in period 1 house price from the equilibrium price can only have a very small impact on the aggregate housing demand through the borrowing constraint.

Consider then top graphs that display the price dynamics after negative income shocks. The most important effect of the borrowing constraint seems to be that it makes the house price increase more rapidly from period 1 to period 2. This means that there are relatively large anticipated capital gains to housing. The intuition behind this result is the following. The house price fall effectively tightens the borrowing constraint for those households for which it is binding. This reduces period 1 housing demand through the liquidity effect that we discussed in subsection 2.2. For the housing market to clear in period 1, that is, for the unconstrained households to be willing to demand sufficiently more, the liquidity effect must be offset by sufficiently large anticipated capital gains to housing from period 1 to period 2.

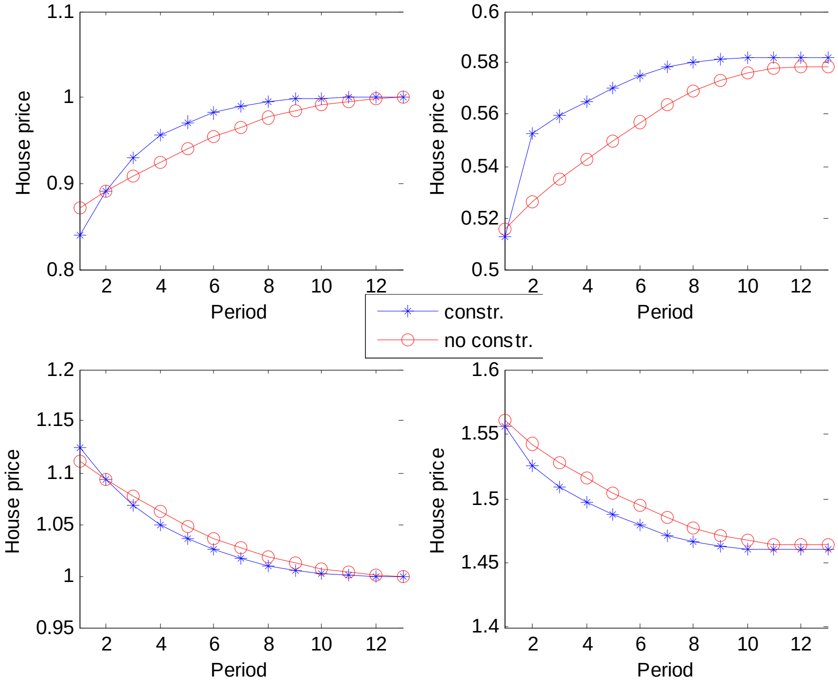

Figure 4. House price dynamics following different income shocks.

One might think that these large anticipated capital gains means that the impact effect of the income shock on the house price is much larger with borrowing constraint than without it. This seems to be true in the case of a temporary negative income shock: In the top-left graph, house prices fall by about 30% more from period 0 to period 1 with the borrowing constraint than with it (from 1 to 0.84 or to 0.88). However, in the case of a permanent shock, the impact effect is almost the same with and without the borrowing constraint. As we showed in subsection 2.2, a fall in the current house price may induce the borrowing constrained households to save more and therefore increase their future housing demand. This is what happens here. In the case of a permanent and negative income shock, the housing demand in period 2 is substantially higher with the borrowing constraint than without it. Hence, the anticipated capital gain that is needed to offset the liquidity effect is created by a relatively high house price in period 2, rather than by a very low house price in period 1.

We next study the effect of different interest shocks on the price dynamics in the same manner as above. We consider a permanent increase (from $R = 1.08$) to $R = 1.12$ and a permanent decrease to $R = 1.04$. The temporary shock lasts for two periods: In the case of an increase in the interest rate, we have $R_1 = R_2 = 1.16$. In the case of a decrease in the interest rate, we have $R_1 = R_2 = 1.0$. Figure 5 displays the house price dynamics following the four different interest rate shocks.

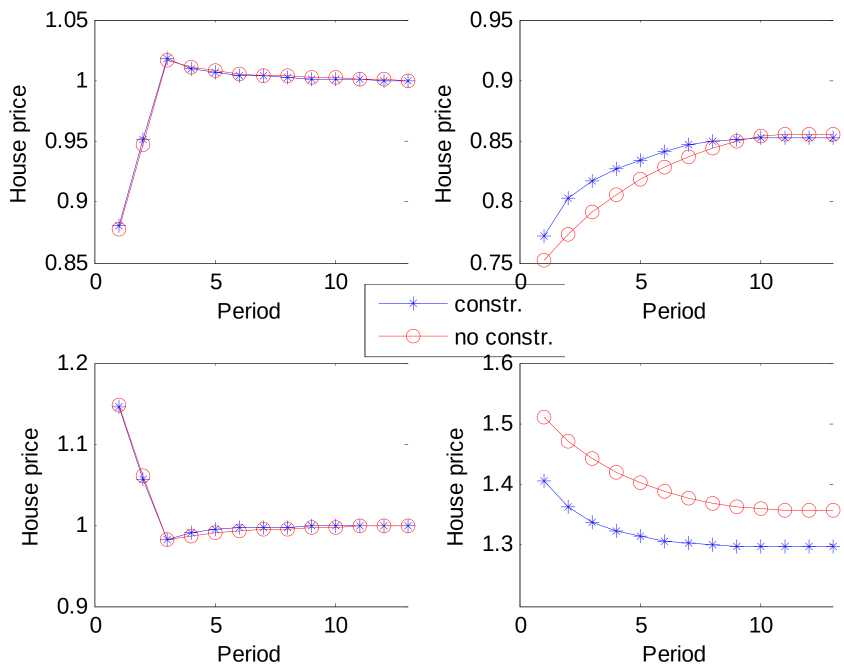

Figure 5. House price dynamics following different interest rate shocks.

Interestingly, with temporary interest rate shocks, the price dynamics are almost identical with and without the borrowing constraint. With permanent interest rate shocks, the borrowing constraint may influence the steady state price (although the difference is substantial only with a decrease in the interest rate). Apart from that, the borrowing constraint has little influence on the price dynamics even with permanent interest rate shocks.

The reason why the borrowing constraint does not affect house price dynamics much following interest rate shocks is that the interest rate shock mitigates the liquidity effect. An increase in the interest rate, for instance, induces a fall in the house price, which tightens the borrowing constraint through the liquidity effect. However, this effect is offset by households' increased willingness to save (due to the higher interest rate), which makes the borrowing constraint less important (or 'less binding') for the households. A similar argument holds for the decrease in the interest rate: a lower interest rate reduces the cost of housing, thereby increasing demand and pushing up the house price. This tends to relax the borrowing constraint for all households. At the same time, however, a lower interest rate makes borrowing more attractive. This makes the borrowing constraint more important.

4.2 Alternative preference parameters

We now illustrate how the price dynamics depend on the preference assumptions in the case of negative income shocks. We vary parameters γ and σ, which determine the intratemporal and intertemporal elasiticities of substitution. Recall that in the benchmark calibration we have $\gamma = -1$ and $\sigma = 2$. We vary one of these two parameters at a time and set γ at 0 or -2 and σ at 1 or 3. In each case, we recalibrate the model so as to match the same three targets as in the benchmark calibration. We recompute the dynamics following the temporary negative income shock that we considered above.

Figure 6 displays the results. Although the house price effect of the shock is very sensitive to the intratemporal elasticity of substitution between housing and consumption, the effect of the borrowing constraint is similar with all these parameter combinations: With the borrowing constraint, house prices initially fall more than without a borrowing constraint but converge after the impact more rapidly towards the steady state level.

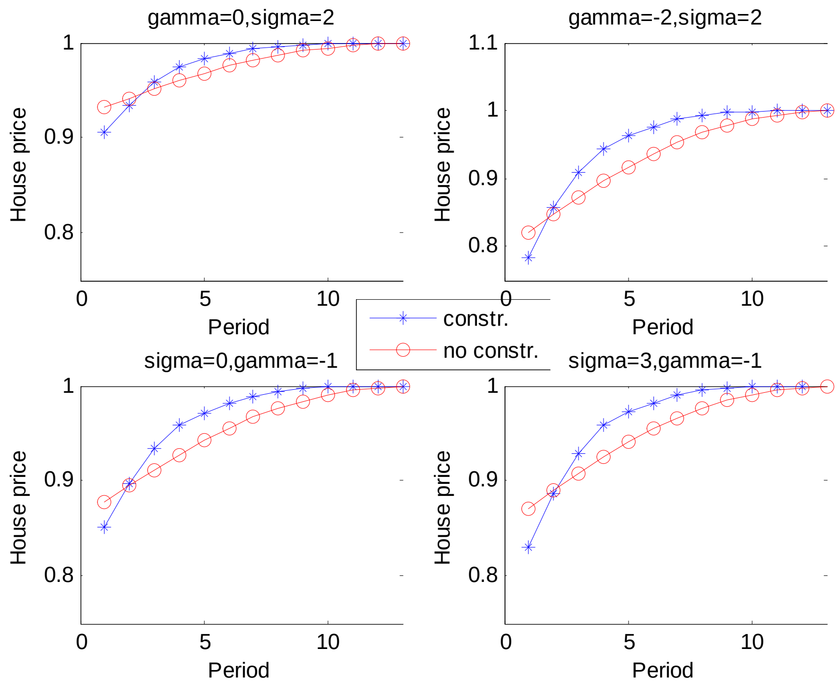

Figure 6. House price following a temporary and negative income shock with different elasticities.

However, closer inspection of this figure suggests that the borrowing constraint matters more for house price dynamics when we either decrease γ (meaning a lower intratemporal elasticity) or increase σ (meaning a lower intratemporal elasticity) in the sense that relative to the case without a borrowing constraint, both the impact effect and the subsequent capital gains are bigger. This is because with a lower elasticity of substitution, the demand response to any given price change is smaller. Therefore, with a lower elasticity, bigger price reduction is needed for the demand increase to be sufficient to clear the market.

4.3 The effects of a marginal house price change

The multiplier effect stressed in Stein (1995) relates to the fact that a fall in the house price may reduce buyer liquidity through the down payment requirement. This leads borrowing constrained households to demand less housing when the price falls. If this effect dominates in the aggregate demand response, there could even be multiple equilibria.

We investigate this issue in our setting in the following way. First, we compute the equilibrium house price sequence following a permanent income shock in period 1. This is the house price dynamics shown in the top-right part of figure 4. We then decrease period 1 house price by 1% leaving other prices unchanged, and solve again the problem of all households. This is a partial equilibrium exercise in the sense that with this new house price sequence, the demand for housing will no longer equal supply in every period. Finally, we compute the change (from the level related to the equilibrium price dynamics) in housing demand for different household types and cohorts for periods 1 and 2. This gives us a measure of the elasticity of housing demand for different household types and ages around the equilibrium path. For there to be potential for multiple equilibria, for some households at least, the additional price reduction should reduce housing demand in period 1.

Figure 7 shows the results for all type 1 households of different ages in periods 1 and 2. We know that following the negative income shock, households of age 2-5 are borrowing constrained in period 1.[7] To understand the figure, consider a household that is of age 2 in period 1 and, hence, of age 3 in period 2. The figure tells us that the (additional) 1% reduction in period 1 house price increases housing demand of this household in both periods 1 and 2 by about 0.3%. For the household of age 1 in period 2, there is no demand change, as this household did not experience the 'disturbance' in period 1.

[7]Type 2 households get children at age 2 and the children live with the parents for five model periods.

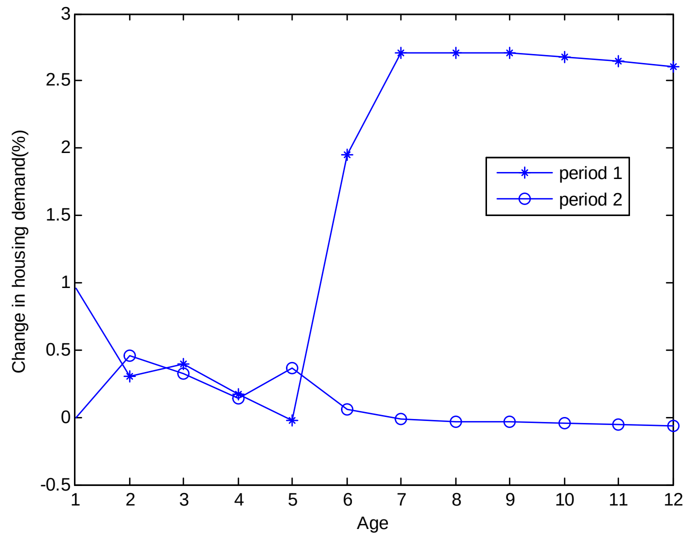

Figure 7. Changes in period 1 and period 2 housing demands following a 1% decrease in period 1 house price.

The figure shows that in period 1 the housing demand of young, borrowing constrained households increases much less than the demand of older, non-constrained households. Hence, the borrowing constraint does substantially reduce the price elasticity of housing demand. In fact, for a household of age 5, the demand actually slightly decreases, that is, for this age group the liquidity effect does indeed dominate the other effects. However, this is not the case for any of the other age groups. In addition, in general, household types 2 and 3 are less likely to be borrowing constrained than households of type 1. All this indicates that there is no scope for multiple equilibria.

Consider then the demand of different cohorts in period 2. The figure shows, consistently with our analytical results in 2.2, that a fall in the current house price induces the borrowing constrained households to demand more housing in the following period. By effectively tightening the borrowing constraint, a fall in the current house prices forces households to save more. This induces them to demand more housing in the future.

4.4 Mimicking the Finnish boom-bust-boom cycle

Our aim in this section is to try to determine to what extent the model can explain the Finnish house price dynamics shown in figure 8. The figure displays real house prices from 1980 to 2006. Real house prices first increased by about 50% from 1986 to 1989 and then fell by about 50% from 1989 to 1993.

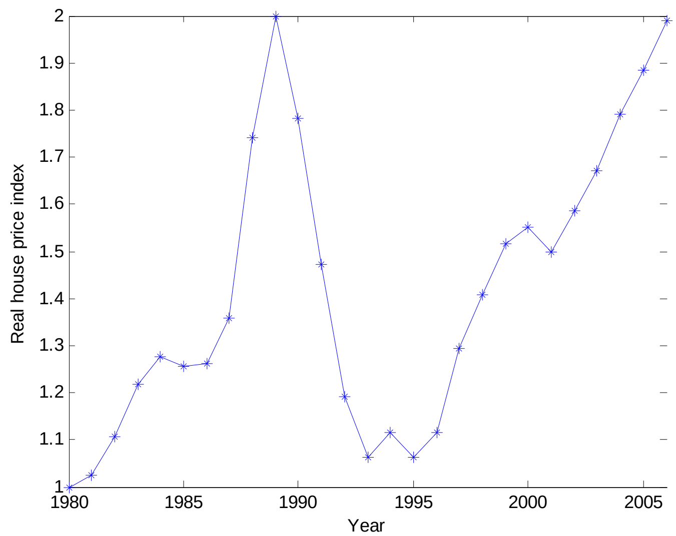

Figure 8: Real house prices in Finland.[8]

Before going to the details of our analysis, we will briefly discuss the economic situation in Finland during the period in question. A comprehensive description and a thorough discussion of the recent Finnish experience can be found in Honkapohja et al. (2009). First of all, the housing market boom of the late 1980s was associated with a credit market liberalization. Before that, the Finnish banking system was highly regulated with tightly controlled and low lending rates which resulted in credit rationing. During the process of financial deregulation, the regulation of lending rates was abolished in 1986. This induced a huge growth of credit (see Koskela et al., 1992 and Laakso, 2000).

Second, GDP growth was rapid in the late 1980s but in 1990 Finland entered a period of deep depression. Figure 9 shows the growth of real GDP per capita over

[8]The source for the house price index series is Bank of Finland.

the period of 1980-2006 and the real after-tax mortgage interest rates. Real GDP decreased by over 10% from 1990 to 1993. After tax real interest rates also varied a lot. High interest rates during the depression aggravated the house price fall. Because of regulation lending rates the real interest rate was very low (negative) during 1980s but soared in the early 1990s. This was partly because of the defence of Finnish currency, but real interest rates were also affected by a tax reform. Until 1992, interest payments were deductible in general income taxation where the average marginal tax rates were close to 50%. In 1993, Finland moved to a dual income tax system, where labor and capital income are taxed separately. In the new system, a tax payer could deduct 25% (the new capital income tax rate) of the mortgage interest payments from taxes. For most households, this reform increased real after tax mortgage interest rates.

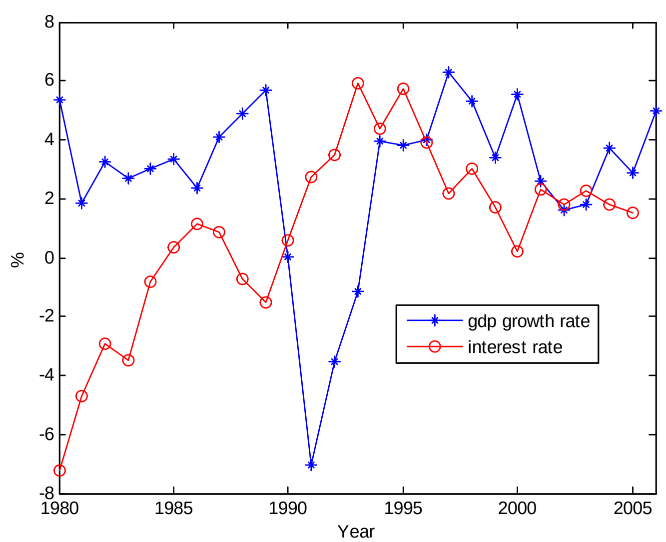

Figure 9: The growth rate of real GDP per capita and real after tax mortgage interest rate.[9]

We try to see to what extent the model can explain the large house price movements in figure 8 as a response to the credit market liberalization and the income and interest rate shocks discussed above. Specifically, we consider a sequence of shocks such that

[9]The source for the GDP series is Statistics Finland and the source for the interest rate series is Oikarinen (2007).

in period 1 the borrowing constraint is relaxed and in period 2 households are hit by income and interest rate shocks. We interpret period 1 as years 1986-1989 and period 2 as years 1990-1993.

We first solve for the steady state with a very tight borrowing constraint. In Finland, before the credit market liberalization, there was effectively credit rationing, and households were constrained by very short mortgage maturities. Based on discussions with market experts, our understanding is that for most households, it was impossible to get a mortgage with a maturity above 8 years. In addition, households typically needed to pay a down payment of around 30% of the house value. (See also Koskela et al. 1992). Although we don't have a maturity constraint formally in the model, we can partly capture it by increasing the borrowing constraint parameter θ. For a typical mortgage contract, a mortgage maturity of 8 years together with a down payment constraint of 30% means that a household needs to pay about 80% of the value of its new house investment during the first four years. This translates into $\theta = 0.80$.

Figure 10 shows the price dynamics following a sudden relaxation of the borrowing constraint from $\theta = 0.80$ to $\theta = 0.25$. On impact, house prices increase by about 25%. The reason why the house price increases on impact is simple: initially, the only thing that changes is that young, borrowing constrained households can buy more housing. Hence, the house price must go up. After the impact effect, the house price starts to decrease but remains higher than in the initial steady state. The reason to this gradual reduction in house price is the following: After the credit market liberalization, the young households borrow more than previously. Hence, they will be less wealthy at old age than the previous generations. Therefore, future generations demand less housing when old than the current old. Therefore, house prices must decline after the impact effect.

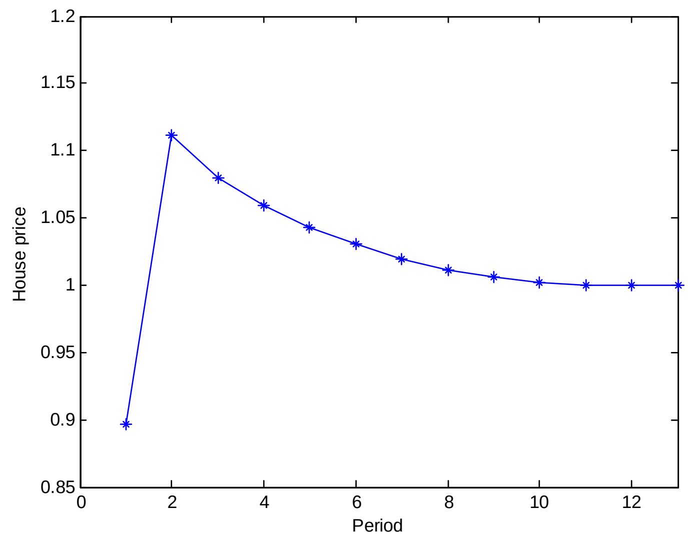

Figure 10: House price dynamics following a relaxation of the borrowing constrained.

As for the income shock, we feed into the model the difference between the actual path of disposable household income and its trend growth path (which is based on average income growth) and assume again that the income shocks are the same for households of all age and type. Disposable household income decreased by 5% from 1990 to 1993 whereas the average growth would have meant an increase of about 10%. Since that, household income has grown quite fast and by 2007 household income had almost converged back to its pre-depression trend growth path. We multiply y_t^j by 0.85,0.90, 0.95 in periods 2, 3, and 4. Households learn this new income path in the beginning of period 2.

Roughly based on figure 9, we assume that the yearly real interest rate increases by 2 percentage points in periods 2 and 3. That is, we set $R_1 = 1.08$, $R_2 = R_3 = 1.16$, and $R_t = 1.08$ for $t > 3$. Households learn also about the temporary interest rate shock in the beginning of period 2.

Figure 11 displays the house price dynamics following a sequence of shocks, where first the borrowing constraint is relaxed and then, in the following period, the economy is hit by the depression which consists both of an income and an interest rate shock. House prices first increase by about 25% and then fall by about 25%. This is about half of the variation we observe in figure 8.

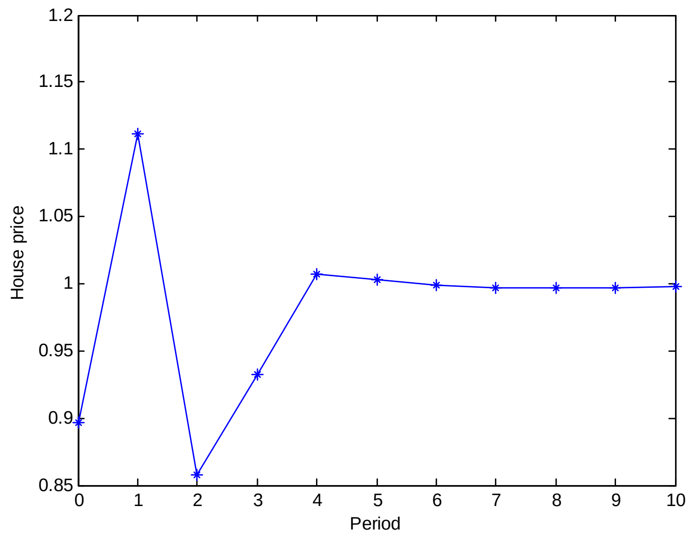

Figure 11: House price dynamics following a relaxation of the borrowing constrained and income and interest rate shocks.

As the figure shows, house prices in the model first increase by about 25% and then fall by about 25%. This is about half of the variation we observe in figure 8. This figure is obtained using our benchmark calibration, with the elasticity of intertemporal substitution equal to one. As figure 6 indicates, the price dynamics are likely to be more sensitive to the borrowing constraint with lower values of the elasticity of intertemporal substitution.

5 Conclusions

We have analyzed how a down payment constraint affects house price dynamics in an OLG model with standard preferences. The effect of the constraint is very different with different shocks. In certain situations, the down payment constraint can shape house price dynamics substantially. In particular, because of the constraint, the house price dynamics following large positive and negative income shocks can be quite asymmetric.

However, it seems very unlikely that a down payment constraint could create multiple equilibria in house prices.

Appendix: Housing demand and the effect of price changes

No borrowing constraint

Assuming that the borrowing constraint is not binding, the first-order conditions related to the household problem (16)-(18) are

$$\begin{aligned} u_c - \lambda &= 0 \\ u_h - \lambda \left(p + \kappa p + \eta - \frac{p'}{R} \right) &= 0 \\ \beta V_b - \lambda \frac{1}{R} &= 0 \end{aligned}$$

Combining the first-order conditions and using the budget constraint gives a system of three equations and three unknowns:

$$\begin{aligned} u_c - \beta R V_b &= 0 \\ u_h - P u_c &= 0 \\ y + p h^{-1} + R a^{-1} - c - P h - \frac{b}{R} &= 0 \end{aligned}$$

where $P = p + \kappa p + \eta - \frac{p'}{R}$. Differentiating this system with respect to p gives

$$\begin{pmatrix} 0 & u_{cc} & -\beta R V_{bb} \\ u_{hh} & -P u_{cc} & 0 \\ -P & -1 & -\frac{1}{R} \end{pmatrix} \begin{pmatrix} \frac{\partial h}{\partial p} \\ \frac{\partial c}{\partial p} \\ \frac{\partial b}{\partial p} \end{pmatrix} = - \begin{pmatrix} 0 \\ -(1+\kappa) u_c \\ h^{-1} - (1+\kappa) h \end{pmatrix}$$

Note first that

$$D = \begin{vmatrix} 0 & u_{cc} & -\beta R V_{bb} \\ u_{hh} & -P u_{cc} & 0 \\ -P & -1 & -\frac{1}{R} \end{vmatrix} = \beta R V_{bb} \left[u_{hh} + P^2 u_{cc} \right] + \frac{1}{R} u_{hh} u_{cc} > 0$$

Then we have that

$$\frac{\partial h}{\partial p} = \frac{\begin{vmatrix} 0 & u_{cc} & -\beta R V_{bb} \\ (1+\kappa)\, u_c & -P u_{cc} & 0 \\ -\left(h^{-1}-(1+\kappa)\, h\right) & -1 & -\frac{1}{R} \end{vmatrix}}{D}$$

$$= \frac{1}{D}\left[\beta R V_{bb}\left((1+\kappa)\, u_c + P u_{cc}\left(h^{-1}-(1+\kappa)\, h\right)\right) + \frac{(1+\kappa)\, u_c u_{cc}}{R}\right]$$

And

$$\frac{\partial b}{\partial p} = \frac{\begin{vmatrix} 0 & u_{cc} & 0 \\ u_{hh} & -P u_{cc} & (1+\kappa)\, u_c \\ -P & -1 & -\left(h^{-1}-(1+\kappa)\, h\right) \end{vmatrix}}{D}$$

$$= \frac{1}{D}\left[-u_{cc} P (1+\kappa)\, u_c + u_{cc} u_{hh}\left(h^{-1}-(1+\kappa)\, h\right)\right]$$

Borrowing constraint

Assume now that the household faces a binding borrowing constraint. Then (16)-(18) can be written as

$$\max_{c,h}\left\{u\left(c,h\right)+\beta V\left(b\right)\right\}$$

subject to

$$c + \left(p + \kappa p + \eta - (1-\theta)\, p\right) h = y + p h^{-1} + R a^{-1}$$

where

$$b = \left(p' - R\,(1-\theta)\, p\right) h.$$

The first-order conditions then become

$$u_c - \lambda = 0$$

$$u_h + \beta S V_b - \lambda T = 0$$

where $T = p + \kappa p + \eta - (1-\theta)\,p$ and $S = p' - R\,(1-\theta)\,p$. Combining the two first-order conditions and using the budget constraint gives two equations with two unknowns:

$$\begin{aligned} u_h + \beta S V_b - u_c T &= 0 \\ y + ph^{-1} + Ra^{-1} - c - Th &= 0 \end{aligned}$$

Totally differentiating the first-order conditions give the following:

$$\begin{pmatrix} u_{hh} + \beta V_{bb} S^2 & -u_{cc} T \\ -T & -1 \end{pmatrix} \begin{pmatrix} \frac{\partial h}{\partial p} \\ \frac{\partial c}{\partial p} \end{pmatrix}$$
$$= \begin{pmatrix} R\,(1-\theta)\,\beta V_b + \beta S V_{bb} R\,(1-\theta)\,h + u_c\,(1+\kappa-(1-\theta)) \\ -h^{-1} + (1+\kappa-(1-\theta))\,h \end{pmatrix}$$

Note first that

$$D^c = \begin{vmatrix} u_{hh} + \beta V_{bb} S^2 & -u_{cc} T \\ -T & -1 \end{vmatrix} = -u_{hh} - \beta V_{bb} S^2 - u_{cc} T^2 > 0$$

and therefore, we have

$$\frac{\partial h}{\partial p} = \frac{\begin{vmatrix} R\,(1-\theta)\,\beta V_b + S\beta V_{bb} R\,(1-\theta)\,h + u_c\,(1+\kappa-(1-\theta)) & -u_{cc} T \\ (1+\kappa-(1-\theta))\,h - h^{-1} & -1 \end{vmatrix}}{D^c}$$

Hence, we can write

$$\begin{aligned} \frac{\partial h}{\partial p} = &\ \frac{-u_c\,(1+\kappa) + u_{cc} T\,((1+\kappa)\,h - h^{-1})}{D^c} \\ & -\frac{u_{cc} T h}{D^c}\,(1-\theta) + \frac{u_c}{D^c}\,(1-\theta) - \frac{R\,(1-\theta)\,\beta V_b}{D^c} - \frac{S\beta V_{bb} R\,(1-\theta)\,h}{D^c} \end{aligned}$$

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